



This is a digital copy of a book that was preserved for generations on library shelves before it was carefully scanned by Google as part of a project to make the world's books discoverable online.

It has survived long enough for the copyright to expire and the book to enter the public domain. A public domain book is one that was never subject to copyright or whose legal copyright term has expired. Whether a book is in the public domain may vary country to country. Public domain books are our gateways to the past, representing a wealth of history, culture and knowledge that's often difficult to discover.

Marks, notations and other marginalia present in the original volume will appear in this file - a reminder of this book's long journey from the publisher to a library and finally to you.

Usage guidelines

Google is proud to partner with libraries to digitize public domain materials and make them widely accessible. Public domain books belong to the public and we are merely their custodians. Nevertheless, this work is expensive, so in order to keep providing this resource, we have taken steps to prevent abuse by commercial parties, including placing technical restrictions on automated querying.

We also ask that you:

- + *Make non-commercial use of the files* We designed Google Book Search for use by individuals, and we request that you use these files for personal, non-commercial purposes.
- + *Refrain from automated querying* Do not send automated queries of any sort to Google's system: If you are conducting research on machine translation, optical character recognition or other areas where access to a large amount of text is helpful, please contact us. We encourage the use of public domain materials for these purposes and may be able to help.
- + *Maintain attribution* The Google "watermark" you see on each file is essential for informing people about this project and helping them find additional materials through Google Book Search. Please do not remove it.
- + *Keep it legal* Whatever your use, remember that you are responsible for ensuring that what you are doing is legal. Do not assume that just because we believe a book is in the public domain for users in the United States, that the work is also in the public domain for users in other countries. Whether a book is still in copyright varies from country to country, and we can't offer guidance on whether any specific use of any specific book is allowed. Please do not assume that a book's appearance in Google Book Search means it can be used in any manner anywhere in the world. Copyright infringement liability can be quite severe.

About Google Book Search

Google's mission is to organize the world's information and to make it universally accessible and useful. Google Book Search helps readers discover the world's books while helping authors and publishers reach new audiences. You can search through the full text of this book on the web at <http://books.google.com/>

166(2)

THE JEROME B. HOWARD •
SHORTHAND COLLECTION
PURCHASED FOR
THE NEW YORK PUBLIC LIBRARY
BY THE
NATIONAL SHORTHAND REPORTERS'
ASSOCIATION AND THE LIBRARY
• 1924 •





*Young Americans in the various countries of the world
Humanity is within the reach of every child.*

THE
ACCOMPLISHED TUTOR;

OR,
Complete System



OF
LIBERAL EDUCATION:

CONTAINING THE
MOST IMPROVED THEORY AND PRACTICE
OF THE
FOLLOWING SUBJECTS:

- | | | |
|---|--------------------------------------|--|
| 1. English Grammar,
and Elocution. | 5. Mensuration, and
Architecture. | 12. Astronomy. |
| 2. Penmanship, and
Short Hand. | 6. Optics. | 13. Mechanics. |
| 3. Arithmetic, Vulgar
and Decimal. | 7. Algebra. | 14. Electricity. |
| 4. Stock-holding, and
Merchants Accounts | 8. Doctrine of Annuities | 15. Pneumatics. |
| | 9. Trigonometry. | 16. Hydrostatics. |
| | 10. Logarithms. | 17. Hydraulics. |
| | 11. Geography. | 18. Drawing, Engraving,
and Painting. |

AND OTHER USEFUL MATTER.

Embellished with Twenty Copper-plates and Six Maps, neatly engraved.

By **THOMAS HODSON.**

THIRD EDITION.

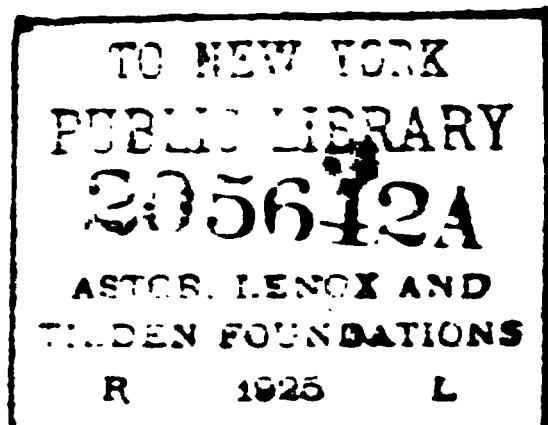
IN TWO VOLUMES.

VOL. I.

LONDON:

PRINTED FOR H. D. SYMONDS, PATERNOSTER ROW; AND
VERNOR, HOOD, AND SHARPE, POULTRY.

1806.



S. Gosnell, Printer, Little Queen Street, Holborn.

P R E F A C E.

THE general utility of a work of this nature supercedes the necessity of a Preface. But it may be thought necessary to say a few words concerning the manner in which the subjects are arranged.

In the rules in Vulgar Arithmetic, I have somewhat departed from the order generally followed, in order to render that science more intelligible, and easy of attainment, to those who choose to go regularly through it. Almost every writer on Arithmetic follows a different order in the disposition of his work; though every one must be sensible of the absurdity of teaching those rules which depend chiefly upon fractions, as Practice, &c. before the learner is acquainted with the doctrine of Vulgar Fractions. It may be observed, that any rule in Arithmetic may with equal propriety be attempted, when the learner is sufficiently skilled in the four single rules. The rules of Interest are also placed under the chapter of Stock-holding; and Tare and Tret in that of Merchants Accompts, being more intimately connected therewith.

In the chapter of Optics, I have endeavoured to render the practical part of that useful science as intelligible as possible, avoiding all algebraical and mathematical calculations (though necessary in complete

scientific treatises upon this subject), in order to render it more perspicuous.

In each of the other subjects, I have retained those parts which appeared to me the most essential, and rejected the superfluous : availing myself of the labours of others on each of the subjects; though I have not quoted every author; as that would be attended with more difficulty than might be at first expected, and partake of a preciseness unnecessary in the present work.

It may be proper to mention, that I have endeavoured to insert all the modern improvements, in each of the subjects, that appeared of any considerable utility. The errors unavoidable in works of this nature, abounding with calculations, I have, as far as they occurred to me, corrected in this edition. The candour with which the former edition of this work was received, is at least some argument in favour of its utility, as also consolation to its author; and tends to alleviate an afflicting dispensation of Divine Providence: and render tolerable an uninterrupted confinement to his room, which has now continued upwards of eighteen years.

Middle Temple,
1802.

CONTENTS
OF THE
FIRST VOLUME.

CHAP. I.
OF ENGLISH GRAMMAR.

	<i>Page</i>
SECT. 1. Of the Division and Sound of Letters . . .	1
2. Of Syllables . . .	4
3. Of the Nine Parts of Speech . . .	5
4. Of Syntax . . .	30
5. Of Punctuation . . .	42
6. Of Elocution . . .	55

CHAP. II.
OF PENMANSHIP.

SECT. 1. Rules for the Attainment of the Art of Writing	69
2. Of Secret Writing . . .	74
3. Of Epistolary Writing, and Superfcription of Letters . . .	77
4. The Art of Stenography, or Short Hand	93

CHAP.

CHAP. III.

OF VULGAR ARITHMETIC.

	<i>Page</i>
SECT. 1. Notation of Numbers . . .	122
2. Of Addition . . .	128
3. Of Subtraction . . .	140
4. Of Multiplication . . .	145
5. Of Division . . .	156
6. Of Reduction . . .	166
7. Of the Rule of Three Direct, single . . .	174
8. Of the Rule of Three Inverse, single . . .	179
9. Of the Rule of Three Direct, double . . .	183
10. Of the Rule of Three Inverse, double . . .	187
11. Of Fellowship, single and double . . .	189
12. Of Alligation, medial and alternate . . .	196
13. Of Vulgar Fractions, Reduction, Addition, Subtraction, Multiplication, Division, and the Rule of Three . . .	203
14. Of Practice . . .	218
15. Of Barter . . .	227
16. Of Loss and Gain . . .	229
17. Of Equation of Payments . . .	231
18. Of the Rule of False, single and double . . .	233
19. Of Exchange . . .	236
20. Of the Relation of Numbers . . .	246
21. Of the Extraction of Roots, Square and Cubic . . .	252

CHAP. IV.

OF DECIMAL ARITHMETIC.

SECT. 1. Of Reduction of Decimal Fractions . . .	259
2. Of Addition of ditto . . .	264

SECT.

CONTENTS.

vii

	Page
SECT. 3. Of Subtraction of Decimal Fractions . . .	265
4. Of Multiplication of ditto . . .	265
5. Of Division of ditto . . .	266

CHAP. V.

OF STOCK-HOLDING.

SECT. 1. The Nature of the Public Stocks . . .	268
2. Of Simple and Compound Interest . . .	270
3. Of Discount and Reversion . . .	280

CHAP. VI.

OF MERCHANTS ACCOUNTS.

SECT. 1. Of Book-keeping, the Italian Way, or by Double Entry . . .	284
2. Precedents of Receipts, Promissory Notes, Bills of Exchange, Bills of Debt, Bills of Lading, Bills of Parcels, &c. with the Laws concerning them . . .	305

CHAP. VII.

OF MENSURATION.

SECT. 1. Of Superficial Measure, with the Method of measuring Mason's, Bricklayer's, Carpenter's, Sawyer's, Plasterer's, Plumber's, Painter's, and Glazier's Work, and the Price of each. Also, the Price of each Commodity, and the Wages of Journey-men . . .	321
--	-----

SECT.

	<i>Page</i>
SECT. 2. Of measuring Land	353
3. Of Solid Measure, measuring Timber, Digging, and Gauging	374
4. The Description and Use of the Carpenter's Rule, Gunter's Line, and Coggeshall's Sliding Rule	397
5. A View of the Five Orders of Architecture	407

CHAP. VIII.

OF OPTICS.

SECT. 1. Of Vision	414
2. Of Single, Compound, and Solar Micro- scopes	428
3. Of Refracting, Reflecting, and Solar Tele- scopes	434
4. Dr. Herschel's Improved Telescope	456



No 1.

ROUND TENT.

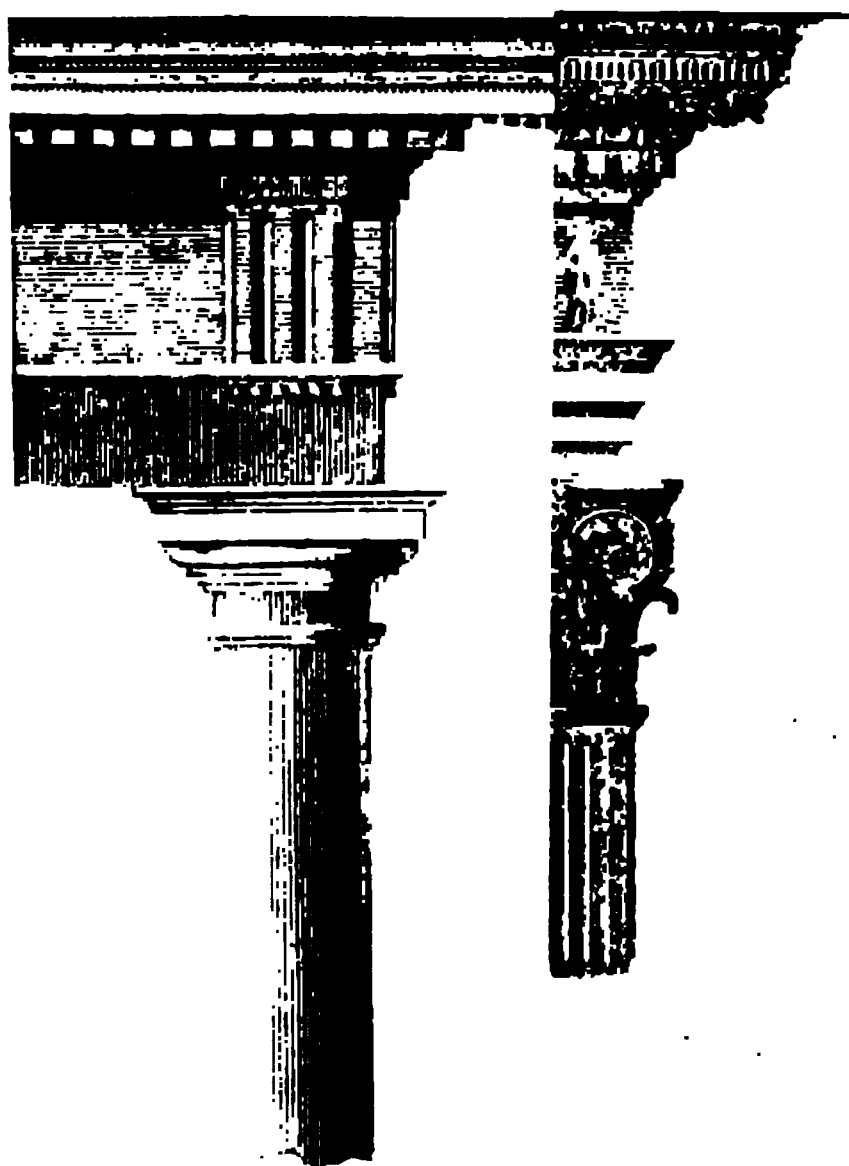
Weymouth

Weymouth

[illegible]



Tuscan



Doric



Ionic



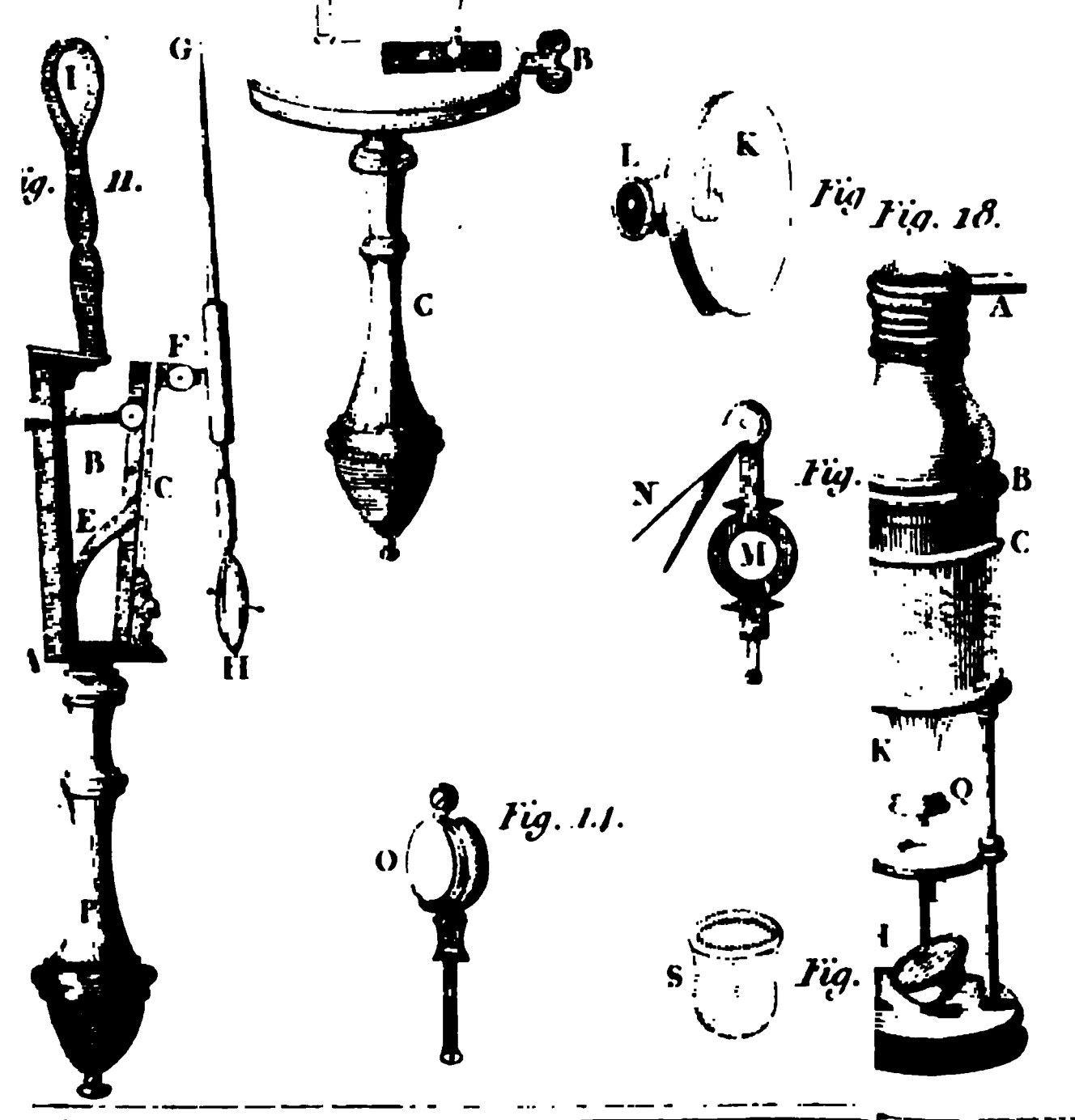
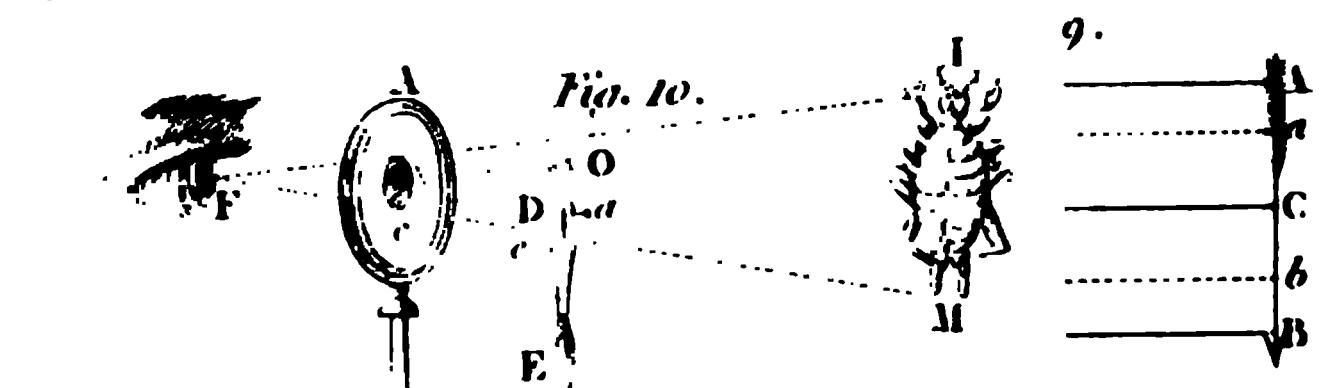
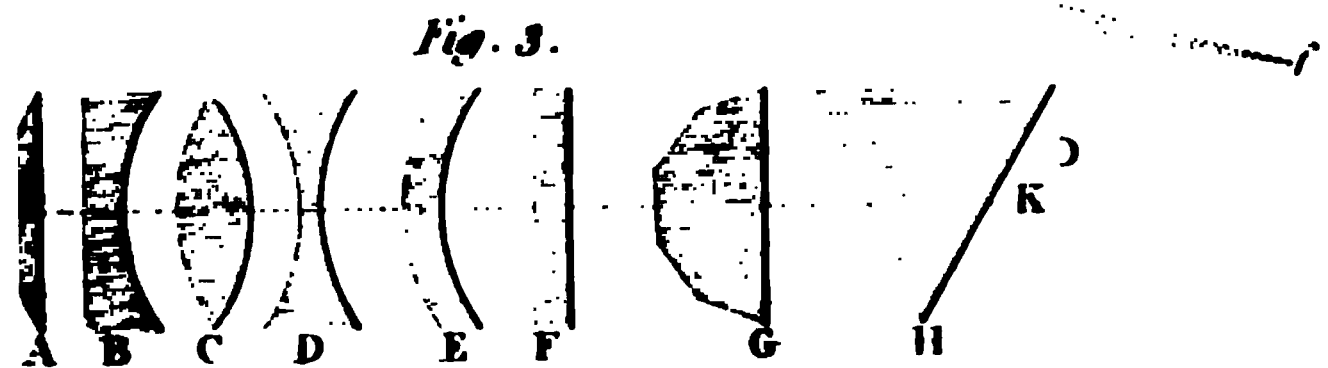
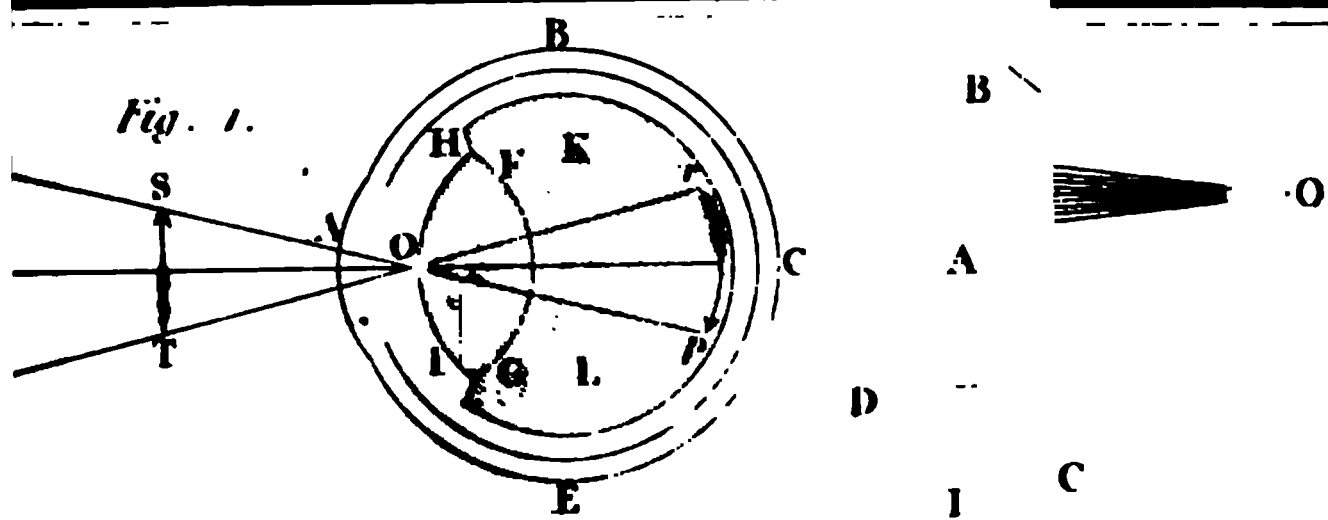


Fig. 1.

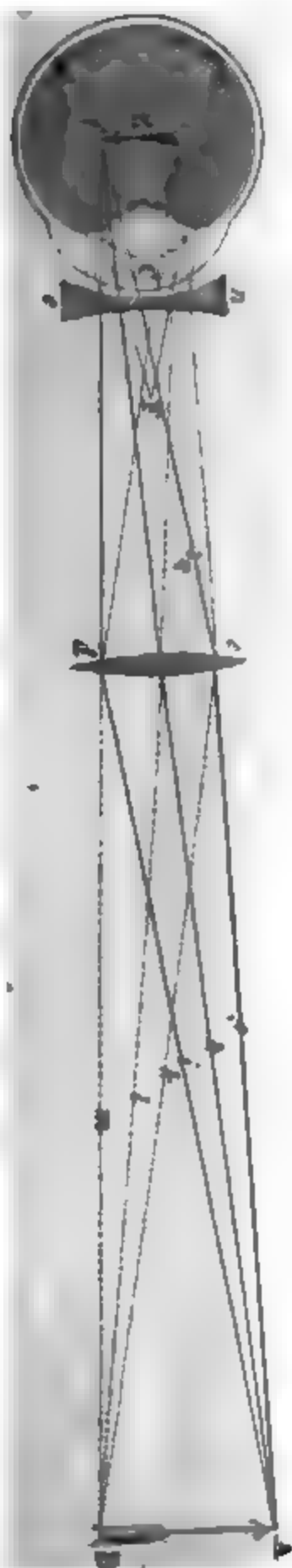


Fig. 5.



D. HERB



Fig. e.

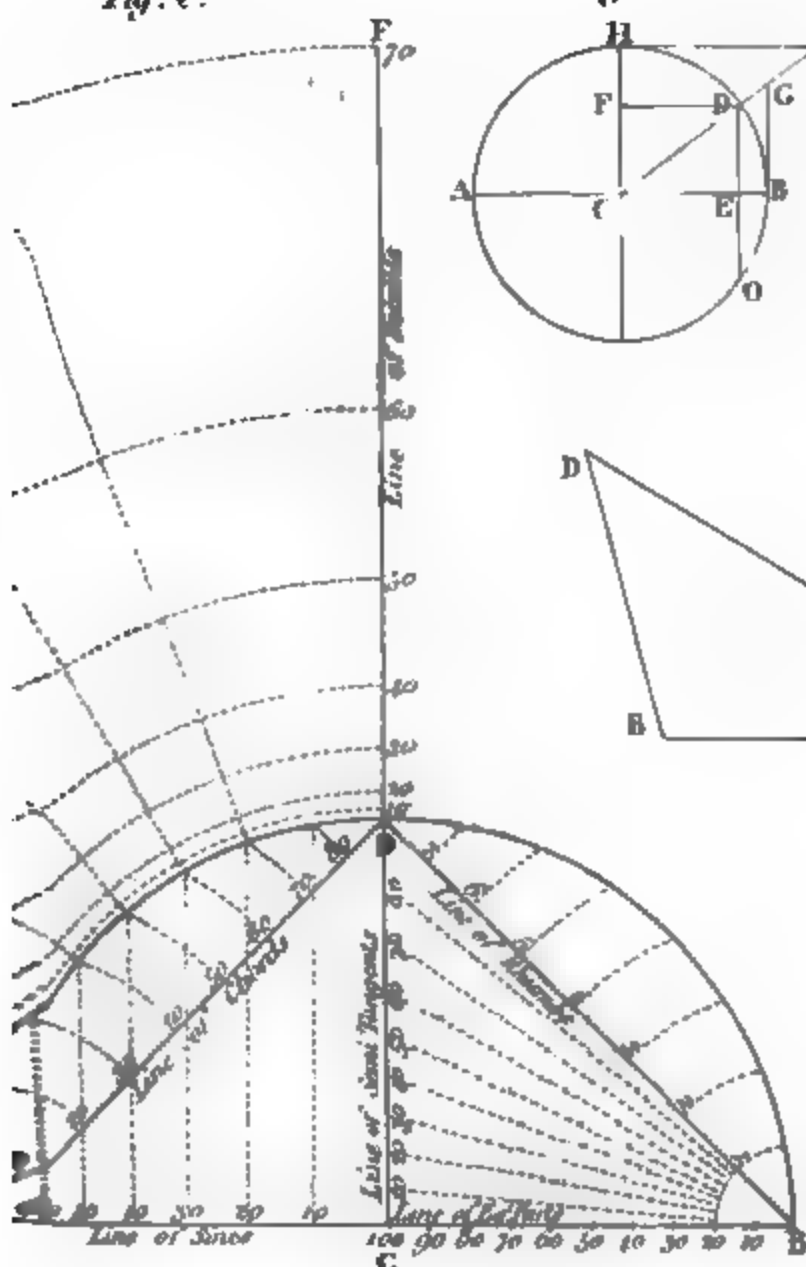
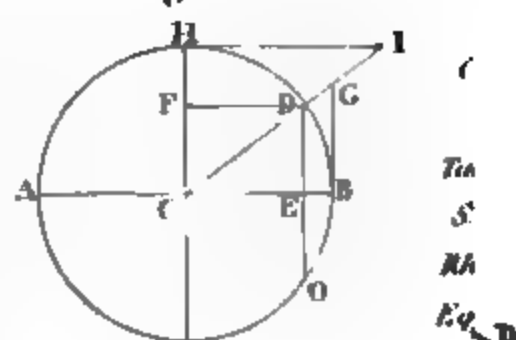


Fig. 1.



Tan
S
H
Eq
D



Fig.

Fig. 11.



Fig. 13.

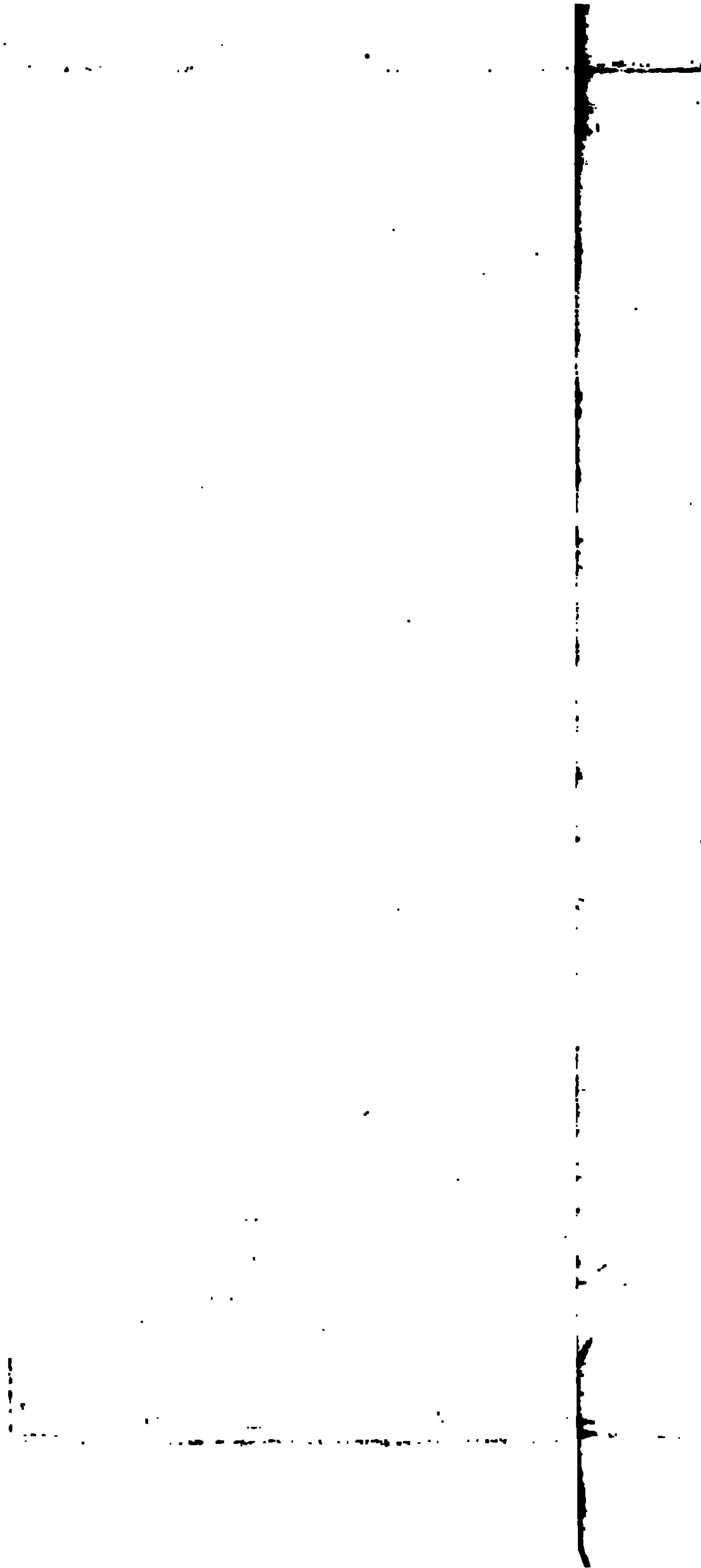


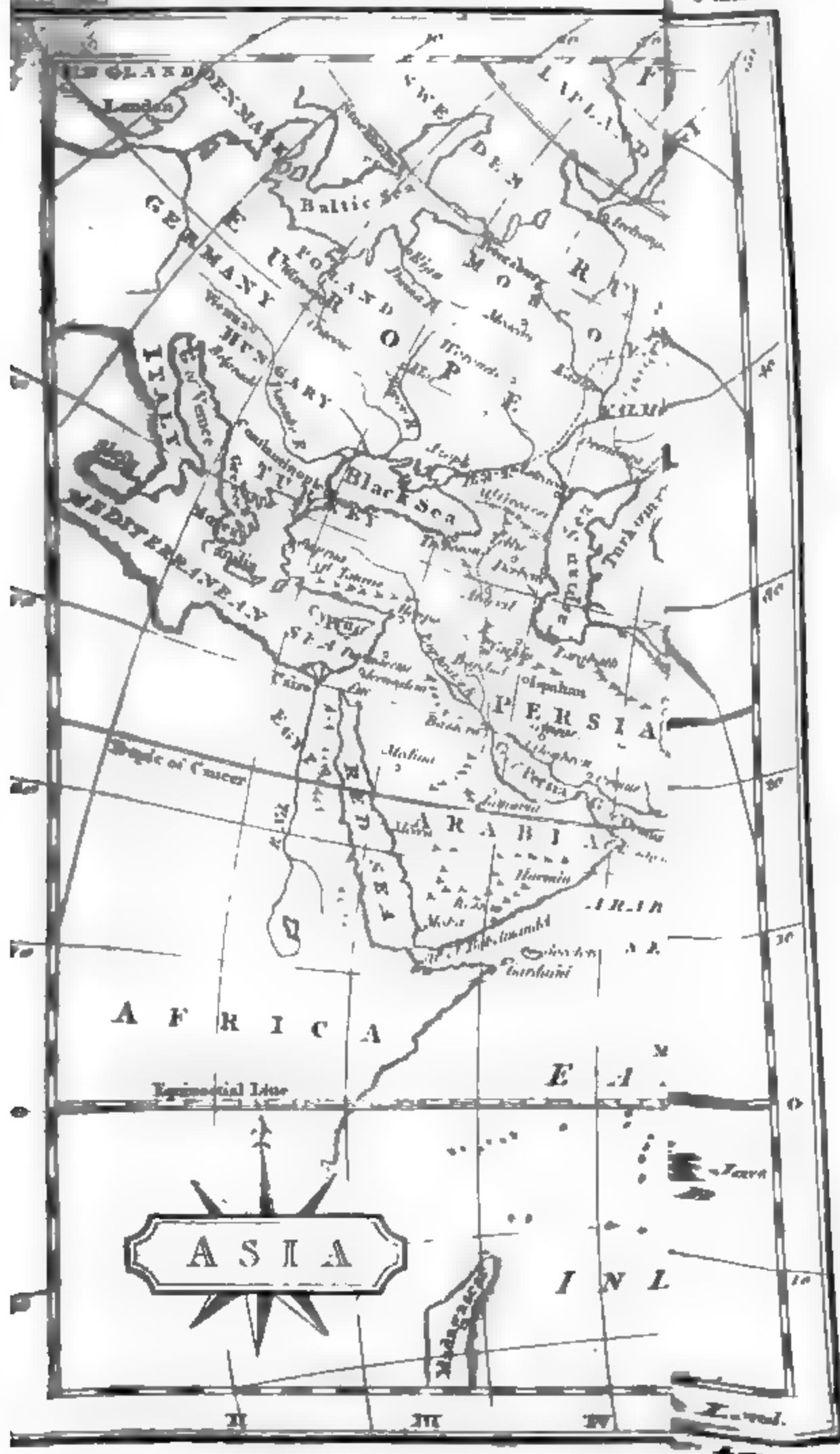


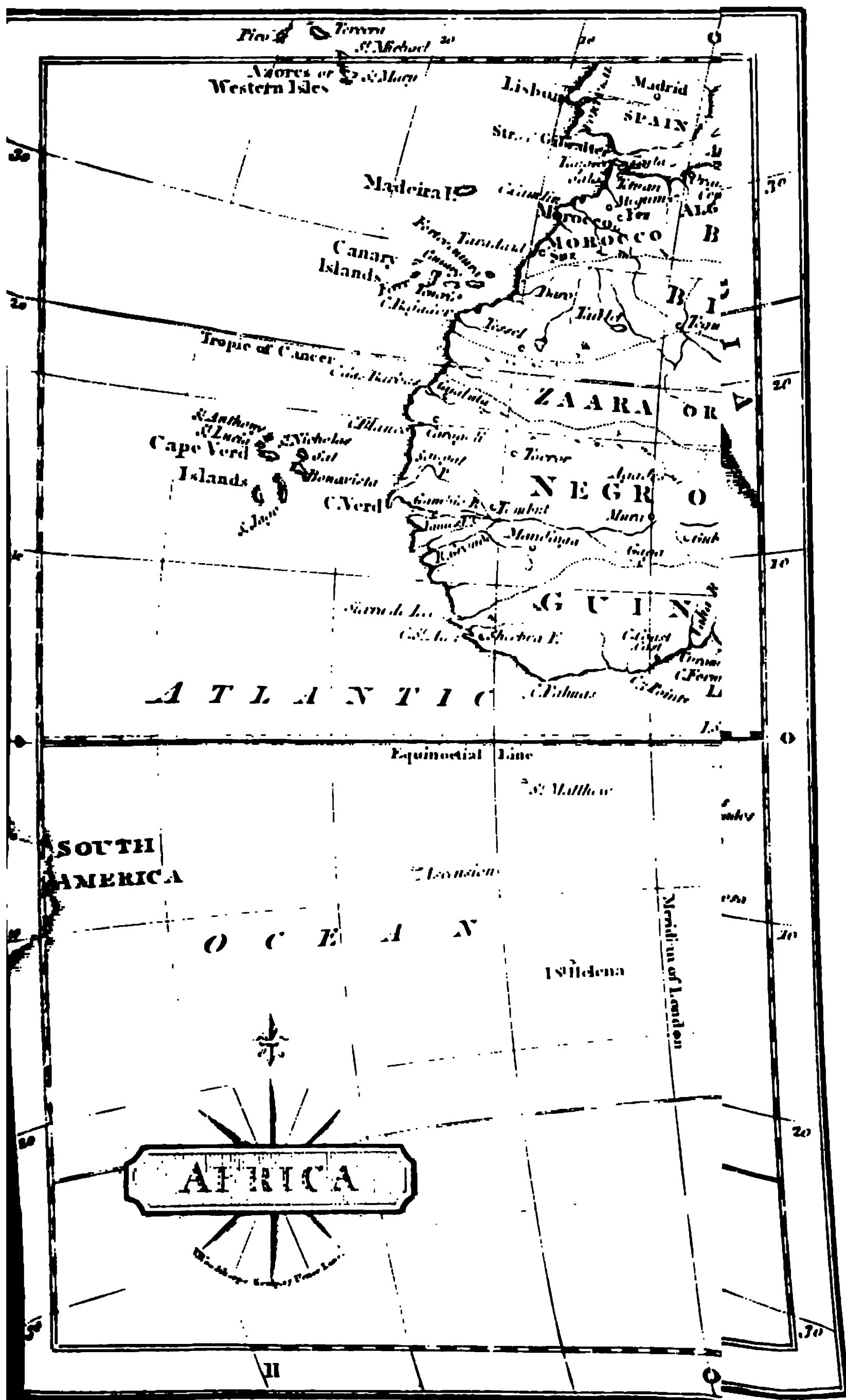


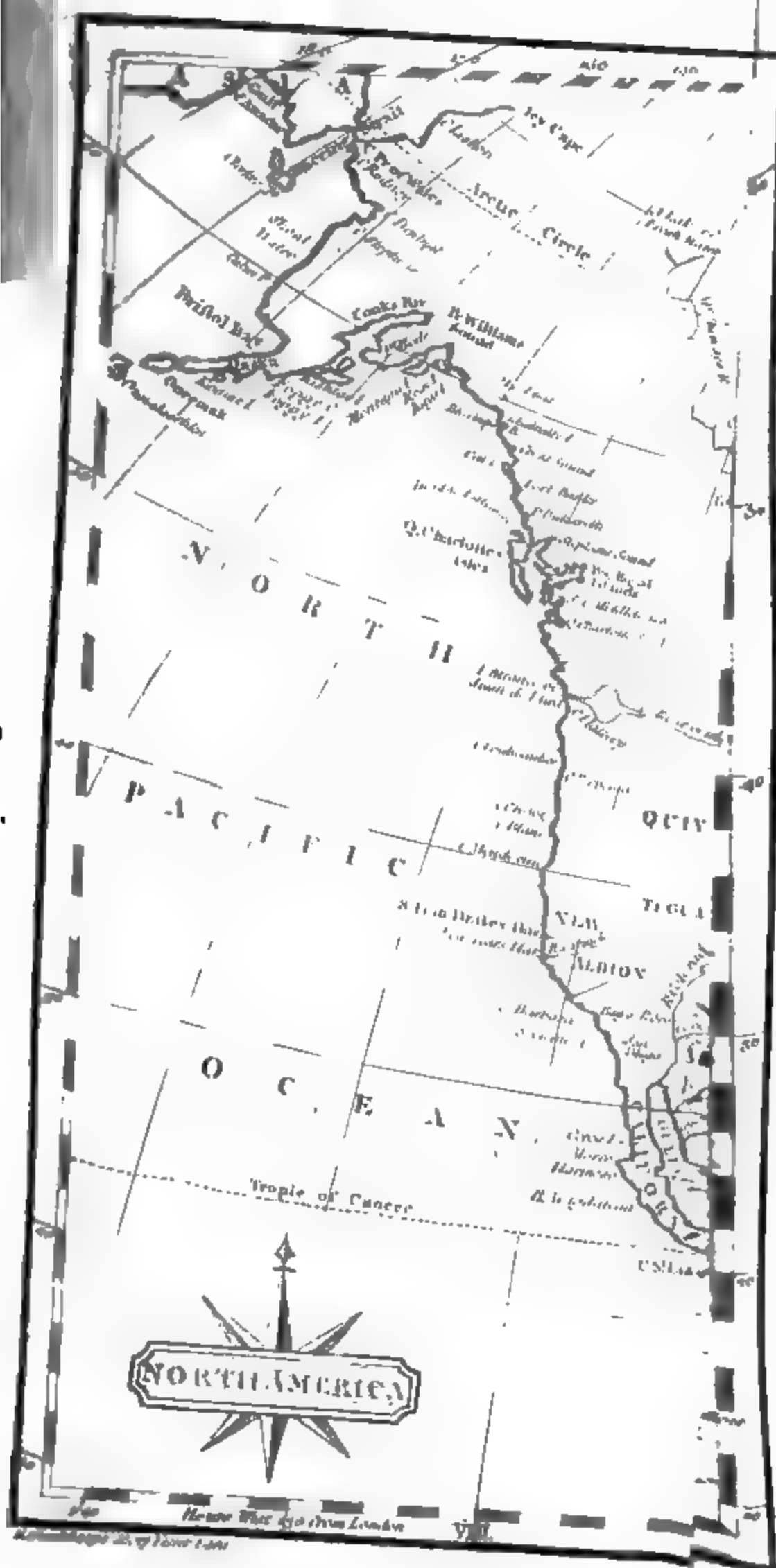


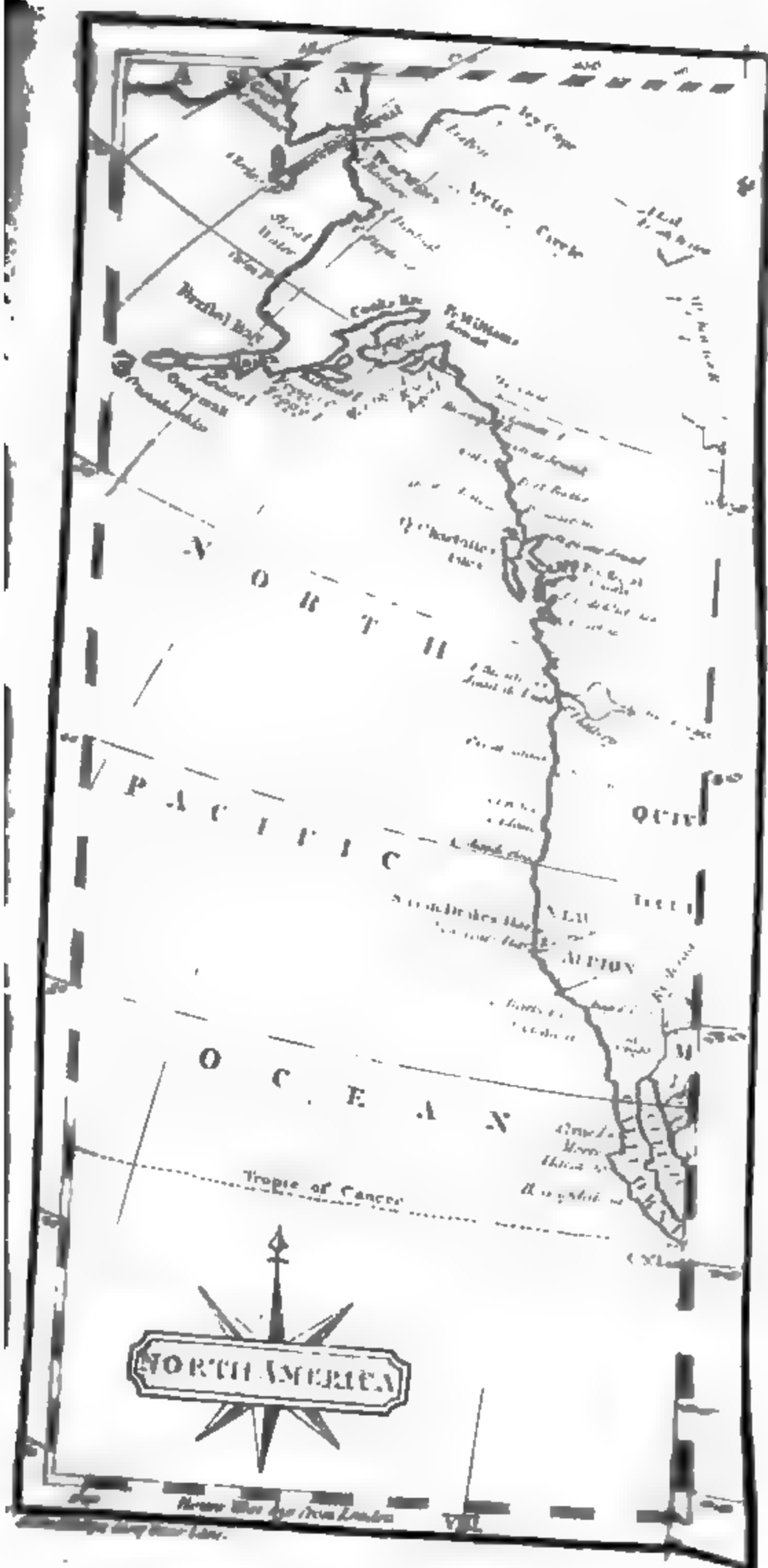


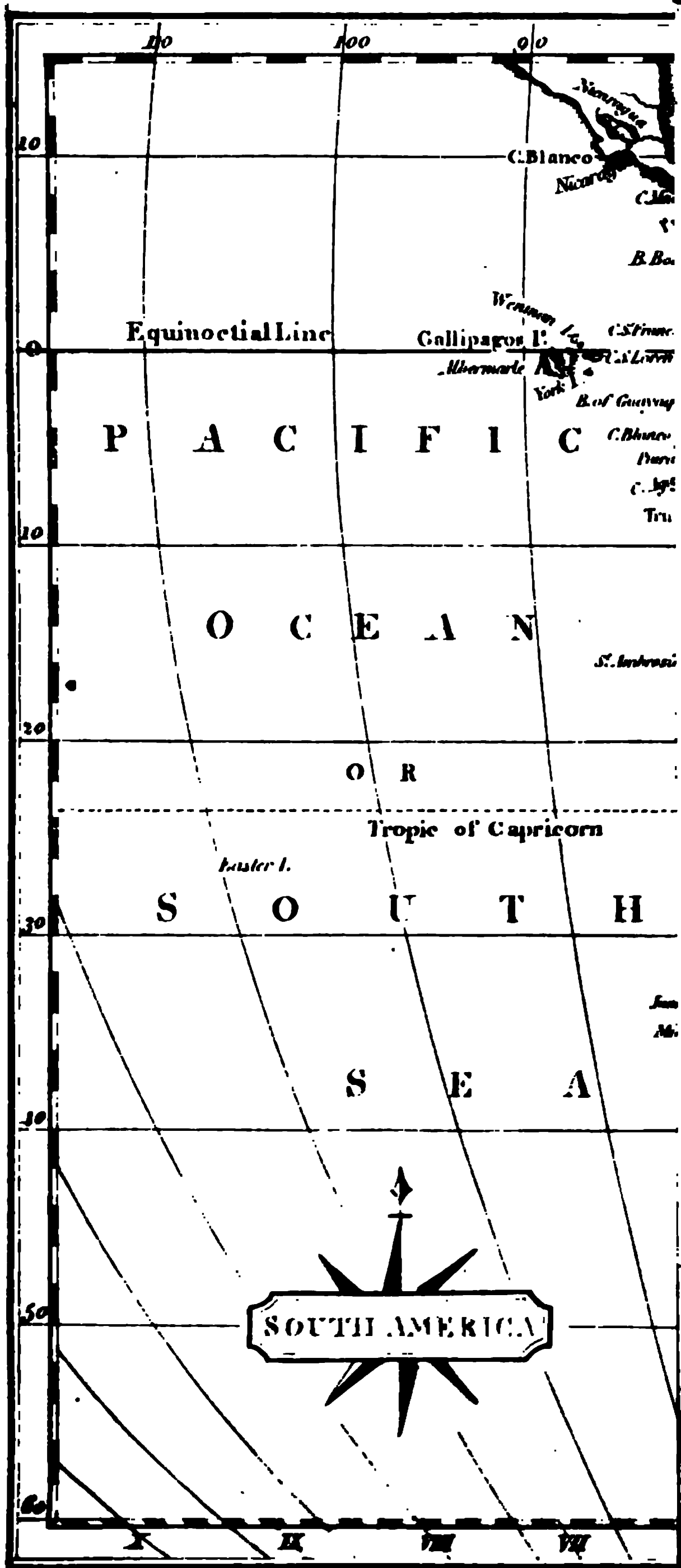


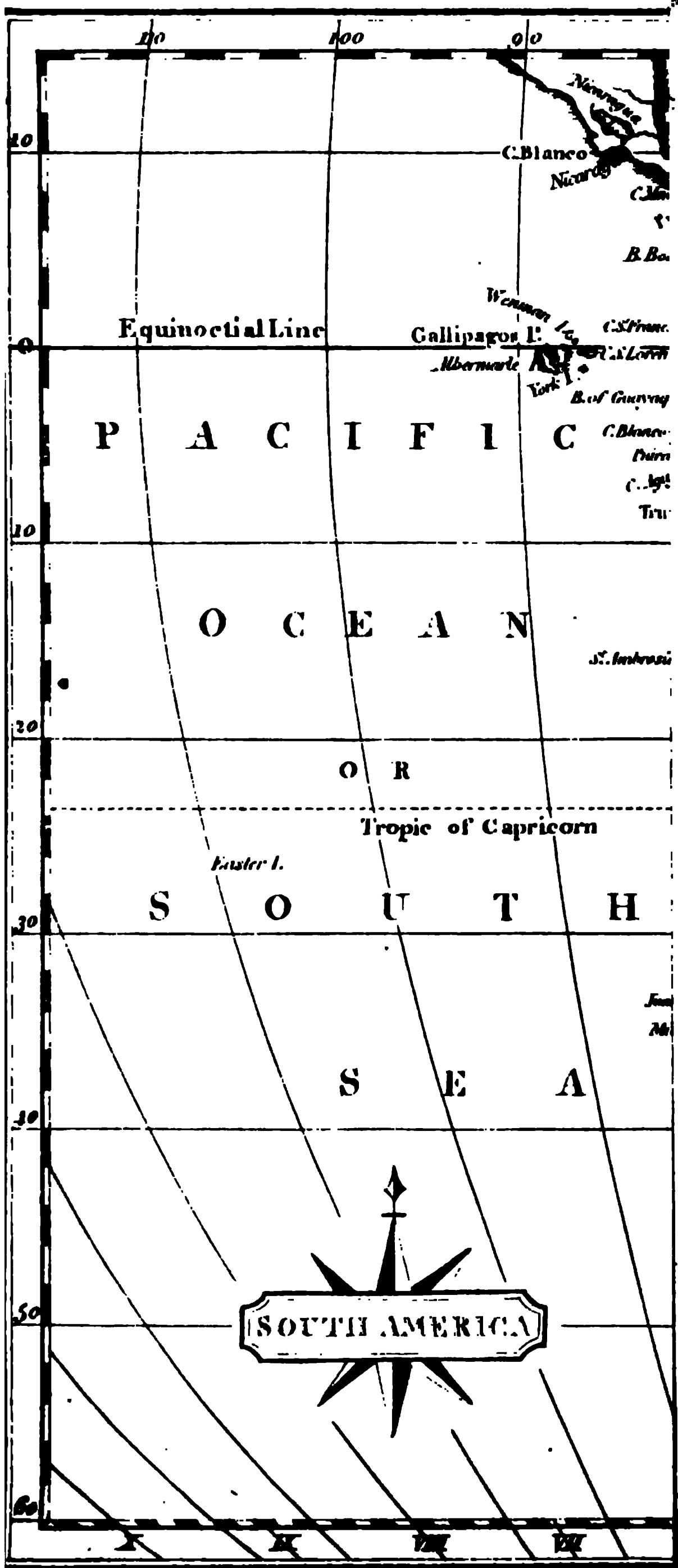


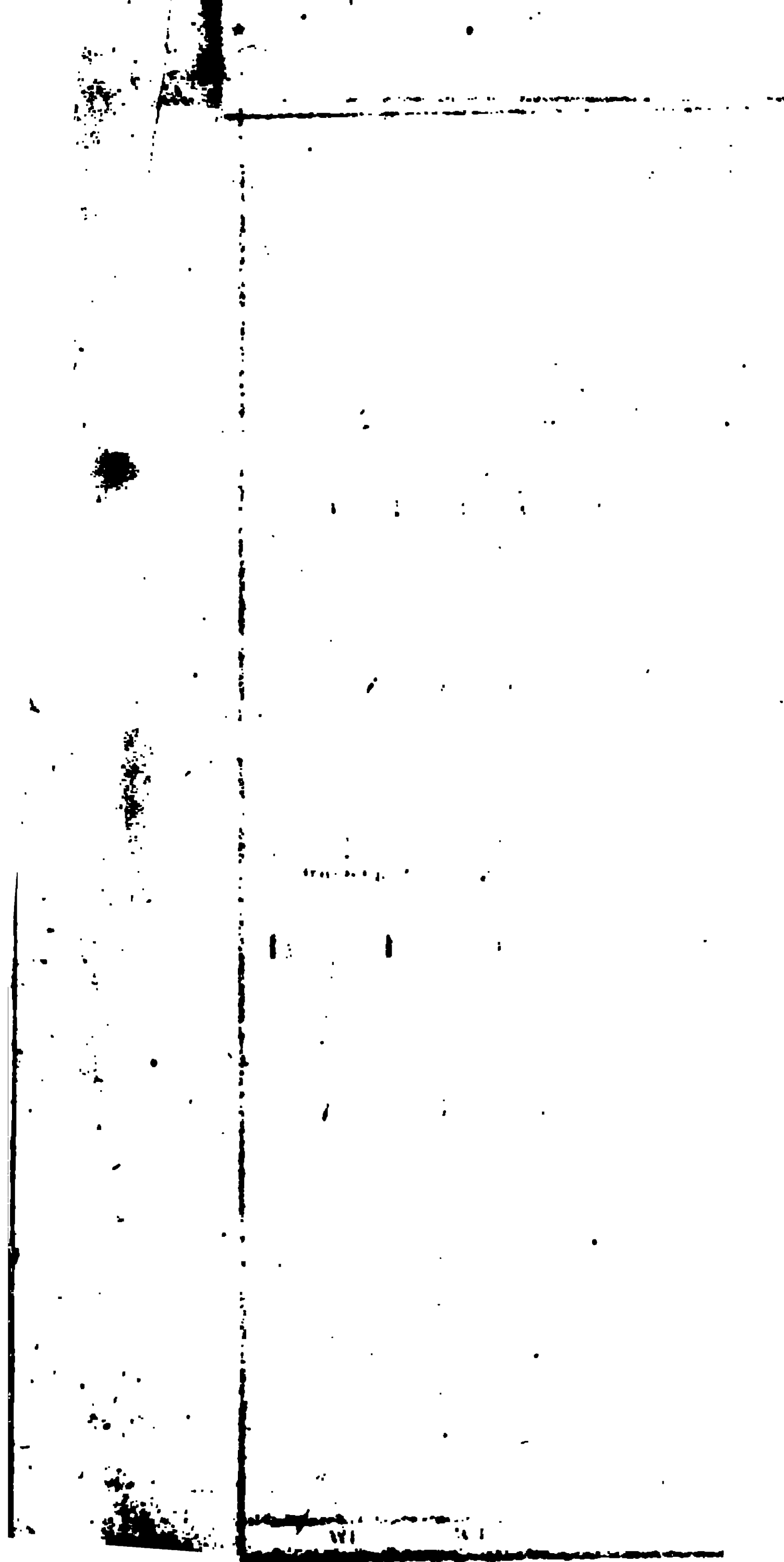












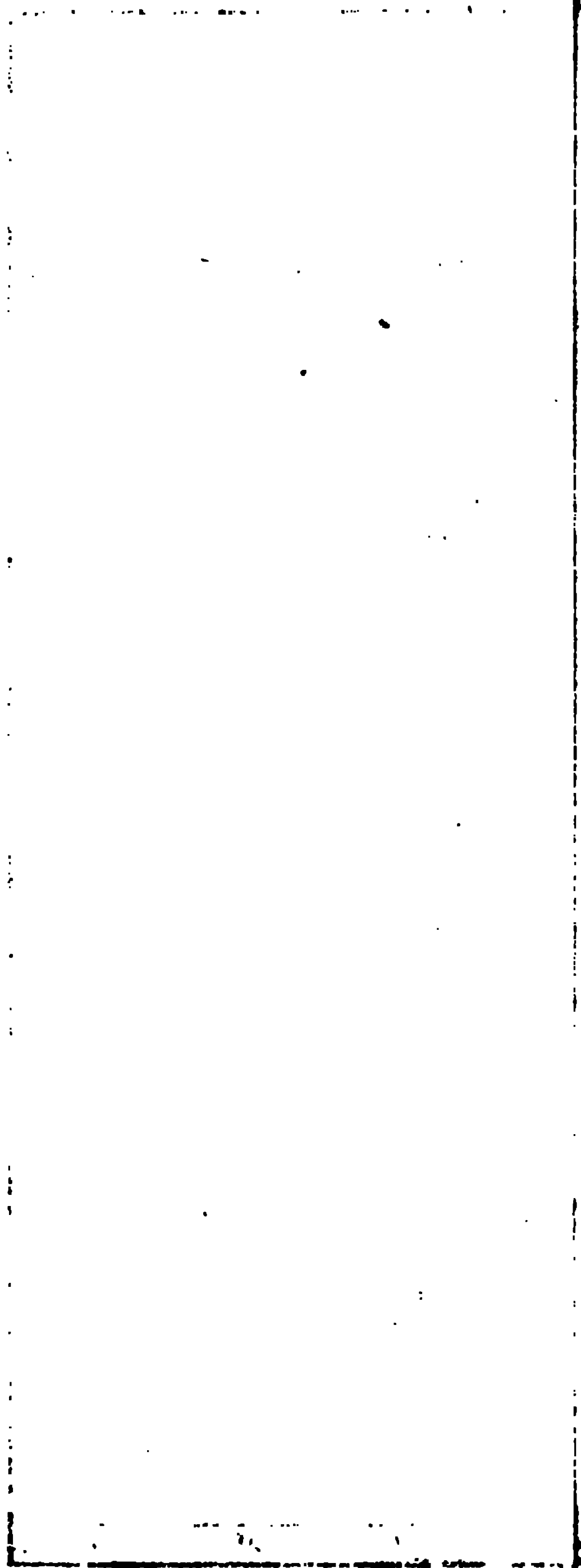


Fig. 1.

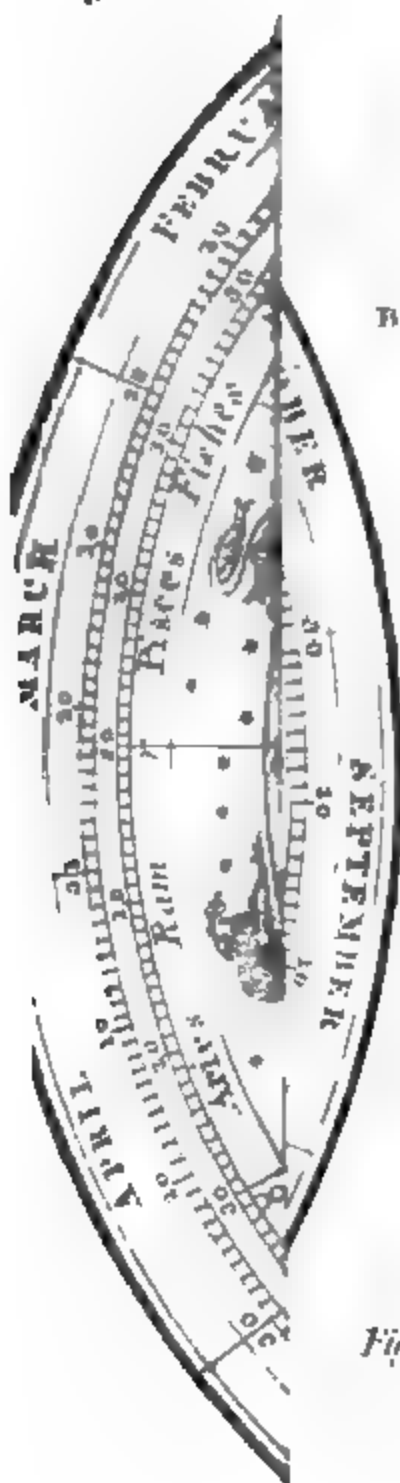


Fig. 3.

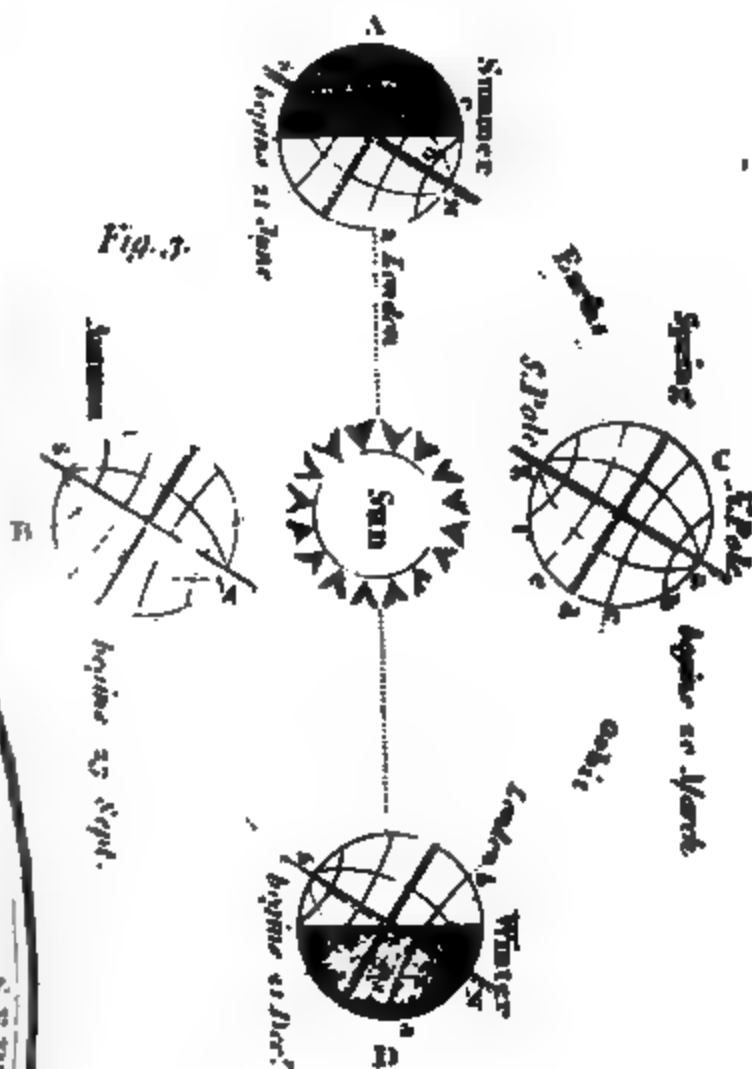


Fig. 2.

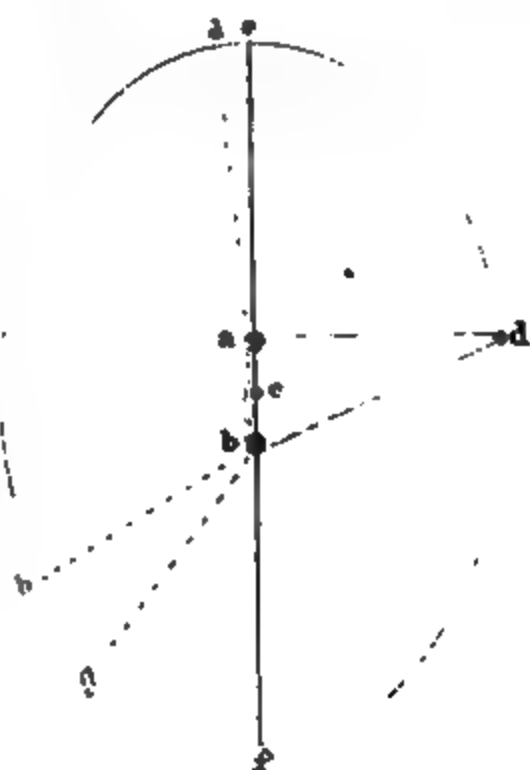




Fig 1.

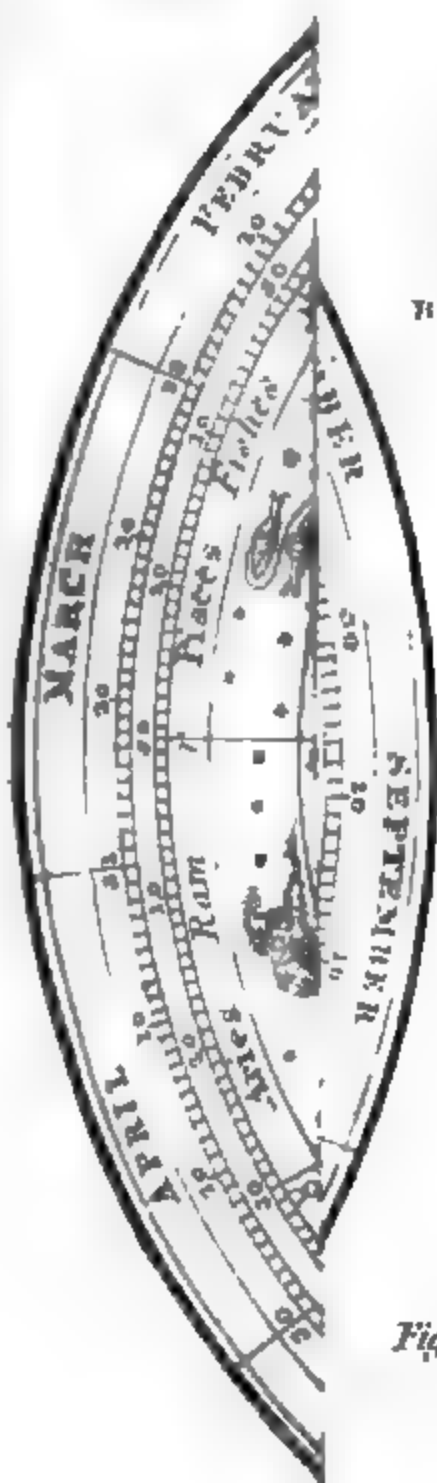


Fig 3

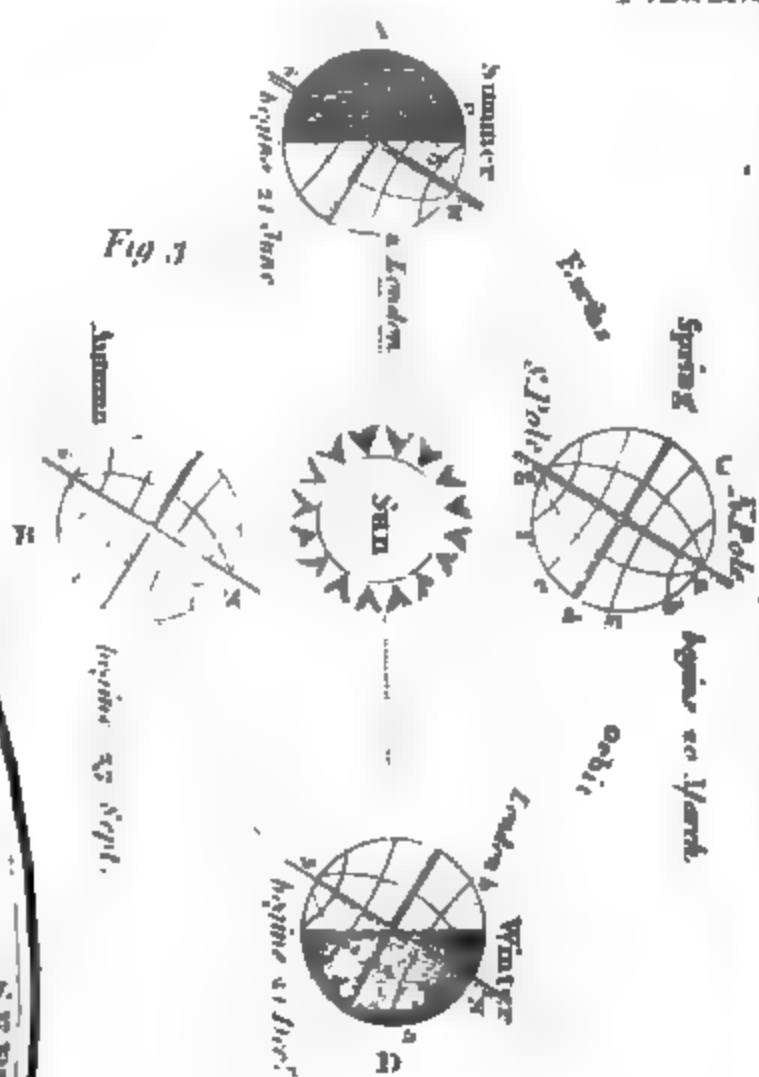


Fig 2.



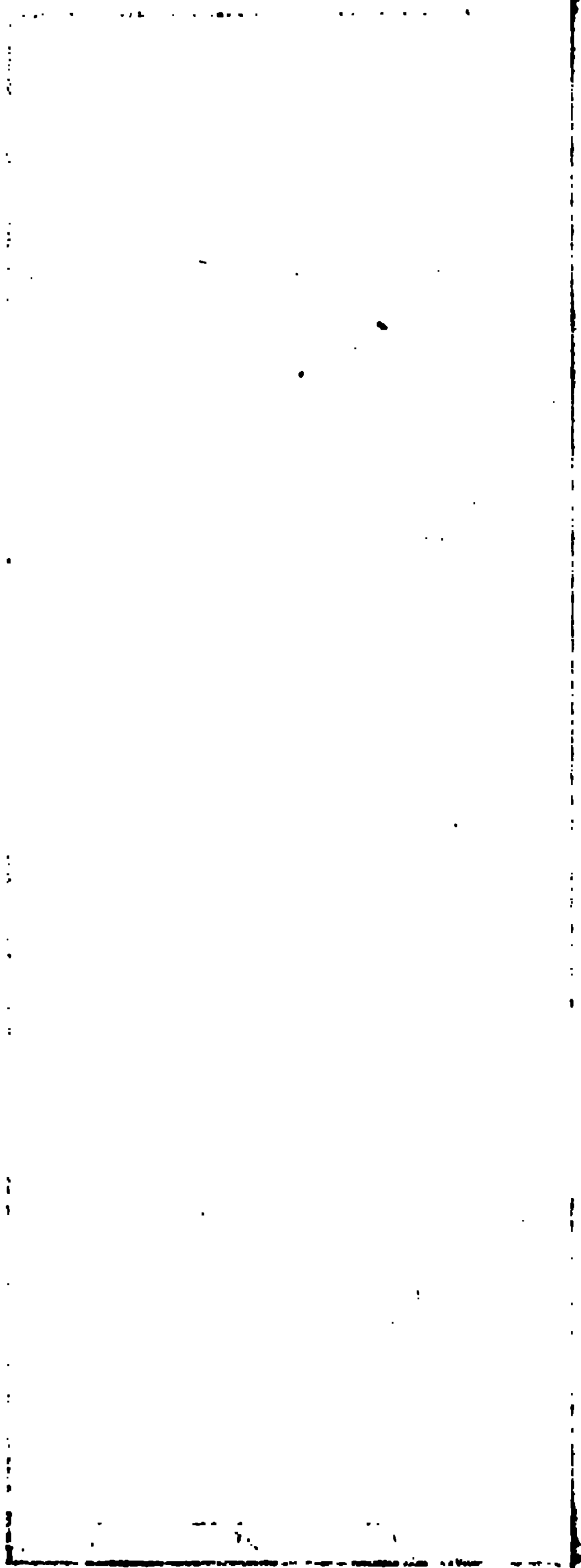


Fig. 1.

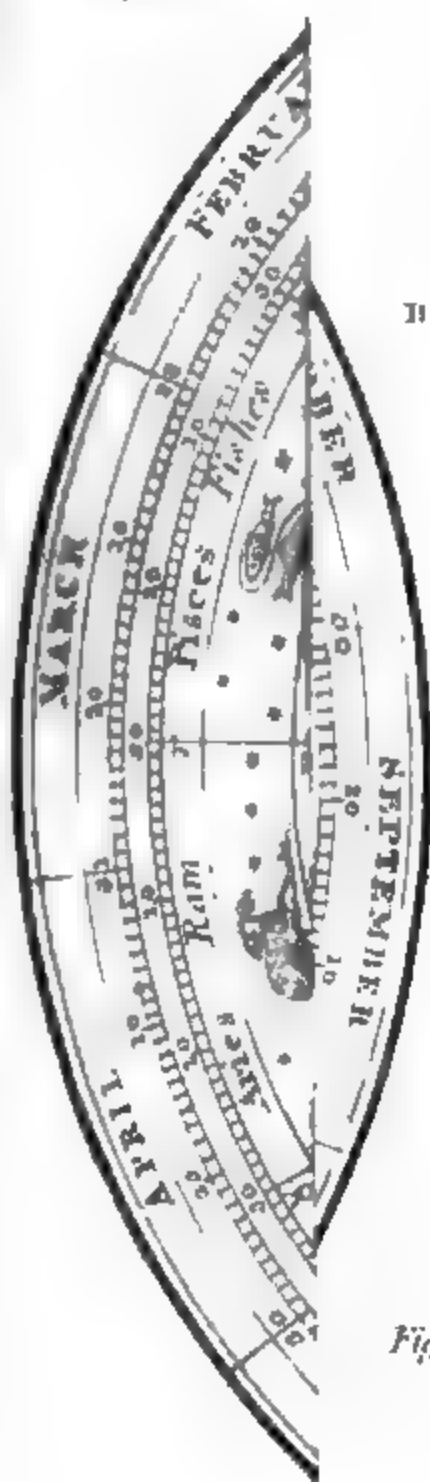


Fig. 3.

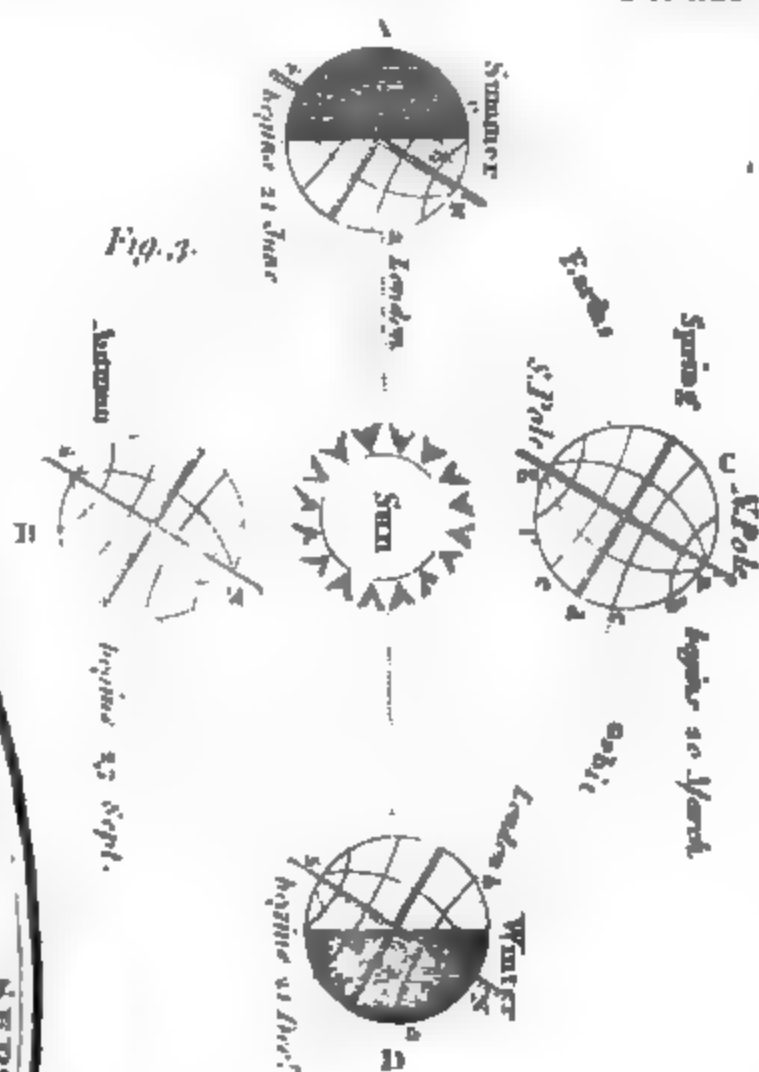
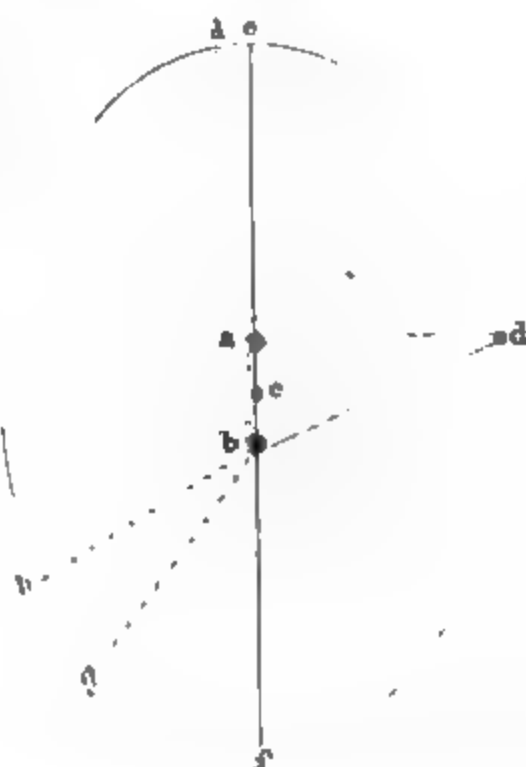
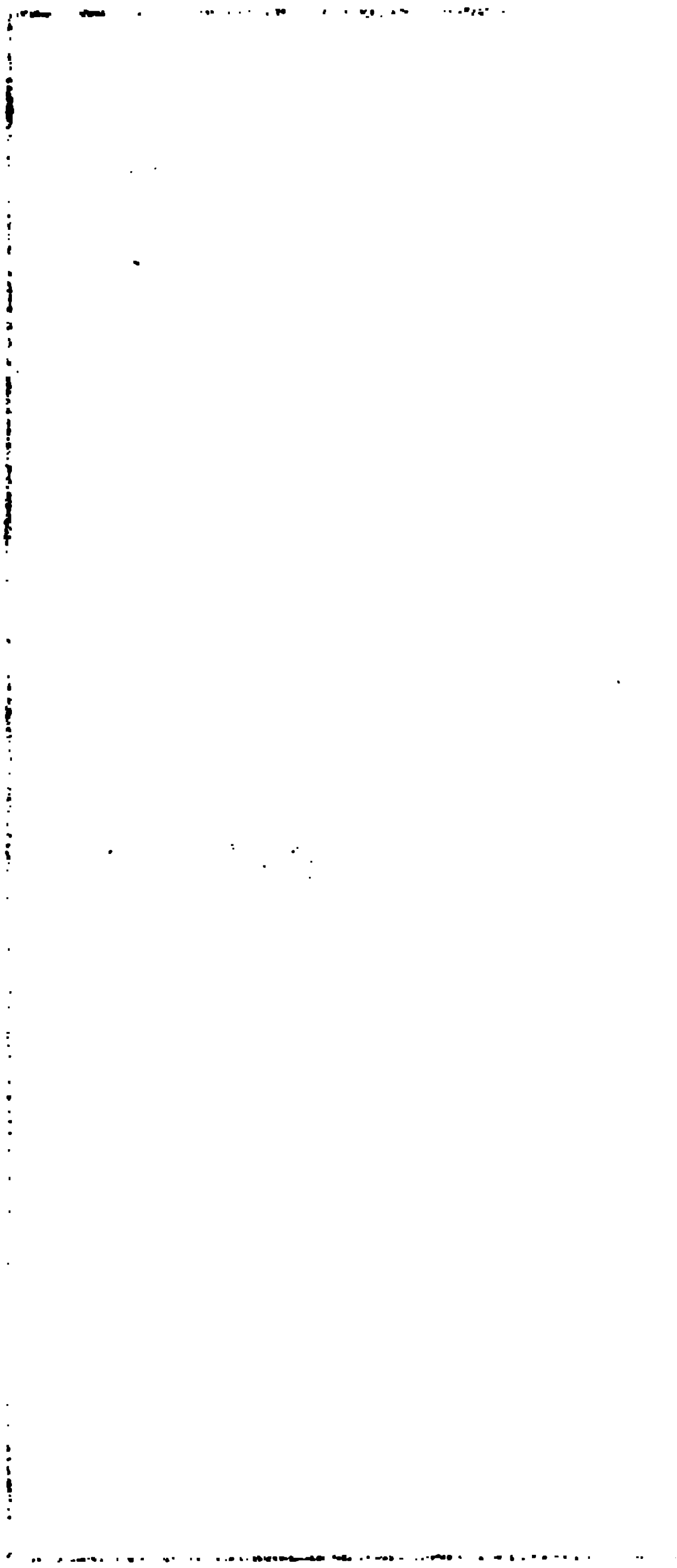


Fig. 2.





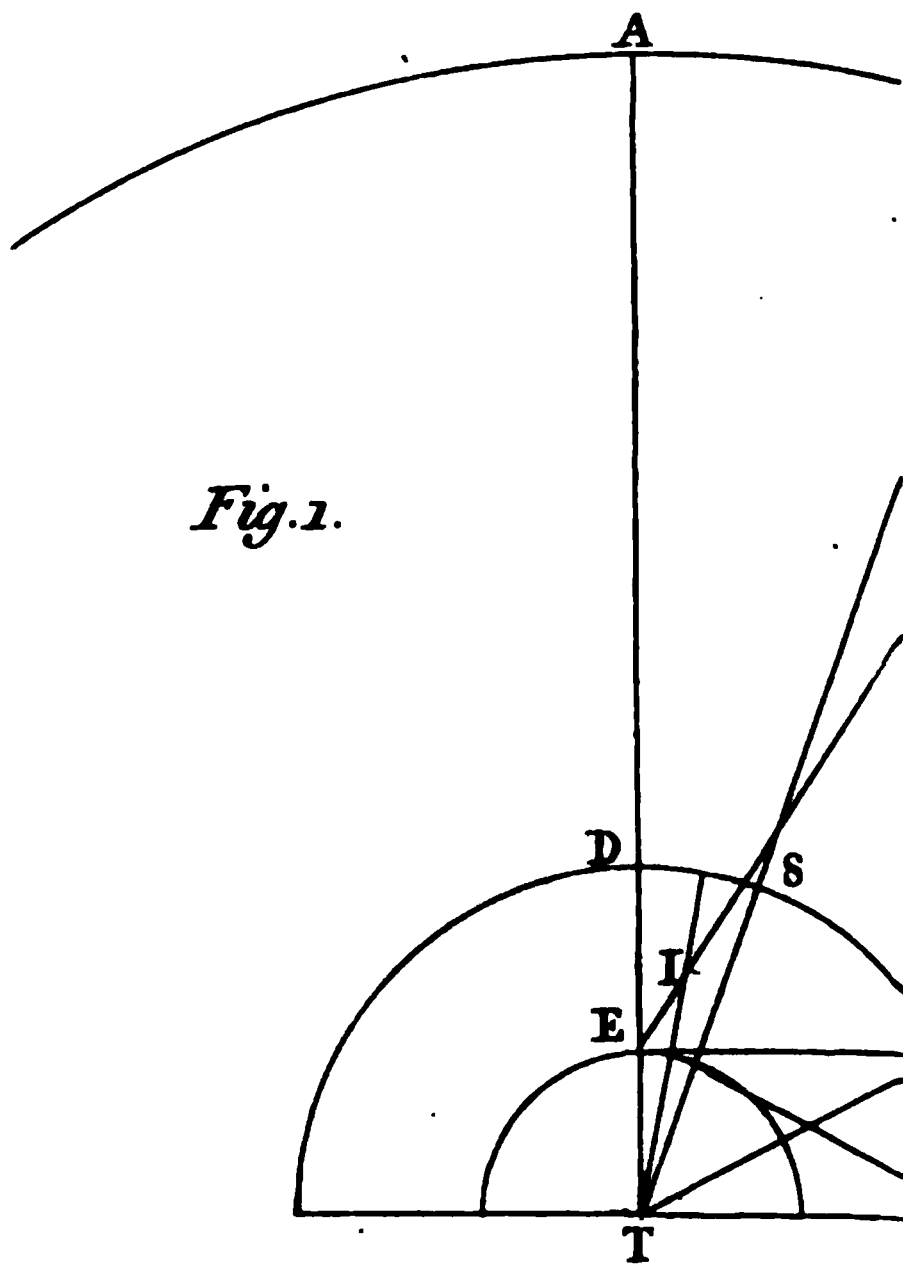
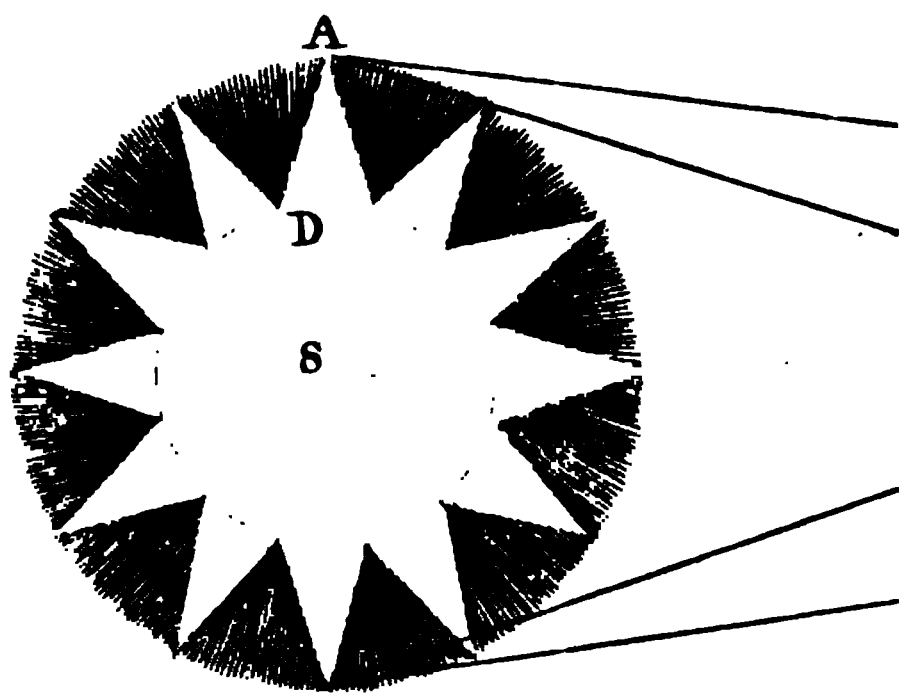


Fig. 1.

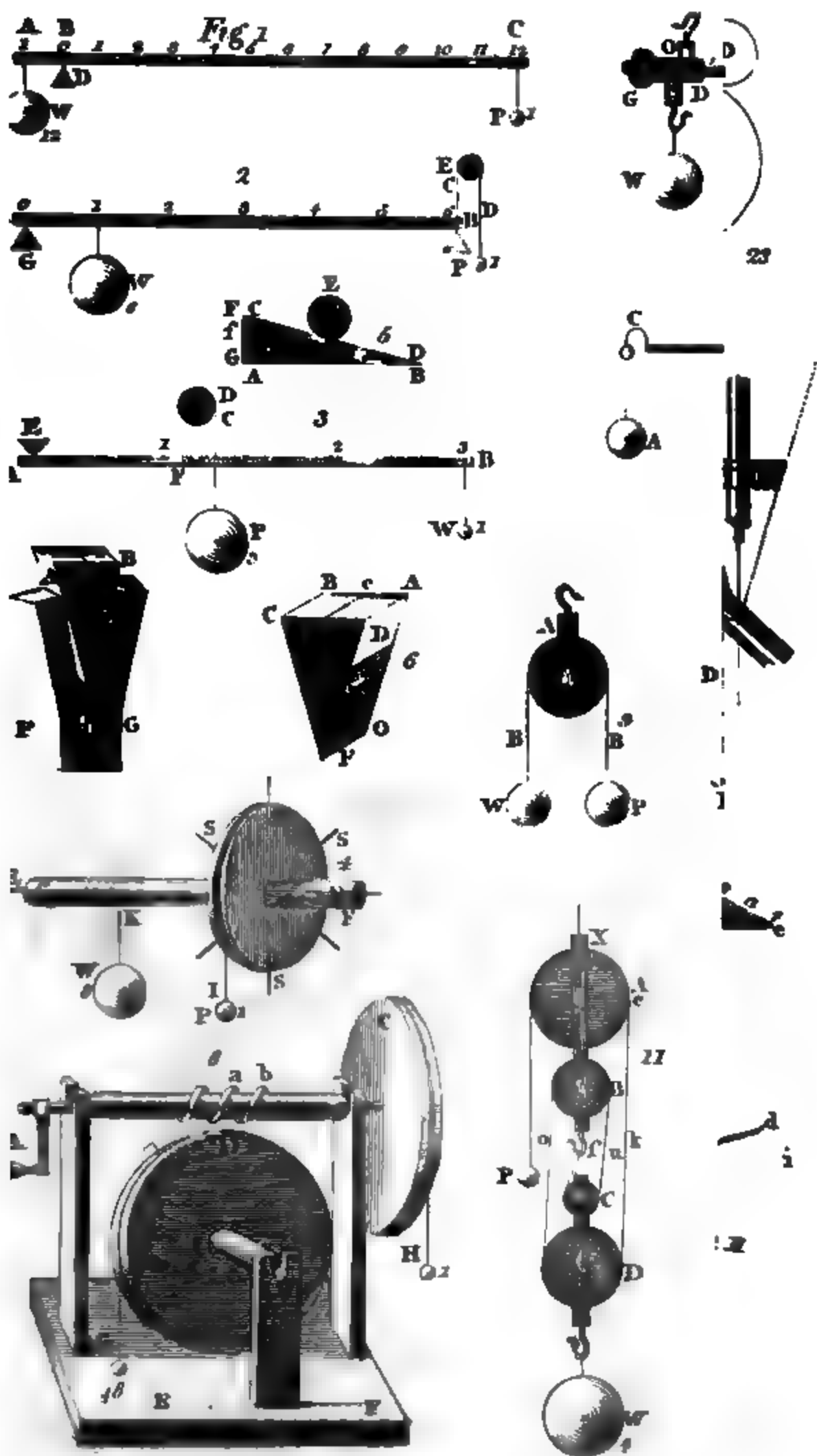


Fig. 3.

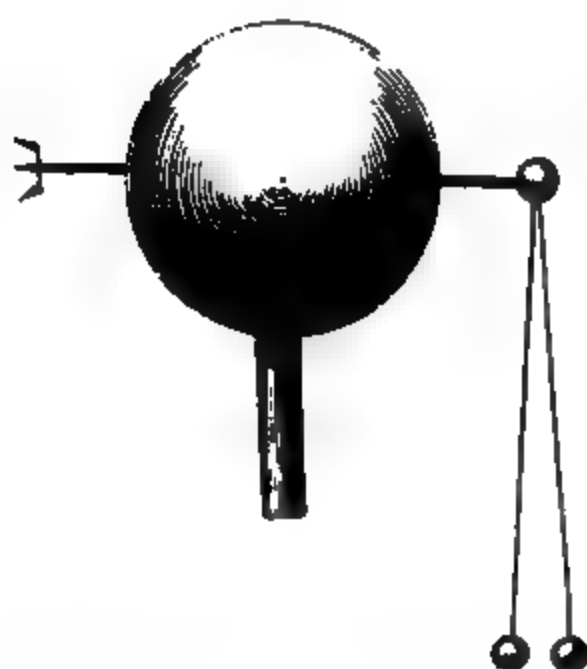


Fig. 2.



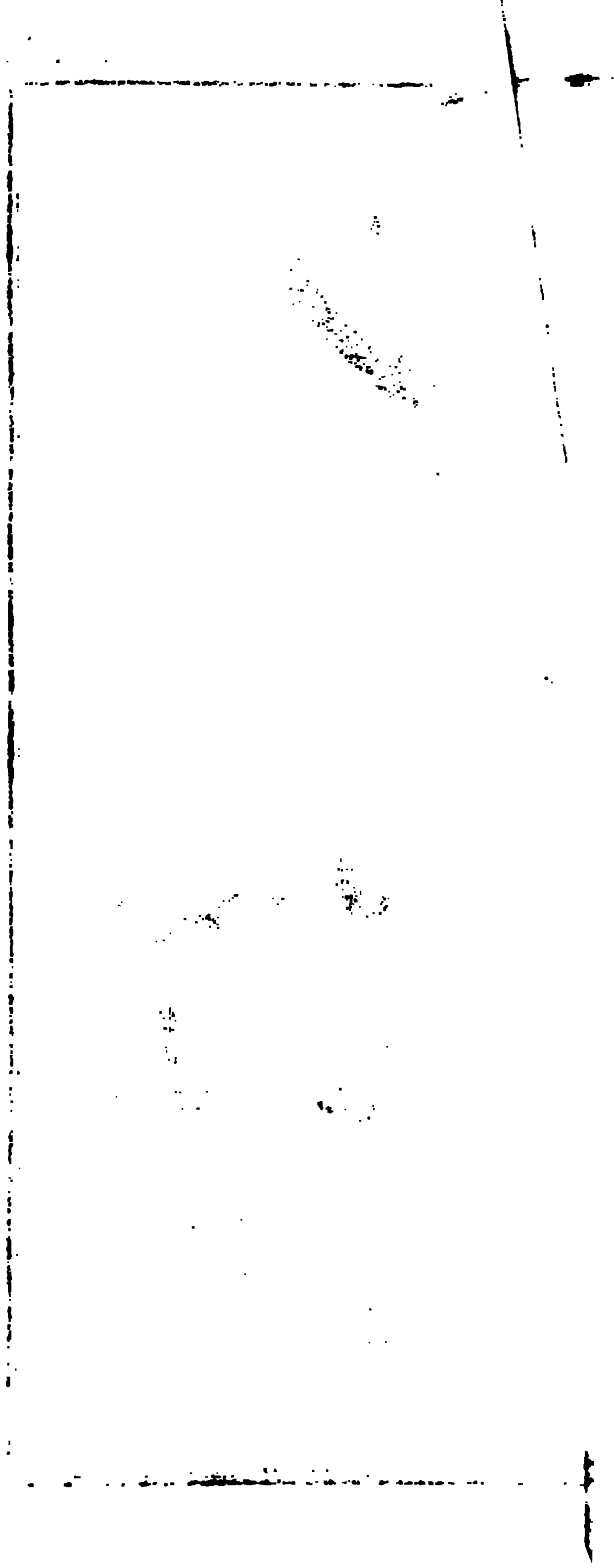


Fig. 20.



Fig. 2.

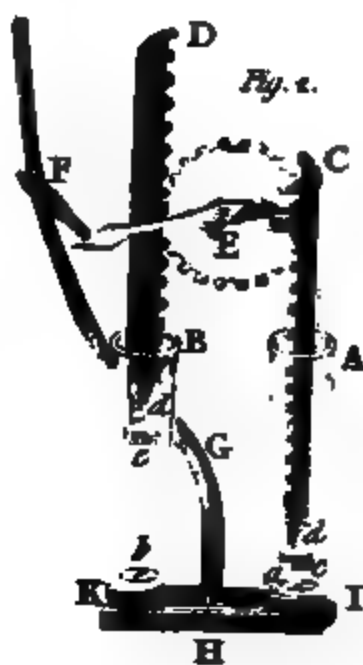


Fig. 29.

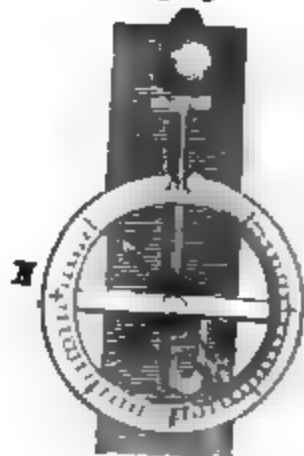


Fig. 4.



Fig. 7.



Fig. 10.



Fig. 3.



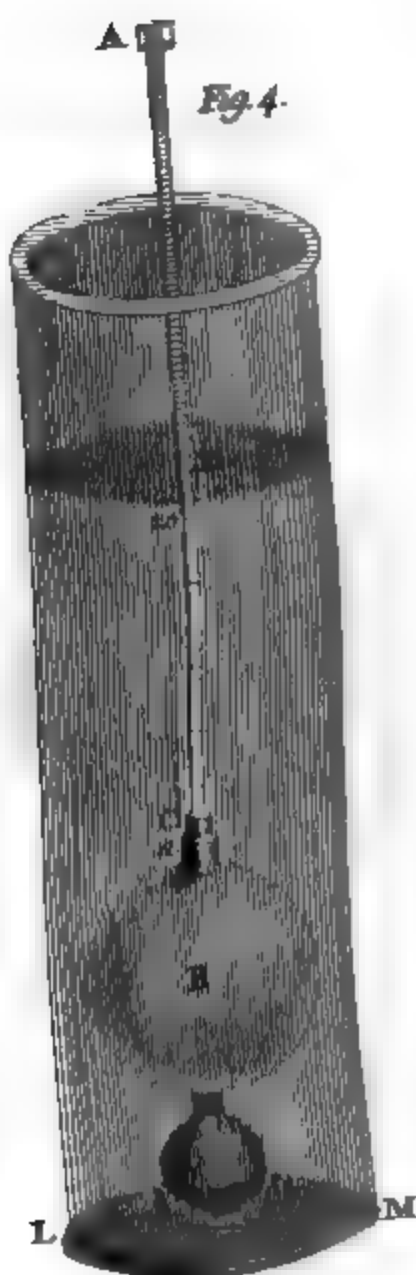


Fig. 4.

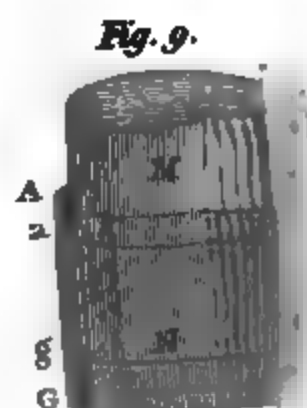
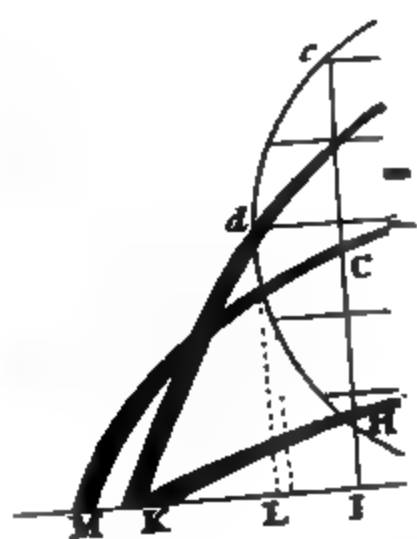


Fig. 9.

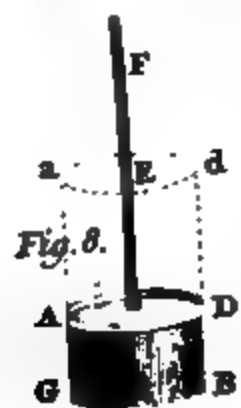


Fig. 8.



Fig. 5.

Fig. 9.

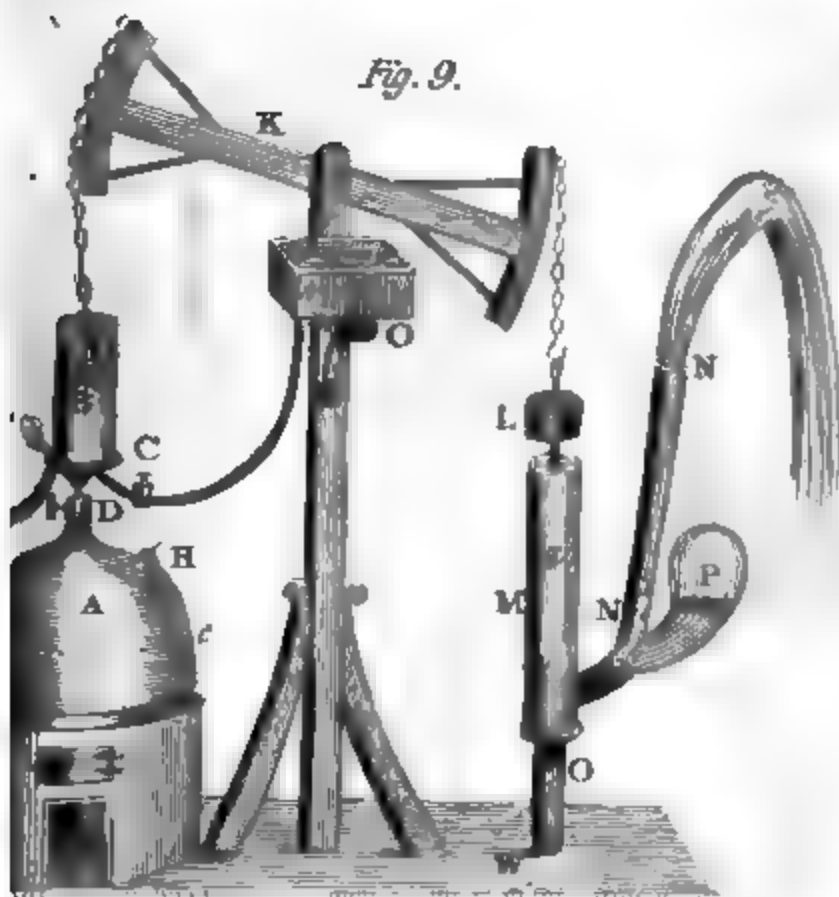


Fig. 8.

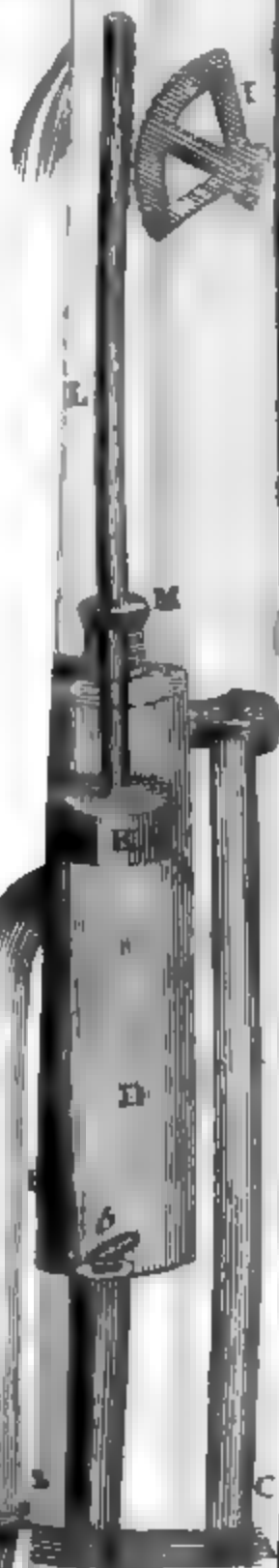
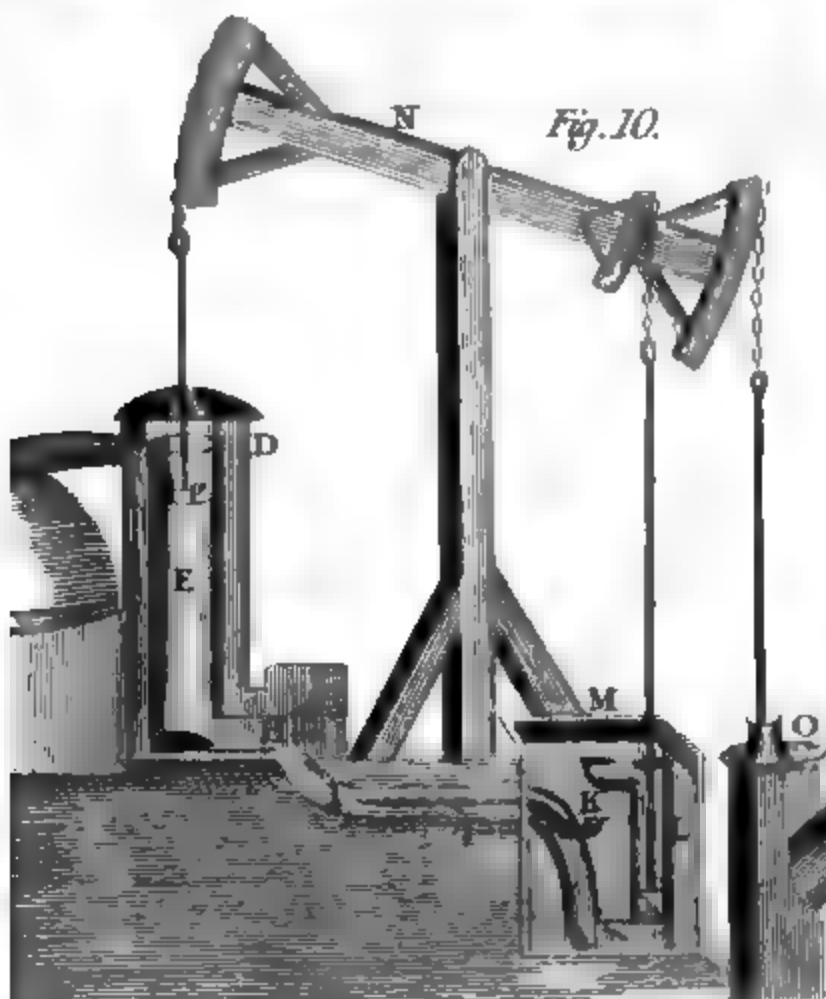
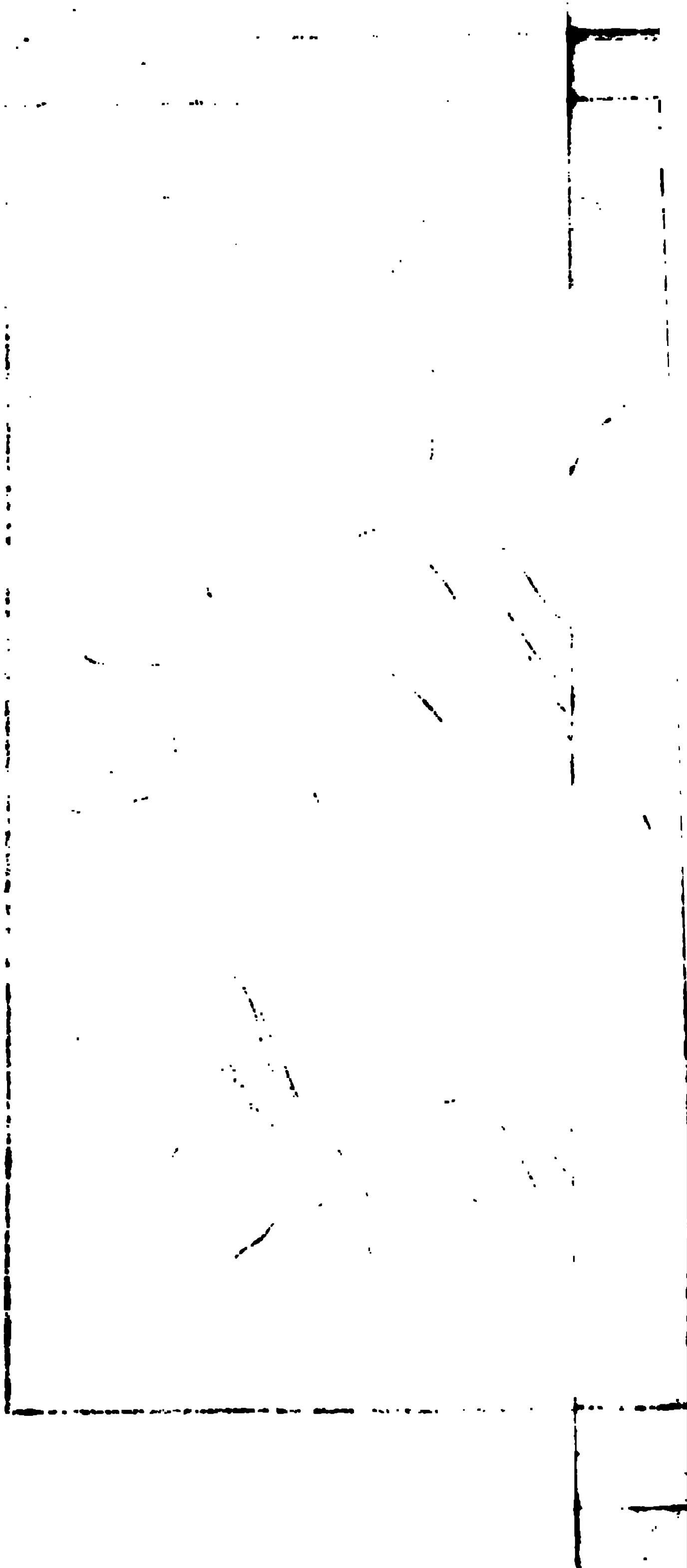
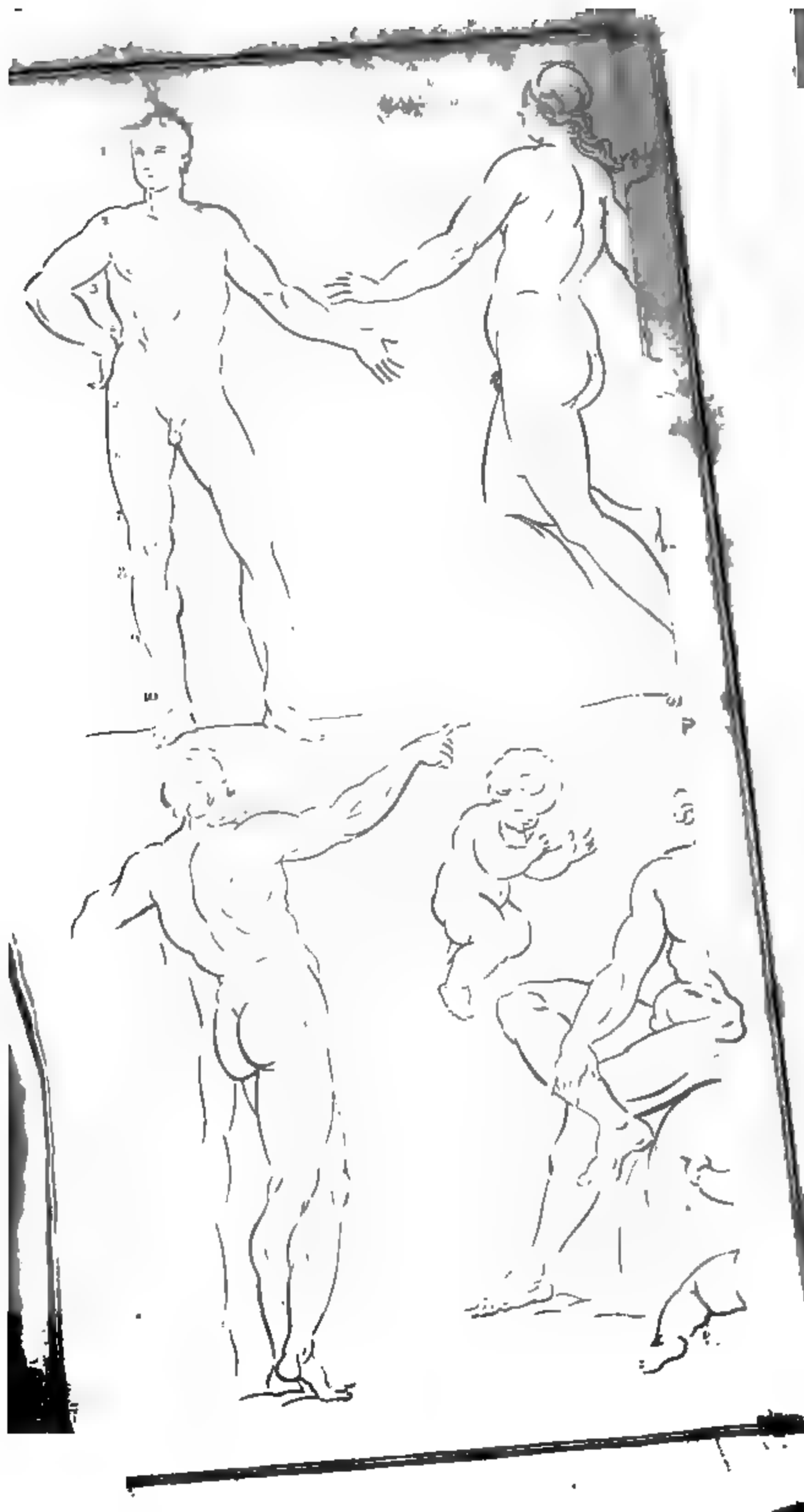


Fig. 10.







THE
ACCOMPLISHED TUTOR.

CHAP. I.
OF ENGLISH GRAMMAR.

SECT. I.

OF THE DIVISION AND SOUND OF LETTERS.

IN the early ages of antiquity, before alphabets were invented, mankind, sensible of their want of some means of recording historical events and scientific discoveries, had recourse to various arts for these purposes; the first of which was painting. That partiality for pictures, so evident in all ages and countries, afforded the ancients a method of perpetuating their transactions. To commemorate that one man had killed another, they painted the figure of a dead man, with another man standing over him, having an hostile weapon in his hand. On the first discovery of America, this was the only kind of writing used by the Mexicans.

The first improvement made by our ancestors in the art of writing (if it might then be called an art) was by the introduction of hieroglyphical characters. These consisted of

certain symbols, which were made to represent certain invisible objects, to which, in some particulars, such symbols were supposed to bear some resemblance. An eye was the symbol of knowledge; a circle, of eternity, as having neither beginning nor end. The figures of animals were also much employed in this kind of writing, on account of some quality with which they were supposed to be endowed, and in which they resembled the object signified. Thus, imprudence was represented by a fly; wisdom by an ant; and victory by a hawk. These hieroglyphics flourished most in ancient Egypt (as did all other learning at that time), where the knowledge of these characters was reduced into a regular art: and many specimens of them are still extant in relics of Egyptian antiquities. Hieroglyphics, though an improvement upon the former mode of writing, was a very imperfect one, and often confused and perplexed its most skilful professors.

In a few succeeding ages, hieroglyphics gave place to simple arbitrary marks, which were introduced to represent objects, without having the least resemblance or affinity to the objects represented. The Chinese still use characters of this nature: they have no alphabet of letters, but every single mark or character signifies one perfect idea or object. The number of these characters are therefore great:—near seventy thousand. To be perfectly acquainted with them, constitutes the business of a whole life; which must be an insurmountable obstacle to the improvement of science. Our common figures, 1, 2, 3, 4, &c. afford us an example of this sort of writing; where each figure or character conveys the idea of the number for which it stands as clearly and intelligibly as the words themselves, one, two, three, &c. But when marks or characters come to be used for all our ideas, in exclusion to an alphabet of letters, they then, from their number, become inconvenient.

The next improvement in the art of writing was by the invention of signs or marks, which stood, not directly for the objects themselves, but for the words or names whereby they

they were distinguished. This was an *alphabet of syllables*. An alphabet of this kind is still in use in Æthiopia, and some countries of India.

But the noble and sublime discovery of an *alphabet of letters* superseded every other improvement in this art. Who was the first in this invention is uncertain. An alphabet of letters was, however, brought into Greece by Cadmus, the Phœnician, who was contemporary with king David. This alphabet consisted of only sixteen letters: the rest were added afterwards, as signs for proper sounds were found to be wanting. The Phœnician, Hebrew, Greek, and Roman alphabets are so much alike in the figures and names of the letters, as plainly to evince they were originally derived from the same.

By the use of the alphabet, we are now, therefore, enabled to express our ideas with the same clearness and precision, as in conversation.

The English alphabet consists of twenty-six letters: A, a; B, b; C, c; D, d; E, e; F, f; G, g; H, h; I, i; J, j; K, k; L, l; M, m; N, n; O, o; P, p; Q, q; R, r; S, s; T, t; U, u; V, v; W, w; X, x; Y, y; Z, z; and is divided into vowels and consonants, mutes and semi-vowels.

The names of the twenty-six letters are as follow: a, bee, cee, dee, e, ef, gee, aitch, i, ja, ka, el, em, en, o, pee, cue, ar, efs, tee, u, vee, double u, ex, y, zad.

The vowels are six in number, viz. a, e, i, o, u, y; all the rest are consonants.

The mutes are those letters which are begun, when they are spelled, by a consonant: as, b, bee; c, cee; d, dee, &c.; those which are begun with a vowel are called semi-vowels: as, l, el; m, em; n, en; r, ar; s, efs, &c.; l, m, n, r, are also called liquids.

When two vowels meet together, they are called a diphthong: of these there are thirteen, viz. ai, ei, oi, ui, au, eu, ou, ee, oo, ea, eo, oa, and ie.

When three vowels meet, they are called a triphthong: as in the word *beauty*.

—With regard to the sound of these double vowels, there are no rules, that can be given, which will hold good in all cases, as words are founded according to the caprice or affectation of the age, or the speakers: a knowledge of them must, therefore, be acquired by experience and observation.

And concerning the sound of single letters, the following rules are all that can safely be depended upon.

C is pronounced hard like *k*, before *a*, *e*, *u*; and soft like *s*, before *e*, *i*, and *y*.

G is also sounded hard before *a*, *e*, *u*; sometimes hard, and sometimes soft, before *i* and *y*; and generally soft before *e*.

E is mostly silent at the end of a word; but in that case it lengthens the foregoing vowel; as, *hid*, *hide*; and that sometimes in the middle of a word; as *ungrateful*. But sometimes it only softens a preceding *g*, as in *lodge*, *judge*.

H is only an aspiration of the breath, and sometimes at the beginning of a word is not sounded at all: as, an *hour*, an *honest* man.

W is either a vowel or a diphthong; its proper sound is the same as *u* in the Italian, *ou* in the French, or *oo* in the English. Sometimes it is not sounded at all after *e*, sometimes like *oo*.

X is a double consonant, composed of a hard *c* or *k*, and *s*; and at the beginning of a word mostly sounded like *x*.

Y has exactly the same sound as *i*; and is only a substitute for it at the end of a word, or before *i*: as, *cry*, *flying*. It is a perfect vowel; and when used as a consonant at the beginning of a word, it answers to the ancient Saxon *i*: as, *yew*, *iw*; *young*, *ioug*.

Z is a double consonant; it sounds as much coarser and thicker than *s*, as *v* does than *f*.

J and *v*, though confounded by some old writers with *i* and *u*, are entirely different letters; the former having the sound of a soft *g*, and the latter that of a coarser *f*. The former is called *ja*, and the latter *ve*.

SECT. II.

OF SYLLABLES.

A SYLLABLE is a constituent part of a word, or a whole word; it consists of one or more letters, and is formed by a single impulse of the voice.

Spelling is the art of rightly naming the letters singly that constitute a word, and dividing them into syllables.

The art of spelling perfectly is not to be acquired (particularly in the English) but by practice: but a few rules for the dividing of syllables may possibly be of service.

1. A syllable in the beginning, or middle, of a word ends in a vowel, except such vowel be followed by *x*, or two or more consonants: as in *re-li-gi-on*.

2. When two or more consonants follow a vowel, which is pronounced short, they must be separated; and one, at least, always belongs to the preceding syllable: as in *ab-si-nce*.

3. When two or more consonants follow a vowel, which is pronounced long, they sometimes belong to the following syllable: as, *di-gress*.

4. A particle, though placed immediately before a vowel, is seldom divided: as, *un-equal*.

5. A mute with a liquid following are seldom divided; but a liquid or a mute, with a mute following, are mostly divided.

6. When *le* or *re* follow a mute they are never divided.

These are the fundamental rules for the dividing of syllables: but some grammarians recommend them to be divided as they are sounded in a just pronunciation.

SECT. III.

OF THE NINE PARTS OF SPEECH.

WHEN mankind had arrived at some perfection in the art of writing, they soon discovered the propriety of reducing language

language into different forms of words; or, as they are commonly called, parts of speech.

In English we have nine forms of words, or parts of speech.

1. The **ARTICLE**—placed before nouns to help to determine their signification: as, *a* man, *the* man.

2. The **SUBSTANTIVE**, or noun—which is the name of any object whatever, of which we have any idea: as, *a* man, *a* horse, *a* spirit, *grief*, *love*.

3. The **PRONOUN**—used as a substitute for the noun: as, *he*, *she*, *it*.

4. The **ADJECTIVE**—added to the noun to denote its quality: as, *an honest* man, *a white* horse, *an evil* spirit.

5. The **VERB**—which signifies a state of existence, acting, or suffering: as, “the centinels *slept*, the enemy *entered*, and the fort *was taken*.”

6. The **ADVERB**—used to qualify or enforce the meaning of other words. It is added sometimes to a verb: as, he reads *well*. Sometimes to an adjective: as, “an *exceeding* high mountain.” Sometimes to another adverb: as, “*most* divinely fair.”

7. The **PREPOSITION**—placed between words to connect them together: as, “the path *to* fame lies *through* the road *to* danger.”

8. The **CONJUNCTION**—used to connect sentences, as prepositions connect words: as, “fear God *and* honour the King.”

9. The **INTERJECTION**—used to express the surprise or affection of the speaker: as, *O!* *alas!*

These are the nine parts of speech in the English tongue; every word in which is reducible to one of these parts.

The **ARTICLE** is placed before substantives to shew how far their signification extends.

There are in English only two articles; *a* and *the*. *A* is called the indefinite, and *the* the definite article.

The indefinite article *a* is changed into *an* when the next word begins with a vowel, or a silent *h* before the vowel; except such vowel be *y* or *w*. This article, as its name imports, is used in an indefinite and undetermined sense: as, *a* man,

man, means one man, without determining who in particular; leaving that to be explained, if necessary, by the other parts of the sentence.

This article can, therefore, be joined to substantives in the singular number only, except it come before the adjectives *few* and *many*: as, *a few days*, *a (great) many men*; or before the complete numerical adjectives, *a dozen*, *a score*, *an hundred*, *a thousand*, *a million*, *a myriad*: as, *a dozen pound*, *a score of eggs*, *an hundred men*, &c.

The definite article *the* determines which particular thing is meant; or of many, which they are: this is therefore employed both with singular and plural nouns: as, "*the flock in the heavens*."

It is also used sometimes before adverbs in the comparative and superlative degrees; as, *the more*, *the better*, *the least* (of all).

The SUBSTANTIVE is the name of any object whatever of which we have any idea.

There are two sorts of substantives, Proper and Common. Proper names are such as belong to individual objects, whether animate or inanimate; but not to every individual or object of the same sort or species: thus, *John* is the name of a man, but not the name of every man; *Chance* may be the name of a dog, but it is not the name of every dog; *London* the name of a city, but not of every city. These are therefore proper names; but *man*, *dog*, and *city* are common names: for *man* is the common name of every man; *dog*, the common name of every dog; and *city*, the common name of every city.

Proper names have no articles nor plural number, unless by a metaphor: as when a cruel tyrant is called *a Nero*; or when a common name is understood, as the (ship) *Royal George*; or when there are many of the same name, as *the twelve Cæsars*.

There are three attributes belonging to the English noun, viz. number, gender, and case.

The number is either singular or plural. The singular number expresses one object: as, *a king*: the plural, two or more, as, *kings*.

The

The plural number is mostly formed, in English, by the addition of *s* or *es*: as, *boy, boys*; *goat, goats*; *fox, foxes*.

Some change the *f* into *v*: as, *wife, wives*; *leaf, leaves*.

Some plurals end in *en*: as, *ox, oxen*; *child, children*; *man, men*; *brother, brethren*. In the latter, and some others, the *e* in the first syllable is changed into *e*. This form is the remains of the Teutonic language; and the following of the Saxon; *louse, lice*; *mouse, mice*; *tooth, teeth*.

Some nouns ending in *y*, change it into *i*: as, *city, cities*; *gallery, galleries*.

Some nouns have no plural: as, *corn, gold, pitch, sloth, &c.* And others no singular numbers: as, *annals, bellows, scissors, lungs, &c.* The words *sheep* and *deer* have no variation of number.

All substantives in English, whether common or proper, are considered with relation to gender, as being either of the masculine, feminine, or neuter gender. That is, either of the male or female sex, or neither.

The masculine or feminine genders are applied to the names of animals only, and such whose sex is obvious: as, *man, Thomas, ox*, which are the masculine gender: *Mary, woman, hen, cow*, which are the feminine gender. Those whose sex is not evident, with all inanimate objects, are of the neuter gender: as, *oyster, worm, house, tree*; except in poetry, and the elegant species of composition, where inanimate objects, and the human passions, are personified into men and women, and consequently have their genders.

A few substantives are marked by their terminations: as, *prince, princess*; *lion, lioness*; *actor, actress*.

The chief use of gender is to agree with the pronoun.

In English there are three different cases of the noun: the nominative, possessive, and objective case.

The nominative case is the simple name of the noun, without relation to other objects: as, "*John*."

The possessive signifies the relation of possession: as, "*John's* book," and was formerly written "*John's* book," and not "*John's* book," as is vulgarly written. It may also be expressed by
inverting

inverting the phrase, and inserting the preposition : as, “ the book of John.”

When the sign of the possessive case is to be added to a phrase of several words, it is generally joined to the last word : as, “ the Emperor of *Morocco's* dominions.” To the plural noun ending in *s*, the *s* is not added : as, the “ *arrows'* heads.”

The objective case, though founded in grammar, is formed as the nominative, and expresses the object of a sentence : as, “ man loves *society*.”

The PRONOUN is a word used as a substitute for the noun, and to prevent its repetition : as, “ John loves *his* book.” Here the pronoun *his* supplies the place of the noun *John*, and prevents the repetition of it.

In a pronoun there are person, number, gender, and case.

There are three persons in the pronoun. First, the person who speaks may mean himself : as, *I, me* ; these are therefore called pronouns of the first person :—or he may speak of the person to whom he addresses himself : as, *thou, thee* ; these are pronouns of the second person :—or he may intend some other person or thing : as, *he, she, or it* ; these are pronouns of the third person. Each of these persons have the singular and plural numbers, and the three variations of case. The third person has also the variations of gender.

The Pronouns declined according to their Variations of Persons, Numbers, Cases, and Genders.

<i>Singular Number.</i>				<i>Plural Number.</i>				
Nominative.	Possessive.	Objective.		Nominative.	Possessive.	Objective.		
<i>First Person.</i>				<i>First Person.</i>				
I,	mine,	me.		We,	ours,	us.		
<i>Second Person.</i>				<i>Second Person.</i>				
Thou,	thine,	thee.		Ye or you,	yours,	you.		
<i>Third Person.</i>				<i>Third Person.</i>				
<i>Genders.</i>								
<i>Mas.</i>	He,	his,	him.	} They,	theirs,	them.		
<i>Fem.</i>	She,	hers,	her.					
<i>Neu.</i>	It,	its,	it					
VOL. I.				B				
				Thou,				

Thou, in all its cases, generally gives place to *you*, except in a very solemn style, and in addresses to the Deity.

As the personal pronoun is a substitute for the noun, so it has the same nature in grammatical construction, and is capable of forming a sentence without the aid of the substantive: as, “*I* that speak unto thee am *he* [Christ].”—John, iv. 26.

But there are other pronouns, of the nature of adjectives, and, like adjectives, require some substantive to be joined with them, which is either expressed or understood. Those are therefore called Pronominal Adjectives. Of these there are four sorts: possessives, definitives, relatives, and distributives.

The possessives are *my*, *thy*, *our*, *your*, *her*, *their*; and are entirely different words from the possessive pronouns, *mine*, *thine*, &c. In these the substantive is sometimes understood without being expressed: as, “*the shame be yours* ;” whereas, to those the substantive should be added: as, “*thy people shall be my people*, and *thy God my God*.”—Ruth, i. 16.

The definitives are, *this*, *that*, *other*, *any*, *some*, *one*, *none*. These define, or limit, the meaning of the substantive to which they refer, or are joined. The three first have the plural number; as, *these*, *those*, *others*. *This* refers to the latter term or sentence, *that* to the former: as, “*these* are definitives, *those* (mentioned before) are possessives; *other* is used in the plural form only, when the substantive belonging to it is not expressed, as is sometimes the case in definitives: *other* and *one* have also the possessive case: as, “*this is others’ property*,” and “*it startles one’s apprehension* ;” *one* is also used in an indefinite sense, as, “*one thinks*.” These are all the variations definitives admit of.

The relatives are, *who*, *which*, *that*. These refer to some substantive or pronoun going before, which is therefore called the antecedent: as, “*I am the Lord that maketh all things, that stretcheth forth the heavens alone*.”—Isaiah, xliv. 24. They also connect the following and foregoing parts of a sentence together. *Who* is varied to express the three cases: as, *who*, *whose*, *whom*; these relatives refer to all the three persons,

persons, whereas the other pronominal adjectives belong only to the third person. *Who, which, what*, are called interrogatives, when they are used in a question.

Each, every, either, are called distributives, because they relate to objects or persons taken separately.

Besides the foregoing pronominal adjectives, there are two others: *own*, and *self*, in the plural *selves*. Both of them express emphasis, or opposition, and are joined to the possessives, and so form a compound pronominal adjective; as, *my own, thyself, yourselves*.

Ourself, the plural pronominal adjective, with the singular substantive, belongs to the regal style.

All substantives belong to the third person, except when an address is made to a person, then the substantive is of the second person.

An ADJECTIVE is a word added to a substantive, to express the quality of it, or some other property: as, *a good man, an hundred pounds, a burning mountain*.

Adjectives have no variation, except the three degrees of comparison: positive, comparative, and superlative.

Most qualities consist of different degrees, or of *more* or *less*. Thus, when a quality is simply expressed, without reference to a greater or less degree of the same, it is called the positive degree: as, *large, short*. When it is expressed with relation to a less degree, it is called the comparative degree: as, *larger, shorter*. When it is expressed as being of the highest degree in its quality, it is called the superlative: as, *largest, shortest*.

There are two ways of forming the comparative and superlative degrees; the first is by adding *r* or *er* to the positive degree, which forms the comparative; and by adding *st* or *es*, to the positive, and so forming the superlative degree; as in the last example. Most monosyllables are compared in this manner; and dissyllables ending in *y*: as, *happy*; and *le*, when they are followed by a mute: as, *able*; or when they are accented on the last syllable: as, *genteel*. But others, and words

of more than two syllables, seldom admit of these terminations; but are compared according to the following rule.

The second method of forming the comparative and superlative degrees of comparison, is by placing the adverbs *more* or *most* before the adjective of the positive degree: as, *experienced, more experienced, most experienced*.

The superlative is, in a few words, formed by adding the adverb *most* to the end of them: as, *nethermost, foremost, uppermost, undermost, &c.*

The four following adjectives are irregular in the formation of the degrees of comparison in most languages: *good, better, best; bad, worse, worst; little, less, least; much or many, more, most*.

Double comparatives and superlatives are improper in grammar: as, the *more greater*, or the *most greatest*; except the *most highest*, a phrase used in the Psalms of David, and is properly applicable to the Supreme Being only.

The VERB is a word which signifies an action, or a state or condition of being; and is either active, passive, or neuter.

A verb active expresses an action in its natural order, that is, when the agent is placed first, and the object last; as, "*John loves his book.*" Here *John* is called the agent, because he performs the action, which is *love*, and which is the verb; and *book* is the object, upon which the action love is exercised.

A verb passive expresses an action, but in an inverted order: that is, when the object is placed first, and the agent last: as, "*the book is loved by John.*" Here may be seen the use of the cases of nouns and pronouns: for when verb is active, the agent is placed first, and is in the nominative case: as, "*I love thee.*" When the verb is passive, the object being placed first, must be in the nominative case, and the agent being last must be in the objective case, accompanied with a preposition; and the verb assumes a different form: as, "*thou art loved by me.*" Thus an active verb may be transformed into a passive, or a passive verb into an active verb.

A verb

A verb neuter expresses a state, or condition, of being, only, and consequently has no object to be acted upon; but an agent only: as, *I walk, I sleep, I dreamed, he fell, she arose*. The verb neuter is called also the intransitive verb, and the verb active transitive.

Many verbs are used in English both in an active and neuter signification; their form demonstrating of which kind they are.

The verb is varied, first, to correspond with the three different persons of the pronoun; secondly, to agree with the number singular or plural; thirdly, to express the three principal gradations of time,—present, past, and future; lastly, to shew the mode or manner in which the action or state of being is expressed.

In a verb are, therefore, these four attributes; person, number, time, and mode,

To agree with the three personal pronouns, though of the same number, the verb is varied: as, *I love, thou lovest, he loveth or loves*.

Also to agree with the two different numbers of the same person: as, *thou lovest, ye love; he loveth, they love*.

Likewise, to express the different times in which the action is represented: as, *I love, I loved, I have loved*. To express the different times, other verbs are often used, called auxiliaries, as will be seen hereafter.

The verb is moreover varied according to the different manner of expressing the action or being; which variations are called its modes; of which there are generally reckoned four, besides the participle: the indicative, imperative, subjunctive, and infinitive.

When a circumstance is simply declared, or when a question is asked, the verb employed on these occasions is of the indicative mode: as, *I love, he loves, lovest thou?* When any thing is commanded, or solicited, or requested to be done, be it to a superior or inferior, the verb employed in the sentence is of the imperative mode: as, *love thou, "O Thou*

my voice *inspire* !”—Pope. When any thing is expressed in a doubtful manner, or under a condition, or supposition, or the like ; or when it is subjoined as the end or design ; or when it is expressed in the form of a wish ; it is, in any of these cases, and in many others, the subjunctive mode of the verb : as, *if* I love, “ *though* he *slay* me, yet will I put my trust in him.”—Job, xiii. 15. “ Long *may* he *live*.” The verb in this mode generally depends on some other verb, and has a conjunction before it : as, “ *Though* after my skin, worms *destroy* this body, yet in my flesh shall I *see* God.”—Job, xix. 26.

Here it must be noted, that when an application is made, of the nature of a request, the verb is of the imperative mode : as, “ Lord *save* me.”—Mat. xiv. When it is of the nature of a wish, the verb is of the subjunctive mode : as, “ O King *live* for ever.”—Dan. iii. When the verb is expressed without any reference to person or number, it is called the infinitive mode : as, *to love* ; and when it is expressed in such a form, that it may be joined to an adjective, it is called the participle, and has somewhat of the nature of an adjective : as, *loving*, *loved*.

To express the different times of the verb, in English, it is often necessary to make use of other verbs, called therefore auxiliaries, or helpers ; of which there are nine : *be*, *have*, *do*, *let*, *may*, *must*, *can*, *shall*, *will*. The two first are chiefly used for forming the different times of the verb : it is, therefore, necessary to know how to decline them according to person, number, time, and mode.

TO HAVE.

INDICATIVE MODE.—*Present Time.*

<i>Singular Number.</i>		<i>Plural Number.</i>	
Persons.	1. I have,	We	} have.
	2. Thou hast,	Ye	
	3. He hath, or has;	They	

Pass

*Past Time.**Singular Number.*

Persons. { 1. I had,
2. Thou hadst,
3. He had ;

Plural Number.

We
Ye
They } had.

Future Time.

Persons. { 1. I shall, or will
2. Thou shalt, or
wilt
3. He shall, or will } have ;

We
Ye
They } shall, or
will have.

IMPERATIVE MODE.

Persons. { 1. Let me have.
2. Have thou, or do
thou have.
3. Let him have.

Let us have.
Have ye, or do ye
have.
Let them have.

SUBJUNCTIVE MODE.—*Present Time.*

Persons. { 1. If, or
though, I
2. If, or
though,
Thou
3. If, or
though,
He } have ;

If, or though,
We
If, or though,
Ye
If, or though,
They } have.

INFINITIVE MODE.

Present, To have ;—*Past*, To have had.

PARTICIPLE.

Present, Having ;—*Perfect*, Had ;—*Past*, Having had.

As the plural pronoun *you* is mostly employed instead of the singular *thou*, it must, therefore, have the plural verb : as, *you have, you had, you were* ; and not *you hast, you hadst, you was*. Which last is an erroneous solecism, being the plural pronoun of the second person, placed in agreement with the first or third person singular of the verb ; and which error writers of the first eminence have committed.

In

In the third person singular, present time, indicative mode, the termination of the verb is generally in *s*, instead of *th* or *eth*, in the polite and familiar style : as, he *has*, instead of he *hath* ; he *loves*, for he *loveth*. But in the very solemn and serious style the termination is in *th*.

TO BE.

INDICATIVE MODE.—*Present Time.*

<i>Singular Number.</i>		<i>Plural Number.</i>	
Persons.	{ 1. I am,	We	} are.
	2. Thou art,	Ye	
	3. He is ;	They	
<i>Past Time.</i>			
Persons.	{ 1. I was,	We	} were.
	2. Thou wast,	Ye	
	3. He was ;	They	
<i>Future Time.</i>			
Persons.	{ 1. I shall, or will,	We	} shall or will be.
	2. Thou shalt, or wilt,	Ye	
	3. He shall, or will,	They	

IMPERATIVE MODE.

Persons.	1. Let me be,	Let us be.
	2. Be thou, or do thou be,	Be ye, or do ye be.
	3. Let him be ;	Let them be.

SUBJUNCTIVE MODE.—*Present Time.*

Persons.	1. If I	} be ;	If we	} be.
	2. If thou			
	3. If he			

Past Time.

Persons.	1. I were,	} were.
	2. Thou wert,	
	3. He were ;	

INFINITIVE MODE.

Present, To be ;—*Past*, To have been.

PARTICIPLE.

Present, Being ;—*Perfect*, Been ;—*Past*, Having been.

The

*The Variation of the Verb active according to Person,
Number, Time, and Mode.*

TO LOVE.

INDICATIVE MODE.—*Present Time.*

Singular Number.

Plural Number.

Persons. { 1. I love;
2. Thou lovest;
3. He loveth, or loves;

We
Ye } love.
They

Past Time.

Persons. { 1. I loved;
2. Thou lovedst;
3. He loved;

We
Ye } loved.
They

Future Time.

Persons. { 1. I shall, or will,
2. Thou shalt, or wilt,
3. He shall, or will, } love;

We
Ye } shall or
They } will love.

IMPERATIVE MODE.

Persons. { 1. Let me love; Let us love.
2. Love thou, or do thou love, Love ye; or do ye love.
3. Let him love; Let them love.

SUBJUNCTIVE MODE.—*Present Time.*

Persons. { 1. I
2. Thou } love;
3. He

We
Ye } love.
They

Also,

Persons. { 1. I may
2. Thou may } love;
3. He may

We
Ye } may love;
They } and
have loved.

Past Time.

Persons. { 1. I might
2. Thou mightest } love;
3. He might

We
Ye } might love;
They } and
have loved.

Also,

Persons. { 1. I could, should,
or would
2. Thou couldst,
shouldst, or
wouldst
3. He could, should,
or would } love;
and
have
loved;

We
Ye } could,
should, or
would love;
and
have loved.
They

INFINITIVE MODE.

Present, To love;—Past, To have loved.

PARTICIPLE.

Present, Loving;—Perfect, Loved;—Past, Having loved.

In the plural number of the subjunctive mode, the imperfect and perfect times are put together. The verb in the present time, and the auxiliary of the present and past imperfect times of this mode, frequently have a future signification; as, “if he *arrive* (hereafter) we shall do well:” “if he *should*, or *would* come (to-morrow), I *might*, *would*, *could*, or *should* speak to him.” The auxiliaries *should* and *would*, in the past imperfect, are also used to express the three species of time, present, past, and future: as, “it *is* my desire that he *should* or *would* come (now or to-morrow), and it *was* my desire that he *should* or *would* come (yesterday).” Therefore, in this mode, the time of the verb is known mostly by the nature of the sentence.

The three grand divisions of time, into present, past, and future; are called indefinite, or undeterminate time; but each of these are subdivided into imperfect and perfect time, which subdivision is called definite or determined time.

Definite or determined Time.

Active Verb.	Passive Verb.	Neuter Verb.
	<i>Present Imperfect.</i>	
I am (now) loving,	I am loved,	I am entering.
	<i>Present Perfect.</i>	
I have (now) loved,	I have been loved,	I have entered.
	<i>Past Imperfect.</i>	
I was (then) loving,	I was loved,	I was entering.
	<i>Past Perfect.</i>	
I had (then) loved,	I had been loved,	I had entered.
	<i>Future Imperfect.</i>	
I shall (then) be loving,	I shall be loved,	I shall be entering.
	<i>Future Perfect.</i>	
I shall (then) have loved,	I shall have been loved,	I shall have entered.

It

It is not necessary to give the variations of the definite times, as they are formed only by the proper variations of the auxiliaries, joined to the present or perfect participle.

In the formation of these definite times, the active and neuter verbs are alike; the passive verb differs from the active, only in having the perfect participle instead of the present in the imperfect times; and having the perfect participle *been* in the perfect times.

The passive verb is formed through all its variations of person, number, time, and mode, by adding the perfect, or passive participle (both which are the same, and in all regular verbs the same as the indefinite past time active) to the auxiliary verb *to be*, through all its variations: as, *I am loved, I was loved, I shall be loved, thou art loved, &c.*; the object is placed before the verb, and is in the nominative case, and the agent follows the verb, and is in the objective case, accompanied with a preposition: as, *I am loved by him*. Here the pronoun *him* is the agent, as he performs the action, viz. *loves*, and is placed in the objective case; and the personal pronoun *I* is the object, and is placed first, and is in the nominative case.

The neuter verb is formed through all its variations like the active verb; but as it does not express any action or passion, but only a state or condition of being, it, consequently, can have no object to be acted upon: it therefore has only an agent, and that always in the nominative case: as, *I am, I sleep*. But in some instances the neuter verb has a passive form; this is in some verbs which signify a sort of motion, or change of place or condition: as, *I am risen, I was fallen, I am come, I was gone*. These verbs partake somewhat of the nature of the passive verb, though they have still the neuter signification. The auxiliary *am, was*, defines the time of the verb.

The two principal auxiliaries have already been declined through all their variations. The auxiliary *to have*, through its several variations, is placed only before the perfect parti-

ciple; *to be* is also placed before the present and perfect participle. The other auxiliaries are placed before the verb, or another auxiliary in its original form.

The nature of the other auxiliaries should be just mentioned. *Do*, past time *did*, expresses the action, or the time of it, with peculiar force. It is much used in interrogative and negative sentences. It also supplies the place of a verb, and renders the repetition of it unnecessary; as, "go and *do* thou likewise." *Let* expresses permission, but its chief use is in forming the imperative mode, which it does,

when to $\left\{ \begin{array}{l} \text{a superior,} \\ \text{an equal,} \\ \text{an inferior,} \end{array} \right\}$ by $\left\{ \begin{array}{l} \text{petition.} \\ \text{persuasion.} \\ \text{command.} \end{array} \right.$

The *possibility* of performing an action depends upon the power of the agent; and is expressed,

when $\left\{ \begin{array}{l} \text{absolute,} \\ \text{conditional,} \end{array} \right\}$ by the auxiliary $\left\{ \begin{array}{l} \text{can.} \\ \text{could.} \end{array} \right.$

The *liberty* of performing an action depends upon a freedom from all hindrances; and is expressed,

when $\left\{ \begin{array}{l} \text{absolute,} \\ \text{conditional,} \end{array} \right\}$ by the auxiliary $\left\{ \begin{array}{l} \text{may,} \\ \text{might.} \end{array} \right.$

When the agent expresses the *resolution of his own will*, to perform an action; it is expressed,

if $\left\{ \begin{array}{l} \text{absolute,} \\ \text{conditional,} \end{array} \right\}$ by the auxiliary $\left\{ \begin{array}{l} \text{will.} \\ \text{would.} \end{array} \right.$

The *necessity* of performing an action from some *external obligation*, whether it be *natural* or *moral*, and what we call *duty*; is expressed,

if $\left\{ \begin{array}{l} \text{absolute,} \\ \text{conditional,} \end{array} \right\}$ by the auxiliary $\left\{ \begin{array}{l} \text{must, ought, shall.} \\ \text{must, ought, should.} \end{array} \right.$

Some of the auxiliaries vary their import according to the person with which they are joined; thus *will*, in the first person, both singular and plural, promises or threatens; in the second and third persons only foretels: *shall*, on the contrary, in the first only foretels; in the second or third, promises, commands, or threatens. But this rule regards explicative sentences

sentences only ; for in interrogative sentences generally the reverse takes place ; thus, I *will* write, we *will* write, they *shall* write, you *shall* write ; these promise, command, or threaten : but, I *shall* write, we *shall* write, they *will* write, ye *will* write, express event only, and simply foretell. The verb *to will* is quite a different word from the auxiliary *will*, and is formed regularly : as, I *will*, thou *wilhest*, he *willeth*, or *wills*.

The auxiliaries *could*, *should*, *would*, *may*, *might*, are used in forming the subjunctive mode. The auxiliary *must* has no variation.

When two or more verbs or auxiliaries, or both, are joined together, the first only of them is varied according to person, number, time, and mode.

Before we proceed to irregular verbs, it may be necessary to say a few words concerning, what are called, by grammarians, contracted verbs.

Verbs which end in *ck*, *ck*, *p*, *n*, *ll*, *ss*, in forming the past time active, and the participle perfect or passive, often change the *d* or *ed* (which all regular verbs end in, in these forms) into *t* : as, *fetched*, *fetcht* ; *packed*, *packt* ; *snapped*, *snapt* ; *mixed*, *mixt* ; omitting also one of the double consonants ; *dwelled*, *dwelt* ; *pressed*, *prest*.

Some verbs that end in *ve*, besides the contraction, change the *ve* into *f* : as, *believe*, *belieft* ; *leave*, *left*.

Most contracted verbs have the entire as well as the contracted form ; and the entire form is always to be preferred to the contracted ; which latter is only a corruption of the verb, and an error in the rules of grammar, introduced into conversation for the sake of a more agreeable sound. This is the reason of the contraction of the verb of the second person singular, which was originally written *lovedest*, *turnedest* ; but is now contracted into *lovedst*, *turnedst*. And the third person, which was formerly *loveth*, *turneth*, is now in most styles written *loves*, *turns*.

IRREGULAR VERBS are next to be considered. In all regular verbs the indefinite past time active, and the participle perfect, or passive, are formed by adding to the verb *ed*, or *d* only when the verb ends in *e*: as, *turn*, *turned*; *love*, *loved*. Verbs which vary from this rule are called irregular verbs.

The English language being compounded of the Saxon, and Norman French, must necessarily partake of the nature of both; but the formation of all our verbs is derived from the Saxon.

All our irregular verbs are monosyllables, except the compounded ones; and are generally the same verbs which are irregular in the Saxon.

There are three classes of irregular verbs. The first class consists of those verbs which are become irregular by contraction, and either have the present and past time active, and participle perfect, or passive, all exactly alike; or else vary in the formation of the past time, and participle, from the present, by shortening the diphthong, or changing the *d* at the end of the verb into *t*. The second class of irregular verbs are those that in the past time and participle end in *ght*, and change the vowel or diphthong of the present time into *aw* or *ou*. Irregulars of the third class form the past time by changing the vowel or diphthong of the present; and form the participle, by adding the termination *en*, with generally the change of the vowel or diphthong.

Irregular Verbs of the First Class, which have the Present, Past Time, and Participle alike.

Beat,	heat*,	lift*,	read,	shed,	split,
burst,	hit,	light*,	rent,	shred,	spread,
cast,	hurt,	put,	rid,	shut,	thrust,
cost,	knit,	quit*,	set,	slit,	wet*.
cut,	let,				

* The verbs marked thus *, throughout the three classes of irregulars, have the regular as well as the irregular form in use.

The two first have also *beaten* and *bursten* in the participle; and thus they sometimes belong to the irregulars of the third class. The verb *light* is pronounced short in the past time and participle; as, I have *lit* the candle; but the regular form is preferable—*lighted*. The verb *read* is also pronounced short in the past time and participle: as, I have *red*, for *read*.

The following vary in the form of the past time and participle from the present: lead, *led*; sweat, *swet**; meet, *met*; bleed, *bled*; breed, *bred*; feed, *fed*; speed, *sped*; bend, *bent**; lend, *lent*; rend, *rent*; send, *sent*; spend, *spent*; build, *built**; geld, *gelt*; gild, *gilt**; gird, *girt*; lose, *lost*. The following are formed by contraction: have, *had*; make, *made*; flee, *fled*; shoe, *shod*. And some change the vowel also: as, sell, *fold*; tell, *told*; clothe, *clad**; stand, *stood*; dare, *durst*.

Irregular Verbs of the Second Class.

Bring,	brought.
buy,	bought.
catch,	caught.
fight,	fought.
teach,	taught.
think,	thought.
seek,	sought.
work,	wrought.

Irregular Verbs of the Third Class.

Present Time.	Past Time.	Participle.
<i>a</i> changed into <i>e</i> . Fall,	<i>fell</i> ,	<i>fallen</i> .
<i>a</i> into <i>o</i> . awake,	<i>awoke</i> ,	(<i>awaked</i>).
<i>a</i> into <i>o</i> . forsake,	<i>forsook</i> ,	<i>forsoaken</i> .
shake,	<i>shook</i> ,	<i>shaken</i> .
take,	<i>took</i> ,	<i>taken</i> .
<i>aw</i> into <i>ew</i> . draw,	<i>drew</i> ,	<i>drawn</i> .
<i>ey</i> into <i>ew</i> . slay,	<i>slaw</i> ,	<i>slain</i> .
<i>e</i> into <i>a</i> or <i>o</i> . get,	<i>gat</i> or <i>got</i> ,	<i>gotten</i> .
help,	(<i>helped</i>),	<i>holpen</i> *.
melt,	(<i>melted</i>),	<i>molten</i> *.
swell,	(<i>swelled</i>),	<i>swollen</i> .
eat,	<i>ate</i> ,	<i>eaten</i> .
<i>ae</i> into <i>a</i> or <i>o</i> . bear,	<i>bare</i> or <i>bore</i> ,	<i>borne</i> .
break,	<i>brake</i> or <i>broke</i> ,	<i>broken</i> .
		cleave

	Present Time.	Past Time.	Participle.
ee into e or o.	cleave,	clave or clove*,	cloven or cleft.
	speak,	spake or spoke,	spoken.
	swear,	sware or swore,	sworn.
	tear,	tare or tore,	torn.
	wear,	ware or wore,	worn.
	heave,	hove*,	hoven*.
	shear,	shore,	shorn.
	steal,	stole,	stolen or stola.
	tread,	trode,	trodden.
	weave,	wove,	woven.
er into e.	creep,	crope*,	crept*.
	freeze,	froze,	frozen.
	seethe,	sod,	sodden.
ee into ew.	see,	saw,	seen.
i long into i. short.	bite,	bit,	bitten.
	chide,	chid,	chidden.
	hide,	hid,	hidden.
	slide,	slid,	slidden.
e long into o.	abide,	abode,	no participle.
	climb,	clomb,	(climbed).
	drive,	drove,	driven.
	ride,	rode,	ridden.
	rise,	rose,	risen.
	shine,	shone*,	(shined).
	shrive,	shrove,	shriven.
	smite,	smote,	smitten.
	stride,	strode,	stridden.
	strive,	strove*,	striven.
i long into u.	thrive,	throve,	thriven.
	write,	wrote,	written.
i long into u.	strike,	struck,	strucken or stricken.
i short into a.	bid,	bade,	bidden.
	give,	gave,	given.
	sit,	sat,	sitten.
	spit,	spat,	spitten.
i short into u.	dig,	dug*,	(digger).
ie into ay.	lie,	lay,	lien or lain.
e into e.	hold,	held,	holden.
e into i.	do,	did,	done, from does.
ee into o.	choose,	chose,	chosen.
ew into ew.	blow,	blew,	blown.
	crow,	crew,	(crowed).
	grow,	grew,	grown.
	know,	knew,	known.
y into ew.	throw,	threw,	thrown.
	fly,	flew,	flown.

The following verbs have the participle now formed, without the termination *en*; except two of them, viz. *drunken* and *bounden*; though this was undoubtedly their original form.

	Present Time.	Past Time.	Participle.
	Begin,	began,	begun.
	cling,	clang or clung,	clung.
	drink,	drank,	drunk or drunken.
	fling,	flung,	flung.
	ring,	rang or rung,	rung.
	shrink,	shrank or shrunk,	shrunk.
	sing,	sang or sung,	sung.
	sink,	sank or sunk,	sunk.
<i>i</i> short into <i>a</i> or <i>u</i> .	sling,	slang or slung,	slung.
	slink,	slunk,	slunk.
	spin,	span or spun,	spun.
	spring,	sprang or sprung,	sprung.
	sting,	stung,	stung.
	stink,	stank or stunk,	stunk.
	string,	strung,	strung.
	swim,	swam or swum,	swum.
	swing,	swung,	swung.
	wring,	wrung,	wrung.
	bind,	bound,	bound or bounden.
<i>i</i> long into <i>ou</i> .	find,	found,	found.
	grind,	ground,	ground.
	wind,	wound,	wound.
<i>a</i> into <i>u</i> .	hang,	hung*,	hung*.
<i>oo</i> into <i>o</i> .	shoot,	shot,	shot.
<i>i</i> into <i>u</i> .	stick,	stuck,	stuck.
<i>e</i> into <i>a</i> .	come,	came,	come.
<i>u</i> into <i>a</i> .	run,	ran,	run.
<i>i</i> into <i>e</i> .	win,	won,	won.

The following are irregular in the participle only; and even then they do not change the vowel:

Lade,	(laded),	laden.
rive,	(rived),	riven.
shave,	(shaved),	shaven.
show,	(showed),	shown.
wreath,	(wreathed),	wreathen.
writhe,	(writhe),	writen.

The following also are irregular in the participle only, which they form in the same manner:

Vol. I.

D

Bake,

Bake,	hew,	owe,	shew,	strew or strow,
fold,	load,	saw,	sow,	wash,
grave,	mow,	shape,	straw,	wax.

These have also the regular as well as the irregular form ; and the regular form should always be preferred.

Besides the irregular verbs, there are also others called defective verbs ; which are not only irregular in their forms, but also wanting in some of their forms ; some have no past time, some no participle, and others neither.

Most of the auxiliary verbs are of this class.

Present Time.	Past Time.	Participle.
Am,	was,	been.
can,	could,	<i>no participle.</i>
go,	went,	gone.
may,	might,	<i>no participle.</i>
must,	<i>no past,</i>	<i>no participle.</i>
quoth,	quoth,	<i>no participle.</i>
shall,	should,	<i>no participle.</i>
weet, wit, or wot,	wot,	<i>no participle.</i>
will,	would,	<i>no participle.</i>
wis,	wist,	<i>no participle.</i>

The whole number of verbs in the English language, regular and irregular, simple and compounded, taken together, amount to about 4300 : whereof fifty-two are irregulars of the first class, eight irregulars of the second class, and 107 irregulars of the third class ; which, with the ten defective verbs, make 177 : all the rest are regular verbs, and have the past time active, and participle perfect or passive, formed alike, and ending in *d* or *ed*, as was before observed. Of the irregular verbs there are not so many as 100, which have different forms for the past time active, and the participle. This has given occasion to introduce a great corruption, by confounding the different forms of the past time and participle with each other, in irregular verbs. Thus, the participle is sometimes used for the past time : as, *he run*, for *he ran* ; *he drunk*, for *he drank* : but the past time is very frequently used instead of the participle : as, *I have*

I have rode, for I have ridden; I have sat, for sitten; I have got, for gotten. This error is frequently committed by our best writers, and the vulgar translation of the Old and New Testament, which is the best standard of the English grammar, is not free from this fault: *held* is there often used for *holden*; *hid* sometimes for *hidden*; and *bid* for *bidden*; and *begot* is twice used for *begotten*. This error is, however, an enormous solecism; and the impropriety of it will appear by the abuse of some verbs which have not been so corrupted: as, *I have saw, for I have seen; I have did, for I have done; I have went, for I have gone;* which are not a whit more ungrammatical than any other verbs which have the past time, instead of the participle, after the auxiliary *have*.

In the formation of the present participle, it is to be observed, that if the verb end in a single consonant, following a single vowel, and accented on the last syllable, if it consists of more than one syllable, it doubles the last consonant in forming the present participle; also in every other form of the verb in which a syllable is added: as, *cut, cutting, regret, regretting, regretteth, regretted.* Verbs which end in *e* omit the *e* in the present participle: as, *increase, increasing*: all other verbs form the present participle by barely adding *ing* to the verb in its original form: as, *turn, turning*.

The ADVERB, as its name imports, is added to the verb, and also to the adjective, to express some modification, or other circumstance of the action expressed by the verb, or the quality expressed by the adjective: as, the time, he reads, *now*;—distance, the countries lie *wide apart*;—relation, they are *closely* united;—quantity, an *exceeding* high mountain;—quality, to live *soberly*;—comparison, they are much *alike*;—doubt, *possibly, perhaps*;—affirmation, *yes, certainly*;—negation, *no*;—demonstration, *evidently*;—interrogation, *how, what*;—manner, *well, ill*;—order, *regularly*;—place, *here, there*;—motion, *slow, swift*.

The adverb in English has no variation, except a few, which have the degrees of comparison; and in these the

degrees of comparison should be formed by the words *more* and *most*: as, *right*, *more right*, *most right*; and not by the terminations *er* and *est*, like adjectives: as, *right*, *righter*, *rightest*: though the best writers of the seventeenth century have frequently fallen into this error.

Some adverbs, which are derived from irregular adjectives, are also irregular in their degrees of comparison; as the adverb *well* (which is derived from the adjective *good*), and has *better* and *best* in the comparative and superlative degrees.

An adverb is sometimes joined to another adverb, to qualify or enforce its meaning: as, *very well*, *much too large*.

PREPOSITIONS are placed between words, to connect them together, and to shew the relation between them.

Prepositions originally denote the relation of place, but are now used to denote other relations: as, *in*, *with*, *through*, *for*, *from*, *by*, *out*, *under*, *to*, *of*, *over*, &c. *Of* has the same meaning with *from*: learn *of* me, that is, *from* me: *for* signifies in the place or stead of another. All the others evidently convey the idea of place, according to the general meaning of the words.

Prepositions are sometimes placed before verbs, and joined to them so as to form but one word; in which case they always alter the sense of the verb: as, *to stand*, signifies a posture—*to understand*, intends comprehension: also, *to go*, *to outgo*; *to look*, *to overlook*. Sometimes they follow the verb, and are not joined to it; when they no less alter the meaning of the verb: as, *to give*, *to give up*; *to cast*, *to cast off*, *to cast down*, &c.

The preposition *on* is sometimes converted into an *a*, and that chiefly before the present participle: as, *a walking*, *a going*, &c. which are evidently derived from the phrases, I was *on* walking, I was *on* going; that is, employed *on* that action. Also, twelve *a* clock, as it is commonly pronounced, but written twelve o'clock, was originally, twelve *on* the clock.

CON-

CONJUNCTIONS are used to connect sentences together ; so as from two or more simple sentences to form one, which is called a compound sentence.

What a simple sentence is has been shewn in page 12. A compound sentence is formed of two or more such simple sentences : as, " John *and* Thomas love their book, *but* Edward is a dunce." This is a compound sentence, formed by uniting the three following simple sentences together, by the conjunctions *and* and *but* ; John loves his book ; Thomas loves his book ; Edward is a dunce.

Conjunctions are principally divided into copulative conjunctions, and disjunctive conjunctions. They both serve to connect the sentence ; but the disjunctive conjunction expresses an opposition in the sense, as has been seen by the conjunction *but*. Also the conjunctions *or*, *than*, *except*, *unless*, *although* (though), *yet*, *nevertheless*, &c. are disjunctives.

INTERJECTIONS, though reckoned one of the parts of speech, are only a kind of natural sounds, thrown into a sentence by the speaker, as the result of his feeling, and to express his own affections : as, *O ! alas !* &c.

When the interjection *O* is placed before a substantive, it shews that an address is made to that particular person or thing of which the substantive is the name ; and the substantive is, in what in Latin is called, the vocative case.

SECT. IV.

OF SYNTAX.

SYNTAX is the right ordering and framing of words, in order to form sentences with grammatical propriety. And for this purpose, words are said to govern, or agree with each other.

When a word governs another, it causes that particular word which it governs, to be in such a particular number, gender, case, person, time, or mode.

When a word agrees with another, it is in that particular mode, number, case, &c. which is required by the word that governs it. Thus: "And she shall bring forth a Son, and thou shalt call his name Jesus; for he shall save his people from their sins."—Matt. i. 21. In this sentence *she* is a pronoun of the third person, singular, feminine gender, and ought to agree with the subject foregoing, namely, the Virgin Mary;—*shall bring*, the future time of the active verb *bring* (referring to the time of the birth of the child), the third person, singular number, to agree with the pronoun *she*, and indicative mode, as it simply declares the event;—*forth*, a preposition added to the verb *bring*, and which alters its meaning from *bring*, to *bring forth*, which signifies to bear or produce;—*a*, the indefinite article;—*Son*, noun substantive, singular number, masculine gender, objective case;—*and*, a conjunction copulative, connecting the following and foregoing sentences together;—*thou*, the pronoun of the second person, singular, and agent of this sentence;—*shalt call*, the future time of the active verb *call*, second person, singular, being governed by *thou*, indicative mode, as it only foretels or declares the name of the child, but does not command it;—*his*, a possessive pronoun, third person, singular, masculine gender, governed by the noun *Son*;—*name*, a substantive common,

common, and the object of the sentence ;—*Jesus*, a substantive proper, masculine gender, agreeing with the noun *Son*, and nominative case ;—*for*, a copulative conjunction ;—*he*, a pronoun, third person, singular, masculine gender, nominative case, being a substitute for the noun *Jesus*, and governed by it, and the agent of this sentence ;—*shall save*, the future time of the active verb (agreeing in time with the other verbs in the sentence, which are all future); third person, singular, governed by the pronoun *he*, indicative mode ;—*his*, a possessive pronoun ;—*people*, a plural noun, neuter gender, objective case ;—*from*, a preposition, shewing the relation between the nouns *people* and *sins* ;—*their*, a possessive pronominal adjective, and as such joined to the plural noun *sins*, objective case.

SENTENCES are either simple or compounded.

A simple sentence hath but one subject, or agent, and one verb in the indicative, imperative, or subjunctive mode ; and consists of three parts, if the verb be active ; the agent, the attribute, and the object : as was seen page 12.

A compound sentence consists of two or more simple sentences united together by the aid of conjunctions, as hath been shewn before : or by relatives, as will be seen hereafter.

As language and style is only an assemblage of sentences, too much attention can hardly be bestowed upon their construction. We will therefore take a view of the rules of English Syntax, as they regard the several parts of speech respectively.

The ARTICLE, as hath been seen, if definite, is placed before both the singular and plural noun : as, *the man*, *the men* : the indefinite article *a* is placed before the singular noun only : as, *a man*.

The SUBSTANTIVE governs both the pronoun and the verb : for if the substantive be plural, it requires both the pronoun and verb to be plural also ; and if singular, they must also be singular : as, “ nothing has so much exposed *men* of learning to contempt and ridicule, as *their* ignorance of things which *are* known to all but *themselves*.”—Johnson’s Rambler.

Rambler. Example of the singular: "*Seneca speaks* in the natural and genuine language of a man of honour, when *he* ~~is there~~, that were there no God to see or punish vice, *he* ~~would~~ not commit it."—Guardian.

There are some nouns called nouns of multitude, which signify many, and have the pronoun and verb agreeing with them, either in the singular or plural number; but if they convey a plural idea, the verb and pronoun should be plural likewise; if they convey a singular idea, the verb and pronoun should be singular: as, "the *assembly* of the wicked *have* enclosed me"—Psalm xxii. 16. instead of *hath* enclosed me. "My *people* *is* foolish"—Jer. iv. 22. instead of *are* foolish.

Two or more singular nouns, joined by copulative conjunctions, have the verbs, nouns, and pronouns agreeing with them in the plural number: as, "Shakespeare and Milton *were* the most eminent *poets* of the English nation."

But sometimes the verb follows in the singular number, and refers to each of the preceding nouns taken separately; as, "*grain, and salt, and a mass of iron* *is* easier to bear than a man without understanding."

Nouns of number, weight, and measure, are often used in the singular form, when they are joined to numeral adjectives; though they denote plurality: as, an hundred *thousand*, instead of *thousands*; an hundred *pound* weight; six *foot*; forty *fathom*.

Nouns, whether of the masculine, feminine, or neuter gender, always govern the same gender in the pronoun: as, "I have a *friend*, *who*, because *he* knows *his* own fidelity and usefulness, is never willing to sink into a companion." "I have a *wife*, whose beauty first seduced me, and whose wit confirmed *her* conquest."—Johnson's Rambler.

Every noun in the nominative case belongs to some verb, either expressed or understood, except the case absolute, as will be seen hereafter; and when an address is made to a person, called the vocative case. Thus in the answer to this question, Who conquered the Persians?—*Alexander*; that is, Alexander *conquered them*.

Every noun in the possessive case has also some noun belonging to it: as, St. Paul's, that is, St. Paul's *Cathedral*; St. James's, St. James's *Palace*.

The noun has no different form in English for the objective case, though this case be founded in nature and grammar: as, "*books* can never teach the use of *books*."

The PRONOUN, being a substitute for the noun, must consequently have the same nature, with regard to the government and agreement of nouns and verbs.

When two or more pronouns of the singular number are joined together, to make the plural pronoun agree with them in person, the second person is preferred before the third, and the first person before both: as, *he* and *you* did as *you* were commanded; *I*, *thou*, and *he* lost *our* characters by it.

The neuter pronoun *it* is employed in a threefold sense:—first, it expresses the subject of any discourse: as, *it* so happened; who is *it*?—secondly, the state of any person or thing: as, how is *it* with you?—thirdly, the thing that is the cause of any event: as, *it* was I that did it; *it* is these that corrupt the mind.

PRONOMINAL ADJECTIVES have some substantive belonging to them, either expressed or understood.

The definitives *this*, *that*, in the plural *these*, *those*, must always agree with their substantive in number: thus, "By *this* means thou shalt have no portion on this side the river." Ezra, iv. 16. "I have wept not *this* forty years." It should be *these* means; *these* forty years. Again, "*They* are *these* kind of gods which Horace mentions in his allegorical vessel."—Addison's Dialogues, ii. on Medals. Here it should be *these* kinds, or *this* kind.

The relatives *who*, *which*, *that*, have no variation of gender, and therefore must agree with their antecedent in this respect. For every relative must have a noun or pronoun, to which it refers, either expressed or understood; which is therefore called its antecedent: as, "*Who* steals my purse, steals trash."—Shakespeare. That is, the *man who* steals my purse, steals trash.

Who is applied to persons only, and is either masculine or feminine; *which* is now applied to things only, and irrational animals; *that* is used both for persons and things, but it should be confined to the latter, particularly in the solemn style; *what* stands for both the relative and the antecedent: as, this is *what* was spoken of before; that is, the *thing which* was spoken of before.

The relative is the nominative case of the verb, when no other nominative comes between it and the verb; but if another nominative comes between it and the verb, the relative is governed according to the sense of the sentence, as, “the God *who* made me, *whose* I am, and *whom* I ought to serve.” In the different members of this sentence, the relative is used in a different sense; in the first member it stands for the nominative case of the verb, having no other nominative case between it and the verb; in the second member it stands for the possessive case; in the third member it represents the object.

Every relative is of the same person with the antecedent to which it refers, and the verb must therefore agree with it: as, “*I that* (who) speak unto thee *am he*.” John, iv. 22. “*Thou that* (who) dwellest between the Cherubims.” Psalm lxxx. 1.

The relative is often omitted, and understood, without being expressed; as, “the God I serve;” that is, the God *whom* I serve. The relative should seldom be omitted even in the familiar style, and never in the serious and solemn styles.

The proper use of the relative consists in the property of presenting the antecedent to the mind of the hearer or reader, without any ambiguity.

The distributive pronominal adjectives *each*, *every*, *either*, always agree with the nouns, pronouns, and verbs in the singular number only: as, “The king of Israel, and Jehosaphat the king of Judah, *sat each* (king) on *his* throne, having (both) put on their robes.”—1 Kings, xxii. 10. “*Every one* ought to cherish and encourage in *himself* this modesty and assurance I have here mentioned.”—Spectator. *Each* signifies

ties *both* taken distinctly or separately; *either* signifies *only the one or the other*, taken disjunctively.

The ADJECTIVE, having no variation of number or gender, must agree with its substantive.

The adjective is always placed immediately before the noun: as, a *good man*; except in the following instances: first when something depends upon the adjective: as, "feed me with *food convenient* for me." Here the relative and the verb are understood: as, food *that is* convenient.—Secondly, when the adjective is emphatical: as, George the *Third*; St. John the *Divine*.—Thirdly, when the verb *to be*, or any auxiliary belonging to it, is placed between the adjective and the noun: as, "How *beautiful are* the tabernacles of the Lord of Hosts!" *happy shall* he be.—Fourthly, when two or more adjectives belong to the same noun: as, "a general *brave and skilful*."

The article is placed mostly before the adjective, except the adjectives *all*, *such*, and *many*: as, *all the men*; *such a man*; *many a man*. Or when an adjective is joined to the adverbs *so*, *as*, *how*: as, not *so large a concern*; *as good a man*; *how fine a sight is this*!

Every adjective has some substantive belonging to it, either expressed or understood: as, "the *twelve*," for the twelve *apostles*.

Sometimes the adjective is used as a substantive, and has an adjective joined to it: as, the *greatest evil*, the *chief good*. At other times the substantive becomes an adjective, and has a substantive added to it, and linked to it by a mark of conjunction: as, *animal-food*, *milk-diet*.

The VERB, or attribute of a sentence, always agrees with the agent or nominative case.

For every verb, except the participle or the infinitive mode, hath a nominative case belonging to it, either expressed or understood: as, "*awake, arise, or be* for ever fallen;" that is, *awake ye, arise ye, &c.* And when the verb is active, it hath, moreover, an objective case: as, "God made *man*."

The nominative case in English is placed before the

verb, and the objective case after the verb: except, first, when the agent or object is expressed by a pronoun, in which case they are often reversed in their order: as, "*Whom* ye ignorantly worship, *him* declare I unto you."—Secondly, when the verb is neuter, the nominative case is sometimes placed after it: as, "presently entered the *troops*."—Thirdly, when the adverb *there* is connected with the neuter verb: as, "*there* was a *man* in the land of Uz." Job, i. 1.—Fourthly, when the relatives are used, though in the objective case, they are always placed before the verb, as are also their compounds, *whoever*, *whichever*, &c. as, "*whomsoever* you choose;" "this is he *whom* you seek."—Fifthly, when a sentence is interrogative, or when a question is asked, the nominative follows the principal verb, or the auxiliary: as, *lovest thou* me? or *dost thou* love me?—Sixthly, when the imperative mode is employed, or when any thing is commanded or requested to be done, the nominative case follows the verb or auxiliary: as, *come thou* here, or *do thou* come here; or the auxiliary *let* with the objective case is used: as, *let me* go; *let us* be gone.—Seventhly, when the conjunctions *if* or *though*, are omitted, the nominative case sometimes follows the auxiliary or the verb, and the verb is in the subjunctive mode: as, "*had he* done this he had been right;" "*sing* *she* ever so sweetly."

The neuter verb expressing no action, but only a state or condition of being, it consequently can have no objective case expressing the object of an action. Whenever a noun follows a neuter verb, it either expresses the same idea with the verb, or some circumstance of the verb: a preposition being understood: as, *to fight a duel*; *to ride a mile*; that is, the space of a mile.

When the pronoun follows the neuter verb *to be*, it should always be in the nominative case: as, *it was I*; *it was he*. Except the verb *be* in the infinitive mode; as, *it was thought to be him*. Thus, "*Whom* say ye that I *am*"—Matt. xiv. is improper; it ought to be, *who* say ye, &c.

When a verb immediately follows another verb, the last
verb

verb should be in the infinitive mode : as, boys *love to play* : but the following verbs admit of other verbs following them without being in the infinitive mode : *bid, dare, need, make, see, hear, feel*, and all the auxiliaries, except *be* and *have*, which have already been spoken of.

The infinitive mode has the nature of a substantive, expressing the action itself in the abstract, without regard to any person as an agent. Thus the infinitive is employed as a substantive in the nominative case : as, *to teach* is a master's duty ; and in the objective case : as, mankind love *to learn*.

The preposition *for*, placed before the infinitive mode, is now become obsolete.

The participle has the nature of an adjective, as the infinitive mode has that of a substantive ; and the participle is sometimes joined to a substantive, like an adjective, merely to denote its quality ; and admits of the degrees of comparison : as, *a learned man, a more learned man, a most learned man* : and the present participle is varied in the same manner : as, *a loving father, a more loving father, a most loving father*.

When the participle has an article before it, and the preposition *of* after it, it then has the nature of a substantive, expressing the action itself which the verb signifies : as, “ by *the preaching of* repentance,” and not by preaching of repentance, as is erroneously written in the Collect for St. John the Baptist. For when the participle has the article before it, the preposition should always follow it ; and when the preposition follows it, the article should precede it. But the participle may have a preposition *before* it ; which answers to what is in Latin called the gerund : as, “ Justice consists *in punishing* the guilty ; and *in protecting* the innocent.”

The case absolute is, when a sentence is formed without a finite verb, by the aid of the participle, or infinitive mode. This case is in English always the nominative ; and the adverbs of time, *when, while, after, &c.* which should otherwise be inserted, are omitted : as, “ the doors *being* shut,
Jesus

Jesus stood in the midst;" instead of, *when* the doors *were* shut, &c.

The infinitive mode is often made absolute, when it supplies the place of the conjunction *that* with the subjunctive mode: as, "*to begin* with the first;" "*to conclude*;" that is, *that I may begin* with the first; *that I may conclude*.

The participle is also made absolute in the same manner as the infinitive mode: as, "this, *strictly speaking*, is the sense;" that is, this, *if I may speak* strictly, is the sense.

ADVERBS have no government. They should always be placed as near as possible to the word which they modify.

Their place is generally before adjectives, after active and neuter verbs, and sometimes between the auxiliary and the verb: as, "He was a *very* wise prince; he governed *mildly*; and was *reverentially* beloved by his subjects."

When two negatives occur in one sentence, they destroy one another, and are equal to an affirmative: as,

Nor did they *not* perceive the evil plight
In which they were, or the sharp pains *not* feel.

MILTON, P. L. i. 335.

There is no adverb so liable to be misplaced as the adverb *only*: most of our classical writers have erred in placing this adverb at too great distance from the word which it modifies: thus, it is commonly said, "they *only* fought *one hour*;" whereas it should be, they fought *only* one hour.

PREPOSITIONS have a government of case, and in English they always require the objective case after them: as, *with* her, *from* me, *to* him, *by* us, &c.

When the preposition governs a relative, it should be placed before the relative; as, "this is he *of whom* it is written;" though some writers have placed the preposition at the end of the sentence, and separate from the relative, which is always ungraceful, and can hardly be reconciled to the rules of grammar: as, "Horace is an author *whom* I am much delighted *with*."

Many

Many of our best writers have committed errors in the use of the preposition, some omitting it entirely when it should be used, others using one preposition for another: as, *into* for *in*, *to*, or *unto*; *for* for *of*; *of* for *on*, &c. as may be seen by an attentive perusal of the classical writers in the English tongue.

The noun mostly requires after it the same preposition as the verb from which it is formed would do: as, "the wisest princes need not think it any diminution *to* (of) their greatness, or derogation *to* (from) their sufficiency, *to* rely upon counsel."—Bacon, Essay 20. Here the nouns *diminution* and *derogation*, being formed from the verbs *diminish* and *derogate*, require the prepositions *of* and *from*.

The prepositions *to* and *for* are often understood, without being expressed, before the objective case of the pronoun: as, get *me* a place; pay *him* the money you owe *them*; that is, get *for* me a place; pay *to* him the money you owe *to* them." This is a relic of the Saxon: in which language, those pronouns which are in the objective case in English, are in the dative case; and consequently have the prepositions *to* and *for* understood. *In* or *on* is also often omitted before nouns expressing time: as, last week; next year; to-morrow: that is, *in* last week; *in* next year; *on* to-morrow.

The preposition subjoined to the adverbs *here*, *there*, *where*: as, *herewith* or *hereof*, *therefore*, *therewith*, *wherewith*, *whereupon*, &c. have the construction or nature of a pronoun; but they are now almost obsolete, except in the very solemn style only.

CONJUNCTIONS have only a government of mode. Some require the indicative, others the subjunctive mode.

The following govern the subjunctive: first, hypothetical conjunctions: as, "if thou *be* the Son of God." Matt. iv. 3. —Secondly, conditional conjunctions: as, "though he *say* me, yet will I put my trust in him." Job, xiii. —Thirdly, distributive conjunctions; "whether it *were* I, or (whether it were) they, so we preach." 1 Cor. xv. 11. —Fourthly, concessive conjunctions;

conjunctions + "Take heed ye will us hear." Lev. xii. 6.—
 Finally, comparative conjunctions. "as power, when it were
 given from above."—Finally, the conjunctions *lest* and *that*,
 introducing a verb in the imperative mood: as, "Let him that
 loveth his wife let it fall." Gen. ii. 24. "Take heed
 that thou dost not to sin." Gen. xlii. 24.

There must be noted that all the foregoing conjunctions
 may be used and often are with the indicative mood; when
 the circumstance expressed is of a more absolute and certain
 nature. It is the circumstance being of a doubtful nature,
 or expressing a wish or condition, or supposition, as in the form
 of a wish that determines the verb to be in the subjunctive
 mood.

Other conjunctions of a more positive nature govern the
 indicative mood.

There are some conjunctions which have other conjunc-
 tions belonging to them, and answering to them: as, *although*
 (*though*), *yet*, *nevertheless*; *whether*—*or*; *either* or *or*—*and*;
either—*or*; *as*—*as*; *as*—*if*; *if*—*so*. *As*—*as* expresses a com-
 parison of equality; "as good as another," that is, equal in
 force or every quality; *as*—*if* expresses a comparison sometimes
 of equality; "as he is, *if* shall we be;" sometimes a compa-
 rison of quality; "as the one dieth, *if* dieth the other;" that
 is, in like manner; *if*—*so* expresses both quality and quantity:
 as, "Pope was not *if* sublime a poet as Milton, nor *if* great
 a man as Johnson;" *whether*, *or*, and *or*, *or*, expresses a
 double negative, as in the foregoing example; *either*—*or*, a
 double distributive, as, *either* choose ye this *or* that.

In comparative sentences, or when different qualities are
 compared, the case of the latter noun or pronoun is governed
 by the verb or preposition, which are sometimes not ex-
 pressed: as, thou art a greater man than *he*; that is, than *he*
is; you shew him more favour than *me*; that is, than (*you*
shew) me. Thus, by supplying that part of the sentence
 that is understood, the case of the latter noun or pronoun will
 be understood.

INTERJECTIONS in English have no government, nor any particular place in a sentence; they are used only to express the speaker's affection; but when they are used too frequently, they rather expose his affectation.

These are the rules of English syntax, in the construction of sentences. But there are also phrases, consisting of two or more words, employed in the formation of sentences: of which the following are the most common:

1st Phrase: when a substantive or pronoun is placed before a verb active, passive, or neuter: as, *I love*; *he is loved*; *thou art*.

2d Phrase: when the substantive follows the verb neuter, or passive; or when the substantive following the verb intends the same thing as the substantive before the verb: as, *I am he*; *Milton is esteemed a classic*.

3d Phrase: when the adjective follows a verb neuter, or passive: as, *life is short*; *exercise is esteemed wholesome*.

4th Phrase: when the substantive follows a verb active; as, *to build a house*.

5th Phrase: when a verb follows another verb: as, *they desire to die*.

6th Phrase: when one thing is said to belong to another: as, *Milton's poems*; or *the poems of Milton*.

7th Phrase: when a substantive is added to another, to explain it more fully: as, *St. John the Baptist*; *King George*; *Robert Brown, &c.* Here the latter noun is said to be in apposition to the former.

8th Phrase: when the adjective or participle is placed before the noun: as, *a fine day*; *a loving friend*.

9th Phrase: when the adjective is placed before a verb in the infinitive mode: as, *worthy to live*.

10th Phrase: when an adverb is used with an adjective, or a verb: as, *he writes quick*; *he is very just*.

11th Phrase: when a substantive with a preposition before it, is added to a verb or an adjective: as, *he acts with prudence*; *good for nothing*.

12th Phrase: when the quality of a subject is compared with that of another subject; the positive adjective having after it the conjunction *as*; the comparative the conjunction *than*; the superlative the preposition *of*: as, as tall *as* you; taller *than* he; tallest *of* all.

SECT. V.

OF PUNCTUATION.

PUNCTUATION is the art of making the several points, used in sentences, to express the degrees of connexion between sentences and their parts; and to express the stops, or pauses, as they are expressed in a just and accurate pronunciation.

Notwithstanding the different pauses in sentences, and degrees of connexion between them, admit of great variety, yet we have but four points by which to express them.

Thus we are often obliged to express pauses of the same quantity, on different occasions, by different points; but more frequently to express pauses of different quantities by the same points.

The doctrine of punctuation must therefore be very imperfect: few rules can be given that will hold good in all cases; but much must be left to the judgment of the writer.

Grammarians have followed the division of the Rhetoricians, who divide all the pauses in writing or discourse into the four following:

The Period, marked thus	-	(.)
The Colon, marked thus	-	(:)
The Semicolon, marked thus	-	(;)
The Comma, marked thus	-	(,)

The

The Period marks a whole sentence, either simple or compound, making a full and perfect sense, and not connected in construction with another sentence.

The Colon marks the greatest division of a sentence, and is a member thereof; containing a perfect sense, but not a perfect sentence.

The Semicolon is a less constructive part of a sentence than the colon, and does not form a perfect sense, but holds a middle place between the colon and the comma, being a greater pause than the latter, and less than the former.

The Comma marks the least constructive parts of a sentence; or it marks a simple sentence.

The precise quantity of time required at each of these pauses or points is uncertain; as the same composition may be rehearsed in a longer or shorter time; but the proportional quantity of time of the points is as follows: the period is a pause of double the quantity of time of the colon; the colon is double of the semicolon; the semicolon is double of the comma.

In order to discover the proper use of these points, we must consider the nature of a sentence, as divided into its constructive parts, and the degrees of connexion between these parts: as also the nature of an imperfect phrase.

An imperfect phrase contains no assertion, or does not amount to a proposition or sentence, as was seen in page 41.

A simple sentence, as was before hinted, consists of an agent, or subject, an attribute, and (if the verb be active or passive) an object: or it consists of one agent or subject, and one finite verb; that is, a verb in the indicative, imperative, or subjunctive mode: as, "God made man;" here *God* is the agent or subject, as he performs the action, viz. *made man*; the verb *made* is the attribute which always expresses the action; and *man* is the object upon which the action is exercised; that is, the action of creation.

But the subject, or agent, and the attribute, and the object, may each of them be accompanied with several circum-

stances or characteristics, called therefore their adjuncts; as the motive, place, time, manner, cause, and the like: and these may be connected either immediately, or mediately, to the parts of the sentence to which they belong.

If the several adjuncts are related to the parts of a sentence in a different manner, they are then only so many imperfect phrases, and the sentence is simple.

But if the several adjuncts belong to the parts of a sentence in the same manner, they then become so many simple sentences; and the sentence is then compound.

For a compound sentence consists of two or more simple sentences connected together; or it hath more than one subject, and one finite verb.

Thus, if several subjects belong in the same manner to one verb, or several verbs belong in the same manner to one subject, the subjects and verbs are to be accounted equal in number: for every subject, except the case absolute, and the vocative case, must have its verb: and every finite verb its subject; and generally has its point of distinction.

Examples of the Use of the Comma.

“ This single consideration will be sufficient to extinguish all envy in inferior natures.” In this sentence *consideration* is the subject or agent, *extinguish* the attribute or verb, *envy* the object: each of which is connected with its adjuncts. The subject does not mean any consideration indefinitely, but a particular consideration, treated of in the former part of the Essay (namely, the consideration of the progress of a human soul towards perfection in knowledge), and here defined by the adjunct, *this single* (consideration). The attribute or verb is also connected with its adjuncts; immediately with *envy*, as the object of the action; and mediately by the intervention of the word *inferior natures*, the subject in which envy is extinguished; the adverb *sufficient* is the adjunct of the verb, denoting its power. It is to be observed, that each of these adjuncts belong to the verb in a different

different manner; *envy* belongs to it as the object; *inferior natures* as the subject in which the object is extinguished; and *sufficient* as the power of the verb to produce the effect. The adjuncts are, therefore, only so many imperfect phrases; and the sentence is simple, and admits of no points to distinguish it into parts.

“Methinks this single consideration, of the progress of a finite spirit to perfection, will be sufficient to extinguish all envy in inferior natures, and all contempt in superior.” Here two new sentences are introduced; the one inserted in the middle of the former sentence, and the other added to the end of it. The former of them, *of the progress of a finite spirit to perfection*, is connected with the attribute in the same manner as the agent, or *this single consideration*, for they both express the same idea; and the verb may be joined with either of them, and the sentence have the same sense: as, “the progress of a finite spirit to perfection, will be sufficient to extinguish all envy in inferior natures.” The latter sentence, *and all contempt in superior*, is connected exactly in the same manner with the attribute as the object in the first sentence is, though it does not convey the same idea; and may be made the object of the sentence in the same manner as the former sentence was made the agent; as, “Methinks this single consideration will be sufficient to extinguish all contempt in superior.” (beings) The first of these newly inserted sentences, may therefore be considered as only another agent; and the latter as another object to the verb, in the first-mentioned sentence. They are therefore so many simple sentences, and should each be distinguished by a comma; and the whole is a compounded sentence.

And again, “A wise man will desire no more than what he may get justly, use soberly, distribute cheerfully, and live upon contentedly.” In this sentence, the phrases, *get justly, use soberly, distribute cheerfully, and live upon contentedly*, are each connected with the subject, *he*, in the same manner, and in effect, form so many distinct sentences; as, “a
wise

wise man will desire no more than what *he may get justly* :
 a wise man will desire no more than what *he may use soberly* :
 a wise man will desire no more than what *he may distribute*
cheerfully : a wise man will desire no more than what *he may*
live upon contentedly. They must each of them, therefore,
 be distinguished by a comma. They are so many simple
 sentences, and the whole is a compound sentence.

As sentences themselves are divided into simple and compound, so the members of sentences may be divided into simple and compound members: for whole sentences, whether simple or compound, may become members of other sentences, by means of a connexion.

The following are marked by a Comma :

First, simple members of sentences, closely connected together in one compound member or sentence: as in the foregoing examples: except, first, when the members are short in comparative sentences: as, in the former part of the last example, "a wise man will desire no more than what he may get justly." There are, in fact, two simple sentences *compared* by the construction *than*: but, being short, they are not separated by a comma. Secondly, when two simple members or sentences are closely connected by a relative, and the subject of the antecedent is related to a particular sense: as, "the man who is master of this excellent frame of mind, is not only easy in his thoughts, but a perfect master of all the powers and faculties of his soul."—Spectator. In this sentence, *the man*, is connected to the following sentences by the relative *who*: which retains the idea of the antecedent to the sense here mentioned.

Secondly, the case absolute: as, "the doors being shut, Jesus stood in the midst."

Thirdly, nouns in apposition, when consisting of many terms: as, "Alexander, the great, cruel, and unjust."

Fourthly, the vocative case, or when an address is made to a person: as, "This said, he formed thee, Adam; thee, O man."—Milton.

Fifthly, when there are more than two nouns, or adjectives, connected by copulatives or disjunctives; or when there are only two, if the conjunction be understood: as, “Raptures, transports, and ecstasies are the rewards which they confer: sighs and tears, prayers and broken hearts, are the offerings which are paid to them.”—Spectator.

Sixthly, a circumstance of importance, though only an imperfect phrase: as, “the principle may be defective or faulty; but the consequences it produces are so good, *that*, for the benefit of mankind, it ought not to be extinguished.” Spectator.

Lastly, the participle with something depending on it: as,

“Now morn, her rosy steps in th’ eastern clime
Advancing, sow’d the earth with orient pearl.” MILTON.

The SEMICOLON is used when a sentence, or member of a sentence, requires a greater pause than a comma, yet neither forms a perfect sense, nor a perfect sentence; but is followed by some other member, or sentence, with which it is closely connected: as,

“To look upon the soul as going from strength to strength, to consider that she is to shine for ever with new accessions of glory, and brighten to all eternity; that she will be still adding virtue to virtue, and knowledge to knowledge; carries in it something wonderfully agreeable to that ambition which is natural to the mind of man.”—Spectator.

This compound sentence is divided into three principal parts by the semicolon; each part requires a greater pause than a comma; but neither of them expresses perfect sense, or forms a perfect sentence, being closely connected in sense with each other.

The COLON is used when a sentence, whether simple or compound, requires a greater pause than a semicolon: it always forms a perfect sense, and would by itself form a complete sentence, but is followed by another member, or sentence, making the sense more full and complete: as,

“Were

“ Were all books reduced to their quintessence, many a bulky author would make his appearance in a penny paper : there would be scarce any such thing in nature as a folio : the works of an age would be contained on a few shelves : not to mention millions of volumes, that would be utterly annihilated.”—Spectator, No. 124.

This sentence is divided into four parts by the colon : the first and last parts are compound members, divided by the comma ; the second and third are simple members.

Each of these parts contains perfect sense, and would also form a complete sentence, if the other parts had not been joined to it.

The colon is also used when a semicolon goes before, and a greater pause is required ; though the sentence be not complete ; also when a speech or an example is introduced.

The PERIOD is used when a sentence is so far finished as not to be connected in construction with the following sentence ; and marks both a perfect sense, and a perfect sentence ; as,

“ Man, considered in his present state, seems only sent into the world to propagate his kind. He provides himself with a successor, and immediately quits his post to make room for him.”—Spectator.

These are two perfect sentences, divided from each other by the period, and subdivided into simple sentences by the comma. The first of these two sentences represents the *apparent* end of man's mission into this present world ; namely, to propagate his kind : the second shews the *real consequence* of his residence here ; begetting a successor, and making room for him. They are entirely distinct sentences ; unconnected in construction with each other, and wanting nothing to make each perfect both in sense and sentence.

Beside the four foregoing points used to express the pauses in discourse, there are three others, which also affect the modulation of the voice : namely,

The Note of Interrogation, marked thus (?)

The Note of Admiration, or Exclamation, marked thus (!)

The Parenthesis, marked thus ()

THE

The **NOTE OF INTERROGATION** is used when a question is asked : as, "Would an infinitely wise Being make such glorious creatures for so mean a purpose? Can he delight in the production of such abortive intelligences, such short-lived reasonable beings? Would he give us talents that are not to be exerted? capacities that are never to be gratified?" Spect. These are all interrogative sentences, and, as such, are marked by the interrogation point.

The **NOTE OF ADMIRATION, or EXCLAMATION**, is used to mark the speaker's admiration, and any sudden passion : as,

"And see!

'Tis come, the glorious morn! the second birth
Of Heaven and earth!"

THOMSON'S WINTER.

Both these points mark an elevation of the voice. The length of time required by them as pauses is uncertain, and may be equal to that of a semicolon, colon, or a period.

The **PARENTHESIS** is used to enclose a word or sentence, inserted in another sentence; which is not necessary to the sense, nor does it at all affect the construction : as, "Pompey on the other side (that hardly ever spake in public without a blush) had a wonderful sweetness of nature." This point marks a moderate depression of the voice, with a pause somewhat greater than a comma.

There are also other marks used in writing, which, though they do not affect the voice as pauses, yet are often necessary to the sense : the principal of which marks we shall just mention.

The **APOSTROPHE**, marked thus ('), is only a comma placed at the head of a letter, and signifies either that some letter or letters are left out for a quicker pronunciation; as, *I'll*, for *I will*; *he don't*, for *he does not* : or else it marks the possessive case : as, *John's book*.

The **ACCENT** (') is placed over a vowel to denote that the word is accented on that syllable; as, *Com-pre-hén-sion*.

“ Were all books reduced to their quintessence, many a bulky author would make his appearance in a penny paper : there would be scarce any such thing in nature as a folio : the works of an age would be contained on a few shelves : not to mention millions of volumes, that would be utterly annihilated.”—Spectator, No. 124.

This sentence is divided into four parts by the colon : the first and last parts are compound members, divided by the comma ; the second and third are simple members.

Each of these parts contains perfect sense, and would also form a complete sentence, if the other parts had not been joined to it.

The colon is also used when a semicolon goes before, and a greater pause is required ; though the sentence be not complete ; also when a speech or an example is introduced.

The **PERIOD** is used when a sentence is so far finished as not to be connected in construction with the following sentence ; and marks both a perfect sense, and a perfect sentence : as,

“ Man, considered in his present state, seems only sent into the world to propagate his kind. He provides himself with a successor, and immediately quits his post to make room for him.”—Spectator.

These are two perfect sentences, divided from each other by the period, and subdivided into simple sentences by the comma. The first of these two sentences represents the *apparent* end of man's mission into this present world ; namely, to propagate his kind : the second shews the *real consequence* of his residence here ; begetting a successor, and making room for him. They are entirely distinct sentences ; unconnected in construction with each other, and wanting nothing to make each perfect both in sense and sentence.

Beside the four foregoing points used to express the pauses in discourse, there are three others, which also affect the modulation of the voice : namely,

The Note of Interrogation, marked thus - (?)

The Note of Admiration, or Exclamation, marked thus (!)

The Parenthesis, marked thus - ()

THE

“Were all books reduced to their quintessence, many a bulky author would make his appearance in a penny paper : there would be scarce any such thing in nature as a folio : the works of an age would be contained on a few shelves : not to mention millions of volumes, that would be utterly annihilated.”—Spectator, No. 124.

This sentence is divided into four parts by the colon : the first and last parts are compound members, divided by the comma ; the second and third are simple members.

Each of these parts contains perfect sense, and would also form a complete sentence, if the other parts had not been joined to it.

The colon is also used when a semicolon goes before, and a greater pause is required ; though the sentence be not complete ; also when a speech or an example is introduced.

The PERIOD is used when a sentence is so far finished as not to be connected in construction with the following sentence ; and marks both a perfect sense, and a perfect sentence ; as,

“Man, considered in his present state, seems only sent into the world to propagate his kind. He provides himself with a successor, and immediately quits his post to make room for him.”—Spectator.

These are two perfect sentences, divided from each other by the period, and subdivided into simple sentences by the comma. The first of these two sentences represents the *apparent* end of man's mission into this present world ; namely, to propagate his kind : the second shews the *real consequence* of his residence here ; begetting a successor, and making room for him. They are entirely distinct sentences ; unconnected in construction with each other, and wanting nothing to make each perfect both in sense and sentence.

Beside the four foregoing points used to express the pauses in discourse, there are three others, which also affect the modulation of the voice : namely,

The Note of Interrogation, marked thus (?)

The Note of Admiration, or Exclamation, marked thus (!)

The Parenthesis, marked thus ()

THE

The **NOTE OF INTERROGATION** is used when a question is asked : as, " Would an infinitely wise Being make such glorious creatures for so mean a purpose ? Can he delight in the production of such abortive intelligences, such short-lived reasonable beings ? Would he give us talents that are not to be exerted ? capacities that are never to be gratified ? " Spect. These are all interrogative sentences, and, as such, are marked by the interrogation point.

The **NOTE OF ADMIRATION, OR EXCLAMATION**, is used to mark the speaker's admiration, and any sudden passion : as,

" And see !

'Tis come, the glorious morn ! the second birth

Of Heaven and earth !"

THOMSON'S WINTER.

Both these points mark an elevation of the voice. The length of time required by them as pauses is uncertain, and may be equal to that of a semicolon, colon, or a period.

The **PARENTHESES** is used to enclose a word or sentence, inserted in another sentence ; which is not necessary to the sense, nor does it at all affect the construction : as, " Pompey on the other side (that hardly ever spake in public without a blush) had a wonderful sweetness of nature." This point marks a moderate depression of the voice, with a pause somewhat greater than a comma.

There are also other marks used in writing, which, though they do not affect the voice as pauses, yet are often necessary to the sense : the principal of which marks we shall just mention.

The **APOSTROPHE**, marked thus ('), is only a comma placed at the head of a letter, and signifies either that some letter or letters are left out for a quicker pronunciation ; as, *I'll*, for *I will* ; *he don't*, for *he does not* : or else it marks the possessive case : as, John's book.

The **ACCENT** (') is placed over a vowel to denote that the word is accented on that syllable ; as, *Com-pre-hén-sion*.

The **NOTE OF INTERROGATION** is used when a question is asked: as, "Would an infinitely wise Being make such glorious creatures for so mean a purpose? Can he delight in the production of such abortive intelligences, such short-lived reasonable beings? Would he give us talents that are not to be exerted? capacities that are never to be gratified?" Spect. These are all interrogative sentences, and, as such, are marked by the interrogation point.

The **NOTE OF ADMIRATION, OR EXCLAMATION**, is used to mark the speaker's admiration, and any sudden passion: as,

"And see!

'Tis come, the glorious morn! the second birth
Of Heaven and earth!"

THOMSON'S WINTER.

Both these points mark an elevation of the voice. The length of time required by them as pauses is uncertain, and may be equal to that of a semicolon, colon, or a period.

The **PARENTHESIS** is used to enclose a word or sentence, inserted in another sentence; which is not necessary to the sense, nor does it at all affect the construction: as, "Pompey on the other side (that hardly ever spake in public without a blush) had a wonderful sweetness of nature." This point marks a moderate depression of the voice, with a pause somewhat greater than a comma.

There are also other marks used in writing, which, though they do not affect the voice as pauses, yet are often necessary to the sense: the principal of which marks we shall just mention.

The **APOSTROPHE**, marked thus ('), is only a comma placed at the head of a letter, and signifies either that some letter or letters are ~~without~~ for a quicker pronunciation; as, I'll, for I will; he don't, for he does not: or else it marks the possessive case: as, John's book.

The **ACCENT**, marked thus (´), is placed over a vowel to denote that it is to be pronounced with a particular tone; as, Com-pre-hen-sion.

VOL. I

The

“ Were all books reduced to their quintessence, many a bulky author would make his appearance in a penny paper : there would be scarce any such thing in nature as a folio : the works of an age would be contained on a few shelves : not to mention millions of volumes, that would be utterly annihilated.”—Spectator, No. 124.

This sentence is divided into four parts by the colon : the first and last parts are compound members, divided by the comma ; the second and third are simple members.

Each of these parts contains perfect sense, and would also form a complete sentence, if the other parts had not been joined to it.

The colon is also used when a semicolon goes before, and a greater pause is required ; though the sentence be not complete ; also when a speech or an example is introduced.

The PERIOD is used when a sentence is so far finished as not to be connected in construction with the following sentence ; and marks both a perfect sense, and a perfect sentence : as,

“ Man, considered in his present state, seems only sent into the world to propagate his kind. He provides himself with a successor, and immediately quits his post to make room for him.”—Spectator.

These are two perfect sentences, divided from each other by the period, and subdivided into simple sentences by the comma. The first of these two sentences represents the *apparent* end of man's mission into this present world ; namely, to propagate his kind : the second shews the *real consequence* of his residence here ; begetting a successor, and making room for him. They are entirely distinct sentences ; unconnected in construction with each other, and wanting nothing to make each perfect both in sense and sentence.

Beside the four foregoing points used to express the pauses in discourse, there are three others, which also affect the modulation of the voice : namely,

The Note of Interrogation, marked thus (?)

The Note of Admiration, or Exclamation, marked thus (!)

The Parenthesis, marked thus ()

THE

The **NOTE OF INTERROGATION** is used when a question is asked: as, "Would an infinitely wise Being make such glorious creatures for so mean a purpose? Can he delight in the production of such abortive intelligences, such short-lived reasonable beings? Would he give us talents that are not to be exerted? capacities that are never to be gratified?" Spect. These are all interrogative sentences, and, as such, are marked by the interrogation point.

The **NOTE OF ADMIRATION, OR EXCLAMATION**, is used to mark the speaker's admiration, and any sudden passion: as,

"And see!

'Tis come, the glorious morn! the second birth

Of Heaven and earth!"

THOMSON'S WINTER.

Both these points mark an elevation of the voice. The length of time required by them as pauses is uncertain, and may be equal to that of a semicolon, colon, or a period.

The **PARENTHESIS** is used to enclose a word or sentence, inserted in another sentence; which is not necessary to the sense, nor does it at all affect the construction: as, "Pompey on the other side (that hardly ever spake in public without a blush) had a wonderful sweetness of nature." This point marks a moderate depression of the voice, with a pause somewhat greater than a comma.

There are also other marks used in writing, which, though they do not affect the voice as pauses, yet are often necessary to the sense: the principal of which marks we shall just mention.

The **APOSTROPHE**, marked thus ('), is only a comma placed at the head of a letter, and signifies either that some letter or letters are left out for a quicker pronunciation; as, *I'll*, for *I will*; *he don't*, for *he does not*: or else it marks the possessive case: as, John's book.

The **ACCENT** (') is placed over a vowel to denote that the word is accented on that syllable; as, *Com-pre-hén-sion*.

... many a
... penny paper.
... folio: the
... have: not to
... utterly annihi-

... the colon: the
... divided by the
... readers.
... and the
... been

[illegible][illegible]

המחיר הנמוך ביותר של המכונה הוא כ-10 אלף שקלים, ויש גם דגמים שונים במחירים נמוכים יותר.

()
(!)
()

The **NOTE OF INTERROGATION** is used when a question is asked : as, "Would an infinitely wise Being make such glorious creatures for so mean a purpose? Can he delight in the production of such abortive intelligences, such short-lived reasonable beings? Would he give us talents that are not to be exerted? capacities that are never to be gratified?" Spect. These are all interrogative sentences, and, as such, are marked by the interrogation point.

The **NOTE OF ADMIRATION, OR EXCLAMATION**, is used to mark the speaker's admiration, and any sudden passion : as,

"And see!

'Tis come, the glorious morn! the second birth
Of Heaven and earth!"

THOMSON'S WINTER.

Both these points mark an elevation of the voice. The length of time required by them as pauses is uncertain, and may be equal to that of a semicolon, colon, or a period.

The **PARENTHESIS** is used to enclose a word or sentence, inserted in another sentence; which is not necessary to the sense, nor does it at all affect the construction : as, "Pompey on the other side (that hardly ever spake in public without a blush) had a wonderful sweetness of nature." This point marks a moderate depression of the voice, with a pause somewhat greater than a comma.

There are also other marks used in writing, which, though they do not affect the voice as pauses, yet are often necessary to the sense : the principal of which marks we shall just mention.

The **APOSTROPHE**, marked thus ('), is only a comma placed at the head of a letter, and signifies either that some letter or letters are left out for a quicker pronunciation; as, *I'll*, for *I will*; *he don't*, for *he does not* : or else it marks the possessive case : as, John's book.

The **ACCENT** (') is placed over a vowel to denote that the word is accented on that syllable; as, *Com-pre-hén-sion*.

“Were all books reduced to their quintessence, many a bulky author would make his appearance in a penny paper : there would be scarce any such thing in nature as a folio : the works of an age would be contained on a few shelves : not to mention millions of volumes, that would be utterly annihilated.”—Spectator, No. 124.

This sentence is divided into four parts by the colon : the first and last parts are compound members, divided by the comma ; the second and third are simple members.

Each of these parts contains perfect sense, and would also form a complete sentence, if the other parts had not been joined to it.

The colon is also used when a semicolon goes before, and a greater pause is required ; though the sentence be not complete ; also when a speech or an example is introduced.

The PERIOD is used when a sentence is so far finished as not to be connected in construction with the following sentence ; and marks both a perfect sense, and a perfect sentence : as,

“Man, considered in his present state, seems only sent into the world to propagate his kind. He provides himself with a successor, and immediately quits his post to make room for him.”—Spectator.

These are two perfect sentences, divided from each other by the period, and subdivided into simple sentences by the comma. The first of these two sentences represents the *apparent* end of man's mission into this present world ; namely, to propagate his kind : the second shews the *real consequence* of his residence here ; begetting a successor, and making room for him. They are entirely distinct sentences ; unconnected in construction with each other, and wanting nothing to make each perfect both in sense and sentence.

Beside the four foregoing points used to express the pauses in discourse, there are three others, which also affect the modulation of the voice : namely,

The Note of Interrogation, marked thus (?)

The Note of Admiration, or Exclamation, marked thus (!)

The Parenthesis, marked thus ()

THE

The **NOTE OF INTERROGATION** is used when a question is asked: as, "Would an infinitely wise Being make such glorious creatures for so mean a purpose? Can he delight in the production of such abortive intelligences, such short-lived reasonable beings? Would he give us talents that are not to be exerted? capacities that are never to be gratified?" Spect. These are all interrogative sentences, and, as such, are marked by the interrogation point.

The **NOTE OF ADMIRATION, OR EXCLAMATION**, is used to mark the speaker's admiration, and any sudden passion: as,

"And see!

'Tis come, the glorious morn! the second birth
Of Heaven and earth!"

THOMSON'S WINTER.

Both these points mark an elevation of the voice. The length of time required by them as pauses is uncertain, and may be equal to that of a semicolon, colon, or a period.

The **PARENTHESIS** is used to enclose a word or sentence, inserted in another sentence; which is not necessary to the sense, nor does it at all affect the construction: as, "Pompey on the other side (that hardly ever spake in public without a blush) had a wonderful sweetness of nature." This point marks a moderate depression of the voice, with a pause somewhat greater than a comma.

There are also other marks used in writing, which, though they do not affect the voice as pauses, yet are often necessary to the sense: the principal of which marks we shall just mention.

The **APOSTROPHE**, marked thus ('), is only a comma placed at the head of a letter, and signifies either that some letter or letters are left out for a quicker pronunciation; as, *I'll*, for *I will*; *he don't*, for *he does not*: or else it marks the possessive case: as, John's book.

The **ACCENT** (') is placed over a vowel to denote that the word is accented on that syllable; as, *Com-pre-hén-sion*.

The **BREVE** (¨), placed over a vowel to signify it should be sounded short: as, *Pärry*.

The **CIRCUMFLEX** (^), placed over some vowel to shew that it is to be pronounced long: as, *disputē*.

The **CARET**, of the same form with the circumflex, but placed beneath the line, to shew that something is omitted which is inserted over the line: as, *this[^] is he*.

The **DIALYSIS** (¨), or two points placed over two vowels in a word, to shew they are to be parted, not being a diphthong: as, *Duäl*.

The **HYPHEN**, or note of conjunction (-), being a straight line, used to connect compound words together: as, *Heart-breaking*; *Book-keeper*. And when inserted at the end of a line, it shews the word is divided according to its syllables: as, *dis-tinguish*.

The **INDEX** (†), or hand, pointing to something remarkable.

The **ASTERISM** (*), or Star, directing to some note in the margin, or at the bottom of the page. Several of them together denote something defective, or improper to be repeated.

The **OBELISK** (†), is used to refer to a note like the foregoing. And in some dictionaries it denotes the word to be obsolete.

The **PARAGRAPH** (¶) marks a division of sentences under one head. It is used chiefly in the vulgar translation of the Bible.

The **SECTION** (§) shews a greater division than the paragraph.

The **CROCHETS** ([]) include a word or sentence explanatory of what was mentioned before.

The **QUOTATION** (“ ”) is a double comma (the first reversed), and placed at the beginning and end of a sentence, or sentences, taken from another author.

A P R A X I S ;

Or Examples of Grammatical Resolution.

JOHN, IV.

1 When therefore the Lord knew how the Pharisees had heard that Jesus made and baptized more disciples than John ;

2 (Though Jesus himself baptized not, but his disciples ;)

3 He left Judæa, and departed again into Galilee.

4 And he must needs go through Samaria.

5 Then cometh he to a city of Samaria, which is called Sychar, near to the parcel of ground that Jacob gave to his son Joseph.

6 Now Jacob's well was there. Jesus therefore being wearied with his journey, sat thus on the well : and it was about the sixth hour.

7 There cometh a woman of Samaria to draw water : Jesus saith unto her, Give me to drink.

1 *When* is an adverb of time ; *therefore*, an adverb with a preposition added to it ; *the*, the definite article ; *Lord*, common noun, singular number, masculine gender, nominative case, being the agent of the sentence ; *knew*, the verb neuter, third person, singular, governed by the noun *Lord*, and agreeing with it, past time, indicative mode, and the attribute of the sentence ; *how*, an adverb of manner ; *the*, definite article ; *Pharisees*, noun common, plural number, nominative case ; *had heard*, verb neuter with the auxiliary *had*, plural, third person, past time, indicative mode, governed by *Pharisees* ; *that*, a copulative conjunction ; *Jesus*, a proper name, masculine, nominative case, and the agent of the sentence ; *made, and baptized*, two verbs active, third person, singular, past time, indicative mode, connected with each other by the copulative conjunction *and*, and are the attribute of the sentence ; *more*, an adverb of comparison ; *disciples*, noun plural, objective case, and the object of the sentence ; *than*, comparative conjunction,

unction, comparing the number mentioned in the foregoing sentence with that of the following : *as, baptized more disciples, than Jesus (did) ;* *John*, a proper name.

2 *Though*, a disjunctive conjunction ; *Jesus*, a proper name, as before ; *himself*, the objective pronoun of the third person singular, masculine gender, joined to the singular substantive *self*, to express an exclusive emphasis ; *baptized*, verb active, third person, singular, past time, and indicative mode, though it be governed by the conjunction *though*, for it is not of a doubtful, but absolute nature, declaring that *Jesus did not baptize any* ; it forms the attribute of the sentence to the agent *Jesus* ; *not*, a negative adverb, added to the verb, and altering its sense, the possessive pronoun *any* being understood ; *but*, a disjunctive conjunction, corresponding to the former conjunction *though*, and expressing an opposition in meaning ; *his*, the possessive pronoun, third person, singular, masculine gender ; *disciples*, the plural substantive, nominative case, the auxiliary (*did*) being understood : *as, his disciples did*. The whole verse is included in a parenthesis, and is not connected in sense or construction, with the rest of the chapter.

3 *He*, a pronoun of the third person singular, masculine gender, nominative case ; used as a substitute for the noun *Jesus*, and governed by it ; *left*, a verb active, third person, singular, governed by the pronoun *he*, past time, indicative mode ; *Jadara*, a proper name of a city, objective case, as it forms the object to the sentence ; *and*, a copulative conjunction ; *departed*, a verb neuter, third person, singular, governed also by the pronoun *he*, past time, indicative mode ; *again*, an adverb ; *into*, a preposition ; *Galilee*, a proper noun, objective case, governed by the preposition *into*.

4 *And*, a copulative conjunction, expressing an addition ; *he*, a pronoun as before, agreeing with its antecedent noun *Jesus* ; *must*, an auxiliary, denoting necessity, and having no variation ; *needs*, an adverb ; *go*, a neuter verb ; *through*, a preposition ; *Samaritis*, a proper name for a country, objective case, being governed by the preposition *through*.

5 *Then*,

5 *Then*, an adverb of time ; *cometh*, a neuter verb, governed by the pronoun *he* in person and number, and placed before it, as is sometimes the case when the agent is a pronoun ; indicative mode, and present time (improperly, for it should be in the past time, as, *then came he*) ; *he*, a pronoun ; *to*, a preposition ; *a*, the indefinite article ; *city*, a common noun, singular, neuter gender, objective case, being governed by the preposition *to* ; *of*, a preposition ; *Samaritan*, a proper name, singular, objective case ; *which*, a relative, agreeing with its antecedent *city* ; *is*, the neuter verb *to be*, present time, singular, third person, indicative mode ; *called*, the perfect participle of the neuter verb *to call*, being placed after the auxiliary *is* : *Sychar*, a proper name of a city, nominative case, being placed after the neuter verb, *called* ; *near*, an adverb of distance ; *to*, a preposition ; *the*, the definite article ; *parcel*, a common noun, singular, objective case, governed by the preposition *to* ; *of*, a preposition ; *ground*, substantive common, objective case, governed by the preposition *of* ; *that*, a relative, agreeing with its antecedent, *the parcel of ground* ; *Jacob*, a proper name of the patriarch, masculine, nominative case ; *gave*, a verb active, third person, singular, past time, indicative mode, governed by *Jacob*, the same as, *Jacob gave that* ; *to*, a preposition ; *his*, a possessive pronoun, agreeing in person, number, and gender, with the noun *Jacob*, and governed by it, and in the possessive case, to shew the relation of possession between *Jacob* and *son* ; *son* is a common noun, singular, masculine gender, objective case, governed by the preposition *to* ; *Joseph*, a proper name, put in apposition to *son* ; that is, in the same case, and governed by the same preposition *to*.

6 *Now*, an adverb ; *Jacob's*, a substantive proper, possessive case ; *well*, a common noun, singular, nominative case, the same as, *the well of Jacob* ; *was*, the neuter verb *to be*, third person singular, past time, indicative mode ; *there*, an adverb of place ; *Jesus*, a proper name ; *therefore*, an adverb with a preposition, the same as *there and for*, it has the construction of

wise man will desire no more than what *he may get justly* :
 a wise man will desire no more than what *he may use soberly* :
 a wise man will desire no more than what *he may distribute cheerfully* : a wise man will desire no more than what *he may live upon contentedly*. They must each of them, therefore, be distinguished by a comma. They are so many simple sentences, and the whole is a compound sentence.

As sentences themselves are divided into simple and compound, so the members of sentences may be divided into simple and compound members: for whole sentences, whether simple or compound, may become members of other sentences, by means of a connexion.

The following are marked by a Comma :

First, simple members of sentences, closely connected together in one compound member or sentence: as in the foregoing example: except, first, when the members are short in comparative sentences: as, in the former part of the last example, “a wise man will desire no more than what he may get justly.” These are, in fact, two simple sentences *compared* by the conjunction *than*; but, being short, they are not separated by a comma. Secondly, when two simple members or sentences are closely connected by a relative, and the subject of the antecedent is confined to a particular sense: as, “the man who is possessed of this excellent frame of mind, is not only easy in his thoughts, but a perfect master of all the powers and faculties of his soul.”—Spectator. In this sentence, *the man*, is connected to the following sentences by the relative *who*; which restrains the idea of the antecedent to the sense here mentioned.

Secondly, the case absolute: as, “the doors being shut, Jesus stood in the midst.”

Thirdly, nouns in apposition, when consisting of many terms: as, “Alexander, the great, cruel, and unjust.”

Fourthly, the vocative case, or when an address is made to a person: as, “This said, he formed thee, Adam; thee, O man.”—Milton.

Fifthly, when there are more than two nouns, or adjectives, connected by copulatives or disjunctives; or when there are only two, if the conjunction be understood: as, “Raptures, transports, and ecstasies are the rewards which they confer: sighs and tears, prayers and broken hearts, are the offerings which are paid to them.”—Spectator.

Sixthly, a circumstance of importance, though only an imperfect phrase: as, “the principle may be defective or faulty; but the consequences it produces are so good, *that*, for the benefit of mankind, it ought not to be extinguished.” Spectator.

Lastly, the participle with something depending on it: as,

“Now morn, her rosy steps in th’ eastern clime

Advancing, sow’d the earth with orient pearl.” MILTON.

The SEMICOLON is used when a sentence, or member of a sentence, requires a greater pause than a comma, yet neither forms a perfect sense, nor a perfect sentence; but is followed by some other member, or sentence, with which it is closely connected: as,

“To look upon the soul as going from strength to strength, to consider that she is to shine for ever with new accessions of glory, and brighten to all eternity; that she will be still adding virtue to virtue, and knowledge to knowledge; carries in it something wonderfully agreeable to that ambition which is natural to the mind of man.”—Spectator.

This compound sentence is divided into three principal parts by the semicolon; each part requires a greater pause than a comma; but neither of them expresses perfect sense, or forms a perfect sentence, being closely connected in sense with each other.

The COLON is used when a sentence, whether simple or compound, requires a greater pause than a semicolon: it always forms a perfect sense, and would by itself form a complete sentence, but is followed by another member, or sentence; making the sense more full and complete: as,

“Were

“ Were all books reduced to their quintessence, many a bulky author would make his appearance in a penny paper : some would be larger and less thing in nature as a folio : the mass of an age would be contained on a few shelves : not to mention millions of volumes, that would be utterly annihilated.” — *Specimens*, Vol. III.

The sentence is divided into four parts by the colon : the first and last parts are compound members, divided by the comma : the second and third are simple members.

Each of these parts contains perfect sense, and would also form a complete sentence, if the other parts had not been present to it.

The comma is also used when a parenthesis goes before, and a further clause is required : though the sentence be not complete, it is when a parenthesis or an example is introduced.

The semicolon is used when a sentence is so far finished as not to be connected in construction with the following sentence; and marks it as a perfect sense, and a perfect sentence : as,

“ Man, considered in his present state, seems only sent into the world to propagate his kind. He provides himself with a successor, and immediately quits his post to make room for him.” — *Specimens*.

There are two perfect sentences, divided from each other by the period, and subdivided into simple sentences by the comma. The first of these two sentences represents the appearance and of man's mission into this present world ; namely, to propagate his kind : the second shows the *real consequence* of his residence here : begetting a successor, and making room for him. They are entirely distinct sentences ; unconnected in construction with each other, and wanting nothing to make each perfect both in sense and sentence.

Beside the four foregoing points used to express the pauses in discourse, there are three others, which also affect the modulation of the voice : namely,

The Note of Interrogation, marked thus (?)

The Note of Admiration, or Exclamation, marked thus (!)

The Parenthesis, marked thus ()

THE

The **NOTE OF INTERROGATION** is used when a question is asked: as, "Would an infinitely wise Being make such glorious creatures for so mean a purpose? Can he delight in the production of such abortive intelligences, such short-lived reasonable beings? Would he give us talents that are not to be exerted? capacities that are never to be gratified?" Spect. These are all interrogative sentences, and, as such, are marked by the interrogation point.

The **NOTE OF ADMIRATION, OR EXCLAMATION**, is used to mark the speaker's admiration, and any sudden passion: as,

"And see!

'Tis come, the glorious morn! the second birth
Of Heaven and earth!"

THOMSON'S WINTER.

Both these points mark an elevation of the voice. The length of time required by them as pauses is uncertain, and may be equal to that of a semicolon, colon, or a period.

The **PARENTHESIS** is used to enclose a word or sentence, inserted in another sentence; which is not necessary to the sense, nor does it at all affect the construction: as, "Pompey on the other side (that hardly ever spake in public without a blush) had a wonderful sweetness of nature." This point marks a moderate depression of the voice, with a pause somewhat greater than a comma.

There are also other marks used in writing, which, though they do not affect the voice as pauses, yet are often necessary to the sense: the principal of which marks we shall just mention.

The **APOSTROPHE**, marked thus ('), is only a comma placed at the head of a letter, and signifies either that some letter or letters are left out for a quicker pronunciation; as, *I'll*, for *I will*; *he don't*, for *he does not*: or else it marks the possessive case: as, John's book.

The **ACCENT** (') is placed over a vowel to denote that the word is accented on that syllable; as, *Com-pre-hén-sion*.

The **Acute** (´) is placed over a vowel to signify it should be pronounced as *acute*.

The **Circumflex** (^), placed over some vowel to show it is pronounced long: as, *â*, *ê*, *ô*.

The **Circumflex** of the line form with the circumflex, but without the line, is used to show that something is omitted: as, *â*, *ê*, *ô*.

The **Diphthong** (¨), or two points placed over two vowels to show they are to be joined, not being a diphthong: as, *â*, *ê*, *ô*.

The **Hyphen** (-) is used in composition: as, *Heart-breaking*. It is also used at the end of a line, to show the word is continued according to its syllables: as, *dis-rupt*.

The **Section** (§) is used, pointing to something re-markable.

The **Section** (¶) is used, directing to some note in the margin, or at the bottom of the page. Several of them are used, denoting interesting, instructive, or improper to be repeated.

The **Section** (¶) is used to refer to a note like the foregoing. It is also used, denoting the word to be omitted.

The **Section** (¶) marks a division of sentences under one head. It is used chiefly in the vulgar translation of the Bible.

The **Section** (¶) shows a greater division than the paragraph.

The **Circumflex** (¨) include a word or sentence explanatory of what was mentioned before.

The **Quotation** ("") is a double comma (the first reversed), and placed at the beginning and end of a sentence, or sentence, taken from another author.

A P R A X I S ;

Or Examples of Grammatical Resolution.

JOHN, IV.

1 When therefore the Lord knew how the Pharisees had heard that Jesus made and baptized more disciples than John ;

2 (Though Jesus himself baptized not, but his disciples ;))

3 He left Judæa, and departed again into Galilee.

4 And he must needs go through Samaria.

5 Then cometh he to a city of Samaria, which is called Sychar, near to the parcel of ground that Jacob gave to his son Joseph.

6 Now Jacob's well was there. Jesus therefore being wearied with his journey, sat thus on the well : and it was about the sixth hour.

7 There cometh a woman of Samaria to draw water : Jesus saith unto her, Give me to drink.

1 *When* is an adverb of time ; *therefore*, an adverb with a preposition added to it ; *the*, the definite article ; *Lord*, common noun, singular number, masculine gender, nominative case, being the agent of the sentence ; *knew*, the verb neuter, third person, singular, governed by the noun *Lord*, and agreeing with it, past time, indicative mode, and the attribute of the sentence ; *how*, an adverb of manner ; *the*, definite article ; *Pharisees*, noun common, plural number, nominative case ; *had heard*, verb neuter with the auxiliary *had*, plural, third person, past time, indicative mode, governed by *Pharisees* ; *that*, a copulative conjunction ; *Jesus*, a proper name, masculine, nominative case, and the agent of the sentence ; *made*, *and* *baptized*, two verbs active, third person, singular, past time, indicative mode, connected with each other by the copulative conjunction *and*, and are the attribute of the sentence ; *more*, an adverb of comparison ; *disciples*, noun plural, objective case, and the object of the sentence ; *than*, comparative conjunction,

junction, comparing the number mentioned in the foregoing sentence with that of the following : *as, baptized more disciples, than John (did) ; John*, a proper name.

2 *Though*, a disjunctive conjunction ; *Jesus*, a proper name, as before ; *himself*, the objective pronoun of the third person singular, masculine gender, joined to the singular substantive *self*, to express an exclusive emphasis ; *baptized*, verb active, third person, singular, past time, and indicative mode, though it be governed by the conjunction *though*, for it is not of a doubtful, but absolute nature, declaring, that *Jesus did not baptize any* ; it forms the attribute of the sentence to the agent *Jesus* ; *not*, a negative adverb, added to the verb, and altering its sense, the possessive pronoun *any* being understood ; *but*, a disjunctive conjunction, corresponding to the former conjunction *though*, and expressing an opposition in meaning ; *his*, the possessive pronoun, third person, singular, masculine gender ; *disciples*, the plural substantive, nominative case, the auxiliary (*did*) being understood : *as, his disciples did*. The whole verse is included in a parenthesis, and is not connected in sense or construction, with the rest of the chapter.

3 *He*, a pronoun of the third person singular, masculine gender, nominative case ; used as a substitute for the noun *Jesus*, and governed by it ; *left*, a verb active, third person, singular, governed by the pronoun *he*, past time, indicative mode ; *Judæa*, a proper name of a city, objective case, as it forms the object to the sentence ; *and*, a copulative conjunction ; *departed*, a verb neuter, third person, singular, governed also by the pronoun *he*, past time, indicative mode ; *again*, an adverb ; *into*, a preposition ; *Galilee*, a proper noun, objective case, governed by the preposition *into*.

4 *And*, a copulative conjunction, expressing an addition ; *he*, a pronoun as before, agreeing with its antecedent noun *Jesus* ; *must*, an auxiliary, denoting necessity, and having no variation ; *needs*, an adverb ; *go*, a neuter verb ; *through*, a preposition ; *Samaría*, a proper name for a country, objective case, being governed by the preposition *through*.

5 *Then*,

5 *Then*, an adverb of time ; *cometh*, a neuter verb, governed by the pronoun *he* in person and number, and placed before it, as is sometimes the case when the agent is a pronoun ; indicative mode, and present time (improperly, for it should be in the past time, as, *then came he*) ; *he*, a pronoun ; *to*, a preposition ; *a*, the indefinite article ; *city*, a common noun, singular, neuter gender, objective case, being governed by the preposition *to* ; *of*, a preposition ; *Samaria*, a proper name, singular, objective case ; *which*, a relative, agreeing with its antecedent *city* ; *is*, the neuter verb *to be*, present time, singular, third person, indicative mode ; *called*, the perfect participle of the neuter verb *to call*, being placed after the auxiliary *is* : *Sychar*, a proper name of a city, nominative case, being placed after the neuter verb, *called* ; *near*, an adverb of distance ; *to*, a preposition ; *the*, the definite article ; *parcel*, a common noun, singular, objective case, governed by the preposition *to* ; *of*, a preposition ; *ground*, substantive common, objective case, governed by the preposition *of* ; *that*, a relative, agreeing with its antecedent, *the parcel of ground* ; *Jacob*, a proper name of the patriarch, masculine, nominative case ; *gave*, a verb active, third person, singular, past time, indicative mode, governed by *Jacob*, the same as, *Jacob gave that* ; *to*, a preposition ; *his*, a possessive pronoun, agreeing in person, number, and gender, with the noun *Jacob*, and governed by it, and in the possessive case, to shew the relation of possession between *Jacob* and *son* ; *son* is a common noun, singular, masculine gender, objective case, governed by the preposition *to* ; *Joseph*, a proper name, put in apposition to *son* ; that is, in the same case, and governed by the same preposition *to*.

6 *Now*, an adverb ; *Jacob's*, a substantive proper, possessive case ; *well*, a common noun, singular, nominative case, the same as, *the well of Jacob* ; *was*, the neuter verb *to be*, third person singular, past time, indicative mode ; *there*, an adverb of place ; *Jesus*, a proper name ; *therefore*, an adverb with a preposition, the same as *there and for*, it has the construction

of

of a pronoun; *sitting*, the present participle of the neuter verb *to sit*; *married*, the perfect participle of the neuter verb *to marry*, being placed after the participle of the auxiliary *to be*, which, as has been shewn, in all its forms requires the present or perfect participle after it; *with*, a preposition; *his*, a possessive pronoun, third person singular, masculine gender, standing for the possessive case of the noun *Jesus*; *journey*, a substantive singular, objective case, governed by the preposition *with*; *sat*, a neuter verb, third person, singular, past time, indicative mode, governed by *Jesus*; *thus*, an adverb of manner, improperly inserted in this place, as it refers to the manner of sitting, or the posture; whereas no particular posture or manner of sitting is mentioned, either before or after this sentence: *on*, a preposition; *the*, the definite article; *man*, a common noun, singular, objective case, governed by the preposition *on*; *and*, a conjunction copulative; *it*, the neuter pronoun, nominative case, standing for the time of the action; *was*, the neuter verb *to be*, third person, singular, past time, indicative mode, governed by the neuter pronoun *it*; *after*, an adverb; *the*, the definite article; *sixth*, an adjective; *man*, a common noun, substantive singular.

is sitting, the neuter verb, third person, singular, present time, indicative mode, and present time, though it should be *was sitting*, as the event here mentioned was evidently past, and in the twentieth verse of this chapter the woman or *Samaritan* is spoken of in the past time: as, “the woman *answered and said*,” a definite article; *woman*, a substantive common, singular, feminine gender, nominative case, being the agent of the sentence, though placed after the verb; *of*, a preposition; *Samaritan*, a substantive common, singular, objective case, governed by the preposition *of*; *to draw*, the infinitive mode of the verb *draw*; *water*, a common substantive, singular; *Jesus*, a proper name as before; *said*, a neuter verb, third person singular, governed by *Jesus*, and present time, improperly for the past; as this should also be as it stands in the tenth verse of this chapter; “*Jesus answered and said* unto

unto her;" *unto*, a preposition; *her*, a pronoun, third person singular, feminine gender, objective case, standing for *the woman of Samaria*; *give*, a verb active, second person singular, imperative mode; *me*, a pronoun, first person singular, objective case, standing for *Jesus*, and governed by the verb in the imperative mode, the word *water* being understood; *to drink*, the infinitive mode of the active verb; it is here made absolute, supplying the place of the conjunction *that* with the subjunctive mode: *as, give to me that I may drink.*

SECT. VI.

OF ELOCUTION.

ELOCUTION is the art of reading or speaking with propriety and elegance; or delivering our words in a just and graceful manner, untainted with pedantry or affectation, and uncorrupted with any provincial sound.

Few people need be told the necessity of a graceful elocution. All will allow, that what a man has occasion daily to do, should be done well; yet so little attention has sometimes been paid to this accomplishment, even by those from whom (from their profession as public speakers) we have been led to expect a perfect model of the art, as hath tended greatly to eclipse all their other merits, however great; while others, of inferior attainments, by the help of a tolerably good style, and a just elocution, have risen to a considerable eminence.

A graceful elocution is to a good style, what a good style is to the subject matter of a discourse, an essential ornament. For if the subject of a discourse be ever so interesting, and the speaker's knowledge ever so profound, without a correct style, the discourse must suffer greatly in its reputation; and though the speaker's abilities be great, and the style good, with a bad elocution,

elocution, it will fare little better. So great an effect have these exterior accomplishments upon public taste.

The first rule in Elocution, as in Rhetoric, is *to follow nature*. All other rules, without a strict regard to this, will only produce the affected declaimer, and not the graceful speaker. The excellency of this rule, and the affectation of some modern examples of eloquence, have induced some to disregard all other rules as superfluous; and to imagine that this rule alone, in a person of good sense, and cultivated taste, is sufficient to form a public speaker. This, however, is an error. But it is necessary to observe the effects of nature, in order to distinguish between them, and those of arbitrary custom, or false taste; and to observe the various ways by which nature expresses the several passions, emotions, and perceptions of the human mind: in order to which, much assistance may certainly be derived from the following particular rules:

1. *A distinct and deliberate articulation* is the first particular rule. This consists in giving a clear and full utterance to all the sounds, both simple and complex. For this purpose, the nature of those sounds should be well understood, and much pains taken to correct any faults in the pronunciation. Sometimes a fault in articulation is ascribed to a natural defect in some of the organs of speech, when it is the effect of bad example, or inattention only. Many of these faults concern the sounding of the consonants. Some cannot pronounce the letter *l*, others the simple sounds, *r*, *s*, *th*, *ph*; others omit the aspirate *h*. These may be corrected by reading sentences chosen for that purpose, wherein these sounds frequently occur; and by constantly guarding against them in conversation.

The other fault in articulation consists in the confusion and clattering of words together. Most people, who have not learned the art of speaking, have a habit of uttering their words so rapidly and indistinctly, that it is impossible for them to give any emphasis to their words, or natural tone to their sentences; though they may pronounce every word plainly.

The

The most effectual means to conquer this habit are, to read aloud passages chosen for that purpose, which abound with long and unusual words; and to read much slower than the sense and just speaking would require. Aim at nothing higher till you can read distinctly and deliberately.

2. *Let every word be pronounced with propriety.* This is a most necessary rule, as errors in pronunciation are more frequently committed, and less guarded against than any others. Faults in pronunciation are innumerable. Some of them are the following: omitting the aspirate *h* where it should be used: as, *our* for *hour*; and using it where there should be none; as, *hegg* for *egg*: confounding and interchanging the *v* and *w*; pronouncing the diphthong *au* like *au*, or *oo*; and the vowel *i* like *ai*; or *e*; and cluttering several consonants together, without regarding the vowels. These errors, and many others too numerous to mention here, must all be carefully guarded against, by every person who would be thought a correct speaker.

It is difficult to point out any standard by which the propriety of pronunciation is to be determined. Profound scholars generally make the etymology of the word the rule of pronunciation, and thus bring upon themselves the charge of pedantry and affectation, by a delivery altogether novel to the rest of mankind. While, on the other hand, the mere man of the world, notwithstanding all his other accomplishments, often retains so much of his provincial dialect, as justly to exclude him from the honour of being the pattern of a just pronunciation. This excellency is perhaps to be found in those only who unite those two characters, and who, to the correctness of true learning, add the elegance and ease of genteel life. *An attention to such models, an intercourse with the polite world, and a regard to the etymology of the words themselves,* are the best rules that can be given, to correct any errors in pronunciation, and to guard against any peculiarities of provincial sound: which every person should endeavour to avoid, who is supposed to have seen too much of the

world, to retain the peculiarities of the district in which he was born.

3. *Let every word of more than one syllable be pronounced with its proper ACCENT.* Many writers have laid it down as a fundamental rule, that the accent of words in the English language should be thrown as far back as possible from the end of the word. A very erroneous rule! and one which has no foundation in the English language; for many words in the English have the accent on the last syllable, and the generality of them have it on the second or third syllable from the end, but a very few on the fourth syllable from the end of the word. The best rule that can be given for this precept, is the foregoing; namely, a strict attention to the best examples; always observing that the accent is not to be determined by any rules of quantity, but by the number and nature of the simple sounds.

4. *The pronunciation should be bold and forcible.* Many public speakers deliver their discourse with such a faint and feeble utterance, and seem so wholly unconcerned with their subject, that their words may be said rather to drop from their lips than to be delivered by them. This seldom fails of producing the same unconcern and indifference among their auditory. Inanimation is a principal fault: a speaker without energy is little better than a lifeless statue.

To remedy this evil, inure yourself, when reading, to take in as much breath as you can with ease contain, and to expel it with vehemence, in uttering those sounds which require an emphatical pronunciation; read aloud in the open air, with all the energy you can command; preserve your body in an erect attitude; let all the sounds, both vowels and consonants, have a free, full, and bold utterance. Practise these rules with perseverance, till you are master of a bold and manly delivery.

But in acquiring an energy of speech, the following caution is necessary, namely, that the speaker avoid the opposite extreme, of running into a passionate and distracting vociferation. This exists among those speakers, who value nothing above the

the admiration of the vulgar. These, as Shakespeare has justly observed, "offend the judicious hearer to the soul, by tearing a passion to rags, to very tatters, to split the ears of the groundlings." Cicero compares such speakers to cripples, who get on horseback because they cannot walk;—they bellow, because they cannot speak.

5. *Learn to acquire a graceful variety, both in the height and tone of the voice.* It is owing to the neglect of this rule, that we daily hear such languid and unaffecting discourses. Speakers, who deliver their sentiments, or read their compositions in a dull, flat, insipid manner, whatever other merits they may possess, seldom convince or affect their auditors. This same unmeaning mode of delivery, so universally prevalent, is called a *monotony*; and consists in one unvaried tone of delivery. Most people, in familiar conversation, can deliver their thoughts with some variety, both in the tone and the height of their voice, which they generally modify according to the nature of their discourse; yet the same persons, when required to read any composition, immediately assume a dry, uniform manner of delivery; as though reading and conversation were to be performed in quite a different manner from each other; whereas, the same sentences, whether delivered spontaneously by the speaker, or read by the lecturer, should always be delivered with the same tone, and on the same key, notwithstanding all that has been said by superficial critics to the contrary.

A person of a moderate capacity will, therefore, be able to alter the tone, as well as the height of his voice, as it may be necessary. Different species of speaking require different modifications of the voice. Different characters, in different situations, speak in very different keys. The vagrant when he begs, the soldier when he gives the word of command, the watchman when he announces the hour of the night, the senator when he harangues, the lover when he whispers his tender tale, differ as much in the key as in the tone in which they speak. The same person also, in the same situation,

may have occasion for all the variety of tone and diversity of voice: as in the support of an argument, the relation of a story, the command given to a servant, a lamentation, an exclamation of anger, and many other cases, require not only different tones, but different elevations of the voice. Reading, therefore, in which all the varieties of expression in real life are copied, must have continual variations in the height of the voice.

To acquire the power of changing the key on which you speak, accustom yourself to pitch your voice in different keys, from the lowest to the highest notes you can command; reading, as exercises on this rule, such compositions as abound with a variety of speakers, observing the height and tone of the voice necessary to each sentence, and changing them as the nature of the subject requires. This will give you such a command of voice, as is not to be acquired by any other method.

6. *Distinguish the more significant words in every sentence by their proper EMPHASIS.* The emphasis serves to point out the precise meaning of a sentence, shews how one idea is connected with another, gives to every part of a sentence its proper sound, and conveys the sense of the whole to the mind of the hearer. It also expresses the opposition between the parts of a sentence where the sense is pointed. Sometimes the emphasis is double, and sometimes treble in a sentence. Emphasis serves likewise to express some particular meaning which does not arise from the words themselves; as in this short sentence: "Did Alexander conquer the Persians?" This simple question may have three different meanings, according to the intention of the speaker; and the emphasis has, consequently, three different places: as when the speaker knew that the Persians were conquered, but did not know by whom, then the emphasis is placed on the word *Alexander*; as, "Did *Alexander* conquer the Persians?" When it is known that Alexander attempted the conquest, but the issue is not known, the emphasis is then placed on the word *conquer*

quer : as, "Did Alexander *conquer* the Persians?" When it is known that he conquered the adjacent countries, but it is not certainly known that he conquered the Persians, the emphasis is placed on the word *Persians* : as, "Did Alexander conquer the *Persians*?"

There are four ways by which the emphasis of a sentence is generally destroyed, or misapplied. First by placing so strong an emphasis on any one word, as to exclude all the other words in the sentence from being in any degree emphatical, which they often require; for, as was before observed, the emphasis is sometimes double, and sometimes treble in a sentence. There are also, of emphases in the same sentence, some which require a more forcible sound than others. This is an error which many animated speakers commit.

Secondly, by placing too great an emphasis on conjunctive particles, and those words of secondary importance, which, though sometimes proper, yet it is but seldom.

Another way by which emphasis is destroyed is, by reading in one uniform musical tone, which is by some erroneously called reading *melodiously*. An agreeable inflexion and variation of the voice, as far as is consistent with just speaking, deserves attention; but to substitute one unmeaning tone of reading, in exclusion to all the grace and propriety of a just elocution, can be the effect only of great ignorance or a corrupt taste.

Lastly, the emphasis is often placed on a wrong word in a sentence. This is the most common fault, and the most liable to be committed, and arises from the want of a thorough knowledge of the sense, and the writer's ideas. For if the reader be not perfectly acquainted with the exact construction, and full meaning of every sentence which he recites, it is impossible he should give those inflexions and variations of the voice which the sense requires. Some persons, finding the difficulty of rightly placing the emphasis, have rejected all emphasis entirely, and read with a dull stupid monotony, which is the worst fault of all.

To speak with a just and forcible emphasis, nothing more is

what is to follow, or enabling the speaker to alter the tone or height of his voice. In doing this, however, it is necessary that the tone of the voice be kept up in such a manner, that the hearer may know that the sense is not completed. Mr. GARRICK and Mr. SHERIDAN often practised this rule with great success.

The principal fault respecting cadence consists in an uniform fall of the voice in the word which precedes every full pause. This, though sometimes proper, yet when strictly adhered to, seldom fails of destroying all grace and energy of speech. The tones and height of the voice, at the close of a sentence, should be infinitely diversified, according to the nature of the discourse, and the particular meaning of the sentence: thus, in plain narrative, argumentative discourses, interrogative sentences, and when the last word of a sentence is emphatical, it is always necessary to close the sentence with an elevation of the voice. This will appear obvious, if we do but consider the manner in which we relate a story, support an argument, or propose a question, in conversation. But when the sense of the sentence does not require the last sound to be emphatical, an easy fall of the voice, sufficient to shew that the sense is finished, will be proper. And in pathetic pieces of the plaintive, tender, or solemn kind, the tone of the passion will often require a still lower cadence of the voice. The best method to correct an uniform cadence is frequently to read select sentences, where the sense is pointed and emphatical; and argumentative pieces, where frequent interrogatories occur; observing at the same time, the manner in which they should be delivered by a just speaker in familiar conversation.

8. The last, though not the least, rule to form a graceful speaker, is *to express the passions and emotions of the mind by tones, looks, and gestures, as well as by words.* To express anger, fear, grief, joy, love, or any other active passion which arises in our minds, nature teaches us to make use of certain tones, looks, and gestures. We naturally discover our feelings
by

by the manner in which we utter our words, the form of the countenance, and other well-known signs. And even when we deliver our thoughts on any less interesting subject, and when the more violent emotions are not excited, some kind of feeling accompanies our words, which should always have its proper external expression. These exterior expressions are the effect of nature alone, and every judicious imitation of it in a speaker will always appear graceful and pleasing; none can deserve the appellation of a graceful speaker, till, to a distinct articulation, a perfect command of voice, and a true emphasis, he can add the various expressions of the passions and emotions of the mind.

It is impossible to lay down any set of rules by which the speaker may form these outward expressions, as they must be as various as the passions they indicate; the following general direction is the only one that can be depended upon:—

In acquiring this accomplishment, as all others in this art, the fundamental rule is, to *follow nature*. Observe in what manner the several emotions, or passions, are expressed in real life, or by those who have with great labour and taste acquired the happy power of imitating nature; and either follow the great original herself, or the best copies to be met with, always observing the caution given by Shakespeare:—
“O’erstep not the modesty of nature.”

More is to be learned by a strict attention to good examples than is perhaps imagined. Example has as much the advantage of precept, as practice has of theory; and many, who have arrived to great perfection in this art, have candidly acknowledged it to be more the effect of imitation, than any theoretical rules.

In order to apply the foregoing rules to practice it is absolutely necessary to go through a regular course of exercises; beginning with such as are most easy, and proceeding by slow advances to those that are more difficult. For if the reader cannot deliver with propriety the plain sentiments of simple narrative, and didactic pieces, how can it be expected he should
do

do justice to the sublime descriptions of poetry, or the animated language of the passions. The student should perform these exercises, at first, with a view of correcting his prevailing faults only, whatever they be, whether they regard the articulation, command of voice, emphasis, or cadence; and content himself with reading with this particular intention, before he aims at any thing higher. This may be irksome and disagreeable; it will require much time and patience; but it is the only way to succeed. It is recorded of Demosthenes, the greatest orator the world ever produced, that he, though not formed by nature either to please or to persuade, struggled with, and surmounted, the most formidable impediments. He shut himself up in a cave, that he might study with less distraction: he declaimed by the sea-shore, that he might be used to the noise of a tumultuous assembly; and with pebbles in his mouth, to correct a defect in his speech: he practised at home, with a naked sword hanging over his shoulder, that he might check an ungraceful motion to which he was subject. Thus, this great pattern of perfection, both in elocution and eloquence, affords the most encouraging example to every student; since it shews how far art and application could avail, in acquiring an excellence which nature appeared willing to have denied.

When the student, by a strict attention to what has been delivered, has perfectly corrected any ungraceful habits, he may then, and not before, begin to recite some of the higher species of composition; such as, sublime poetry, passionate orations, declamations, and the like; with a view to express the passions, feelings, and sympathies of the author. For this intention, he should daily read aloud, and by himself; and as often as convenient, under the direction of some ingenious person. He should also, if possible, recite compositions from memory. A few select sentences, treasured up in the memory, and frequently repeated, has several advantages attending it. It enables the speaker to enter more fully into the meaning and spirit of the piece, by obliging him to dwell upon

the uses which he is to express. It gives him a previous knowledge of the several inflections, emphases, and tones, which the words require. And by not having his eye confined to the words, it relieves him from the school-boy habit of bending over his book, and, consequently, reading in a different way and tone from that of conversation; and leaves him at full liberty to express his own feelings, by all the varieties of countenance and gesture.

It generally requires some time, and many and frequent exercises, before the student can be brought to consider reading in the same light as conversation; and be persuaded that it should be conducted in the same manner. There is, more or less, in all people (except in a very few accomplished persons), an innate uniformity, which always distinguishes reading from conversation: the fixed posture, the bending of the head, and the staid look at the book, which are requisite, are all destructive of that ease, freedom, and variety of tone, expression and action, necessary to a just elocution.

It would be tedious the necessity of most of the foregoing rules, if public speakers would deliver their discourses from written extracts, or from memory. But if this be too much to be expected, especially from preachers of divinity, who are obliged to compute, and are so frequently called upon to speak in public; it is, however, very necessary, that they should make themselves so well acquainted with their discourses, that they may be able to take in a considerable portion with a single glance of the eye.

After the student has acquired a just and natural elocution, it is, perhaps, not the least difficult part of this art to apply this accomplishment to the purposes of real life. Many can deliver their discourses in a graceful manner in private, or before a few select friends, who, when required to speak in a public capacity, at the bar, from the pulpit, or in the senate, generally betray either a timid bashfulness, or an impertinent assurance. The former is apt to lead the speaker into an awkward uniformity, the latter into a disgusting affectation; the

the former arises from an humble diffidence of the speaker's own abilities, and respect for the understandings of his hearers; the latter is the effect of conceit, and a contempt for the opinions of his auditors: the former may soon be overcome by a successful attention to the foregoing rules; the latter can seldom be subdued, till, by repeated disappointments, the rash adventurer is convinced, that he made a false estimate of his own value.

We shall close this section with Hamlet's instructions to the players, taken from Shakespeare; which has always been considered as containing an important lesson on elocution, and may exemplify most of the foregoing rules.

“ Speak the speech, I pray you, as I pronounced it to you, trippingly on the tongue. But if you mouth it, as many of our players do, I had as lieve the town-crier had spoke my lines. And do not saw the air too much with your hand, thus; but use all gently: for in the very torrent, tempest, and, as I may say, whirlwind of your passion, you must acquire and beget a temperance that may give it smoothness. O! it offends me to the soul to hear a robustious periwig-pated fellow tear a passion to tatters, to very rags, to split the ears of the groundlings, who (for the most part) are capable of nothing but inexplicable dumb shows, and noise: I could have such a fellow whipp'd for o'erdoing ter-magant; it out-herods Herod. Pray you, avoid it.

“ Be not too tame neither; but let your own discretion be your tutor. Suit the action to the word, the word to the action, with this special observance, that you o'erstep not the modesty of nature; for any thing so overdone is from the purpose of playing; whose end, both at the first and now, was, and is, to hold, as 't were, the mirror up to nature; to shew virtue her own feature, scorn her own image, and the very age and body of the time, his form and pressure. Now this overdone, or come tardy off, though it make the un-skilful laugh, cannot but make the judicious grieve: the censure of one of which must, in your allowance, o'erweigh

a whole theatre of others. Oh ! there be players that I have seen play, and heard others praise, and that highly (not to speak it profanely), that neither having the accent of Christians, nor the gait of Christian, Pagan, nor Man, have so strutted and bellowed, that I have thought some of nature's journeymen had made men, and not made them well; they imitated humanity so abominably.

“ O, reform it altogether. And let those that play your clowns, speak no more than is set down for them; for there be of them, that will themselves laugh, to set on some quantity of barren spectators to laugh too; though in the mean time, some necessary question of the play be then to be considered: —that's villainous; and shews a most pitiful ambition in the fool that uses it.”

CHAP. II.

OF PENMANSHIP.

SECT. I.

RULES FOR THE ATTAINMENT OF THE ART OF
WRITING.

THERE is no part of literature acquired with less difficulty than the art of writing. Few people, be their capacities ever so mean, are incapable of learning this. Hence we see so many, who, though ignorant of the more early parts of science, such as English grammar, and even spelling good English, yet can write a tolerably good hand. This is a glaring fault; for the more correct the penmanship, the more does it display the orthographical and grammatical errors. I therefore advise all those who may have occasion to write much, to make themselves perfectly acquainted with what has been delivered in the former chapter concerning English grammar. I trust it need not be mentioned, that they should render themselves perfect in spelling; every one knows the necessity of this, and the ridicule and contempt which only one or two words wrongly spelled bring upon the writer.

I shall proceed to give a few directions, by the help of which an inexperienced person may qualify himself in this art;

art; and without which, though perhaps deemed superfluous by some, a work of this kind might possibly appear deficient.

It is necessary that the learner be provided with the implements requisite for writing; a good pen, and good free ink; without which it is impossible to write a fair copy; a round or flat ruler (the round one is used for dispatch, and the flat one for sureness), a leaden plummet, or black lead pencil, to rule the lines, without which the learner will never write straight; and some pounce, or gum sandrack powder, to rub the paper with, if it be too thin to bear the ink, and when bold hands are to be written, as large text, German text, or the like; also when a word or sentence is scratched out with the penknife, in which case, the place must first be rubbed smooth with the haft of the knife, or a piece of clean paper, and then rubbed with the pounce, to enable it to bear the ink. A quarto-sized copy-book is the most proper, as each page will contain a copy of ten or a dozen lines, which will be sufficient to write at one time.

Being provided with these implements, the learner may proceed to practice. The lines should be ruled straight and even, and at the same distance from each other, at the dotted lines, marked No. 1, in the plate. The distance between every two lines of writing should be about twice or three times the distance which there is between the two pencil lines that belong to the same line of writing; though this is often more or less, according to the caprice of the writer.

The pen must be held in the right hand, between the thumb and the fore and middle fingers. The middle finger must be placed on the back of the pen, opposite to the upper part of the cut or cradle of the pen, and the fore-finger close to it, and both held straight. The thumb must be placed against the opposite side of the pen, called the belly of the quill, and must be bent a little in the joint. The top of the pen should point towards the right shoulder. The elbow should be drawn in towards the body, but not too close. The

arm

arm may rest lightly on the edge of the desk, or table, between the elbow and the wrist; but the stomach should not press against the desk. The pen must be held very lightly; for if it be griped hard, the learner will never acquire an ease and expedition in writing.

The learner, having acquired a just habit of holding the pen, may copy the small letters *i, e, o*; having his lines ruled according to the dotted lines in the plate. He may next copy the other small letters; taking care to be perfect in each, before he proceeds to the next. Then the capital letters; ruling his copy in squares, according to the pattern in the plate; and also join-hand copies, as soon as he can make the capitals. When the learner can write an indifferent good round-hand text, he may proceed to the small round-hand, and running hand; in writing which last, the pen should never be taken off the paper till the word is finished.

The learner should imitate the best copies. Copper-plate copies are to be preferred to those written by the pen, as being more correct. Those small letters which have tops or stems, as *b, d, f, h, k, l, s*, must all be of the same height. And those with tails, as *f, g, j, p, q, s, y*, must be all of the same length. A due distance must be observed between the words, and between the letters of the same word. The capitals must be all of the same size. The upright strokes, or those that are formed by the upright stroke of the pen, must be fine or hair strokes; and the downright strokes must be fuller and blacker; but a constant attention to the copy would in a great measure supersede the necessity of most of the foregoing rules. The learner should not sit long at one time, lest he grow tired of learning, in which case he will not improve; nor be ambitious of writing fast, for five or six lines, well written, will improve the learner more than fifty lines, written in the same time, without attention to their correctness.

When the learner has arrived to some proficiency in writing, it is requisite that he know how to make and mend his

his pen. This is sooner to be acquired by an attentive observance of those who can make a pen well than by any verbal directions; the following rules, however, may be of service. Being provided with a good goose quill (those called seconds are the best), scrape the scurf from it, with the back of the penknife, scraping the back of the quill most, that the slit may be clear; then cut the quill half through, near the end, on the back part; and cut the other side of the quill quite through, near half an inch from the end. The quill will now appear forked; next cut away a very short slit in the back notch, where the slit of the pen is to be; and putting the peg of the penknife haft, or the end of another quill, under this slit, and holding the nail of the left thumb pretty hard on the back of the quill, as high as it is intended the slit shall go, with a quick sudden force rend up the slit: it must be very sudden and quick, that the slit may be clear and close; for if the slit be clear and close, that it cannot be seen through, it is done well. Then, by several necessary cuts, the quill is to be brought into the form of a pen; and having brought it to a fine point on each side of the slit, it is to be nibbed in this manner; place the inside of the nib on the left thumb nail, holding the pen between the fore and middle fingers of the left hand, and enter the knife at the extremity of the nib, and cut it through a little sloping; then, with an almost downright cut of the knife, cut off the nib; lastly, by other proper cuts, the pen is to be brought into an handsome form. But the nib is not afterwards to be mended by any cutting or scraping, for that causes a roughness, and absolutely spoils it. If the nib, therefore, be altered or mended with the knife after it is nibbed, it must be nibbed again, as before directed. And observe that the breadth of the nib must be equal to the breadth of the downright strokes of the letters.

Copies for Round Hand Text.

Avoid bad company.	Be wise betimes.
Care destroys the body.	Do the things that are just.
Expect to receive as you give.	Frequent good company.
God is perfect in his works.	Hours fly swift away.
Innocency need not fear.	Join experience to theory.
Keep faith with all men.	Learn in the time of youth.
Money corrupts many.	No task is too hard to learn.
Opportunities are slighted.	Provide against poverty.
Quiet men have quiet minds.	Remember your duty.
Sin produces shame.	Time and tide stay for none.
Value a good conscience.	Understand your trade.
Wisdom is valuable.	Xerxes wept at mortality.
Yield patiently to fate.	Zeal is sometimes proper.

Copies for Round Hand.

All letters even at head and tail must stand :
 Bear light your pen, and keep a steady hand.
 Carefully strive in each line to excel ;
 Do every thing, that is to be done, well.
 Excel in each new line, in every part :
 Faults for the future shun, by rules of art.
 Gripe not your pen, but hold it very slight,
 Hold in your elbow, have a left hand light.
 In all your writing at the copy look ;
 Join all your letters by a fine hair-stroke :
 Keep free from faults, and blots, your copy-book. }
 Learn the command of hand by frequent use,
 Much practice will good penmanship produce.
 Never strive to write too fast at first ;
 Of all a learner's faults, this is the worst.
 Practice alone can produce expedition ;
 Quick writing is most learners' vain ambition.

Rule your lines straight, and make them very fine ;
Set stems of letters fair above the line,
The tops above the stems, the tails below.
Use pounce to paper, if the ink go through.
View well your copy, see how much you mend ;
Wipe clean your pen, your task being at an end.
Your spelling mind, write each word true and well ;
Zealously strive good writers to excel.

A Receipt for making black Ink.

To one quart of soft water, put four ounces of fresh blue galls of Aleppo, bruised pretty small ; two ounces of copperas ; and two ounces of gum-Arabic. Bottle it up, and shake it once a day, and in three or four weeks it will be fit for use.

The green peelings of walnuts, soaked in the water before the ink is made (if they be in season), will render it the stronger, and more beautiful.

A Receipt for red Ink.

Simmer three pints of stale beer, or vinegar, with four ounces of ground Brazil wood, for an hour ; then strain it through a flannel, and bottle it up for use.

Or a little gum-water and vermilion will make a curious red ink for present purpose.

SECT. II.

OF SECRET WRITING.

SECRET writing may be performed several ways. Former ages were very fertile in inventions of this kind ; and, by these means, intelligence has been obtained by countries, from

others, with whom they were in a state of hostility; and that not unfrequently in modern times. It may also serve individual purposes, where secrecy is required.

There are principally three ways of writing, so as not to be read by any, but those who can discover the manner in which it is written. First, writing in cipher, which requires great ingenuity, and of which, my limits will not permit me to speak. Secondly, substituting other arbitrary marks or characters, for words or letters, than the words or letters themselves. And, thirdly, writing with some ink or liquid which will not appear legible, till rendered so by some mechanical operation.

The second method, of substituting one character for another, is easily performed; as any person might make an alphabet of his own, consisting of twenty-six characters, each of which might stand for some one letter of the English alphabet; and thus the writing would be unintelligible to any but those who have the key or index. Or, the numerical figures may be used to the same purpose: as for example *a* may be represented by 1; *b*, by 2; *c*, by 3; &c. as follows:

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26
a	b	c	d	e	f	g	h	i	j	k	l	m	n	o	p	q	r	s	t	v	u	w	x	y	z

According to this index, the following sentence, *riches gain friends*, will be written thus: 18, 9, 3, 8, 5, 19. 7, 1, 9, 14. 6, 18, 9, 5, 14, 4, 19. Or the figures, to correspond to the letters, may be placed in any other order. Or the letters in the alphabet may be transposed. Certain consonants may be substituted instead of the vowels, and the vowels instead of the consonants: as, instead of the vowels, *a*, *e*, *i*, *o*, *u*, *y*, use *l*, *m*, *n*, *p*, *q*, *r*, and *vice versa*, respectively; then the foregoing sentence will stand thus: *ynchms glni fynmids*. And an infinite number of other ways might be invented, by this mode of substitutional writing, which the ingenious reader will discover; such as, dividing the alphabet into two parts, and transposing the letters which stand in the first or

... in one part, with those in the first or second ... in the other part; or, dividing it into three columns, and applying this rule alternately; first, to the first column, and then to the second: and a number of other ways there are, too numerous to mention.

The third way of writing secretly, is, First, by writing with the juice of a lemon, the juice of an onion, urine, or the spirits of vitriol, which will not appear legible till it be holden to the fire.

Secondly, by tracing the letters on the back of the paper, after it is written, with a pen dipped in milk; these letters, so traced, will not appear legible, till the paper be holden to the fire; then they will appear of a bluish colour. But, in this manner of writing, the paper should be very thin.

The last method I shall mention, is, by using sympathetic inks, as they are generally called: there are various preparations under this name. I shall mention only two, and which may safely be depended upon.

1. If a little green vitriol be dissolved in water, with a little nitrous acid, the characters written with it will be invisible, till they are wetted with the following mixture:—

Put two ounces of small Aleppo galls in half a pint of water; when it has stood three or four days, pour it off. A pencil, dipped in this mixture, and drawn over the letters, written with the former ink, will render them of a beautiful black.

Or letters written with the latter ink, will be invisible, till they are wetted with a solution of Prussian blue in water; and letters written with this solution, will also be invisible, till wetted with the above ink of galls, and water.

2. Incorporate one ounce of litharge of lead with two ounces of distilled vinegar; let it stand twenty-four hours, then strain it off, and let it settle.

Put one ounce of orpiment, in powder, and two ounces of quick lime, in a quart bottle, with water sufficient to
cover

cover them about an inch above the ingredients. Place the bottle in a moderate heat, for twenty-four hours: then pour it off, and cork it close.

The letters written with the former of these inks, will not appear till they be exposed to the vapours of this latter ink; when they will appear perfectly plain.

SECT. III.

OF EPISTOLARY WRITING, AND SUPERSCRPTION OF LETTERS.

Of every species of composition, there is none that, in its nature, approaches nearer to familiar conversation (except plain dialogue) than epistolary writing. A letter is a direct address from one person to another, and should, therefore, contain all the ease, elegance, and familiarity of conversation; paying the same regard to the nature of the subject, and the person addressed, as in a personal application. The principal characteristics of a letter are, nature, simplicity, sprightliness, and wit. The style of a letter should be natural; and appear to express the genuine feelings of the mind. It should not indicate the least mark of study. There should be no formal division of the parts, no laboured introduction, nor pathetic conclusion; but all should appear the spontaneous product of the writer's own emotions. It should likewise be wholly devoid of any complexity or ambiguity of expression: for this purpose, the sentences should be short, and the style perspicuous. It should contain all the vivacity of conversation. And, if the writer be master of any wit, a letter (if the subject permit) affords as proper a place for the
display

display of it, as any composition whatever. A gentle satire, a repartee, or a burlesque, may sometimes be introduced with success; nay, it is often expected, in letters on domestic subjects, between familiar friends.

It is needless to give copies of letters on different occasions, in such a number as is usually done; the subjects of letters being as various as those of conversation: any attempt, therefore, to give specimens of letters, to serve for every purpose for which the writer may have occasion, must be absurd. A learner who copies his letter from any precedent, will not be able to express his own thoughts with ease and freedom. By being confined to a copy, from which he will find it difficult to depart, his letters will carry an awkward stiffness and formality; and he will be a long time before he acquires that freedom, and unadorned elegance, always expected in extemporary writing.

To form an epistolary writer no more is requisite than an intimate acquaintance with English grammar; whereby he will be able to deliver his sentiments with propriety in conversation. If he possess this qualification, it will supersede the necessity of any artificial helps; but if this be wanting, other assistances will be of little use, except to serve to publish his deficiency to his correspondents.

Nevertheless, in conformity to general custom, I have added a few examples. It need hardly be mentioned, that the name, by which the person is addressed, be placed on the left hand side of the page, at the top of the letter; and the letter begun just under it: the name of the writer at the bottom of the letter, on the right hand; the date of the letter, either on the right hand, at the top of the letter, or on the left, and at the bottom: and the letter should conclude with the name by which the receiver of it is addressed at the beginning.

From

From a Master to his Scholar, during the Holidays.

Dear George,

I take the first opportunity that has offered, to inquire after your health, and that of your friends. And I expect, that you will regularly answer each of my letters, that, during this time of leisure, I may have an opportunity of observing, whether you remember, or have forgotten, the rules I formerly gave you, concerning writing letters. I now, therefore, call upon you to put those rules into practice. But, willing to grant you every indulgence at this time of festivity, and lest your recollection should not be so clear, as when in constant exercise, I shall briefly repeat those rules, to which, I hope, you will pay a strict attention.

You remember, no doubt, my first direction was, to be very correct and circumspect in your spelling: this is the first, and most essential requisite in all kinds of writing: and make use of no word, of which you do not perfectly understand the sense. The vulgar part of the world, in general, are very much addicted to this absurdity. You will, now, often hear people condemn a work, as ungrammatical, and deficient in the ornaments of style, though themselves be unacquainted with the first form of grammar, and know not the meaning of a flower in rhetoric.—Avoid repetitions: they always offend the judicious ear, and are seldom proper, except when they enforce any particular meaning, or explain it more fully. Parentheses are always inelegant, and should never be used (*but when absolutely necessary*), as they render the sentence too complex. Never use the long *f* in a word, except when two meet, in which case it is necessary for distinction. A letter interlined has a very ungraceful appearance; it is also an affront: for it indicates either laziness or indifference, or both. Use no capital letters, but at the beginning of a sentence, to proper names, and to the first word of every line in poetry. When you wish to lay a strong emphasis on any word, or intend

and that it should be particularly noticed, it is common to draw a stroke under it with the pen, thus; such words, when printed, are put into *Italics*; but when these emphatical words are employed too frequently, they lose their effect, and when used improperly, they puzzle the reader. Beware of using many monosyllables, they are insignificant words; nor use many too long words, lest you exceed comprehension. Shun particles, as much as possible: be very sparing of your *ands*, *fers*, and *bats*. Be not fond of inventing new words; there are enough, already, to express all our ideas; and more, I fear, than you will ever fully comprehend. Be attentive to the rules of grammar, and do not jumble the present, past, and future times of the verb together; as many incorrect writers do: neither confound the genders of the pronoun; nor use the singular, for the plural verb; which is frequently done: as, *you was*, for *you were*. If the sentence be conditional, use the conditional mode. Let your style be simple and perspicuous, and your sentences short: let it be as concise as possible; for a prolix writer tires the patience of his reader. Observe that your points be all placed justly, which will add grace and perspicuity to your writing. These hints I hope will be attended to: let me see the effects of them in your next letter; while I remain, with compliments to your father and mother,

Dear George,

Berlshire,

Your sincere friend, &c. &c.

Jan. 2, 1803.

The Scholar's Answer.

Honoured Sir,

I return you my sincere thanks for the kind attention you shew me. It shall always be my study and ambition to follow your instructions. I never write a letter to
any

any of my friends, but I pay a particular regard to all the directions you have given me, on the subject of letter-writing. I exercise myself daily, in grammar, arithmetic, or some other part of literature; my father insists upon my setting apart two hours every morning for that purpose. My father and mother desire their best respects to you, and return you their kind thanks for the trouble you have taken in your late letter. I remain, with the greatest respect,

Honoured Sir,

Your very much obliged

And humble Servant,

GEORGE * * *.

London,

Jan. 10, 1803.

A Letter from a Father to his Son, at School.

My dear Boy,

I am exceedingly happy to hear of the great progress you have made in your learning. I hope you will continue to pay the same attention to all your master's instructions as you have done hitherto. I have herewith sent you a present of a gold watch; which I design, in the first place, as an encouragement for you to proceed; and in the second place, as a monitor to remind you of the swiftness and importance of time. This little machine travels with the hours, yet keeps pace with months; it keeps time with minutes, yet does not outrun years. Whenever you look upon this little prompter, let it remind you of your time. Every complete revolution of the minute-hand silently informs you, that there has elapsed, what a whole world cannot redeem—*an hour of your precious time*. And, though you may perform in the following hour, or half-hour, the business that should have been done in the foregoing hour, yet you must remember, that one hour is for ever *irrecoverably lost*. The whole life of man, at the longest, is but short; there is no time to lose. Though you make the best use of your

THE MORE TIME WE HAVE, THE MORE WE WILL BENEFIT (if you have
the proper disposition) and you improved
our work by it. But in the use of your time, though
not to mind, to the intellect. But let me give you the
reason why. Our Christian life is in this sub-
ject. The use of the minutes, for the hours will take
care of themselves. I have seen it in Augustine's work, who
lived a long time in his time, for upwards of thirty years,
and he was a man of great power, one of each twenty-
four, in his knowledge of history and religious exercises,
and that is the way. Now, then, in the secondary avocations of
the mind, and now, then, in the secondary work in cloth, or
in the secondary work. But, then, living in a day in
a day in the secondary work, though it is a matter of such con-
sideration as it is in the secondary work during the rest of his life.

... and now, as what I have said, that I would have you
... of your work, and allow you no time for recreation.
... the hours and days of the night, if you find yourself to dis-
... you is the situation of your companions; but if, at
... at such times, you must your own, either to anticipate
... your future tasks, or to rest or any of those you have gone
... through. I will never make the purpose of your residence
... in London, which is to acquire your mind in useful know-
... ledge, and in a suitable manner, it was resolved more to
... your present situation, and conduct and more to your
... future advantage.

And that I mean by a man I have advanced, is that I would never have him pervert his life. When you are not at your place, be at your work: and when you are not engaged at your work, you may just as play at the appointed times, but no more. Beware of idleness and dissipation: Some boys are too great cowards to play, and too great dunces to learn. Hence they have the features of all that pain, and generally, in the future, prove idle men, who are the most unprofitable animals in the creation. The Arabians have a proverb, that "an idle person is a playfellow for the devil." And it

is

is a maxim of the Chinese law, that, "for one idle person, the whole state suffers." Which is as true as it may appear novel to us; for he eats the produce of others' labour, without making an adequate return.

In order to guard against idleness, never give way to procrastination, or defer that till to-morrow which may be as well done to-day. Many people postpone their business till a future day, merely because it is future; but you must remember, that two days ago, yesterday was a to-morrow; and two days hence, to-morrow will be yesterday. Never, therefore, defer that to a future time which may be done at the present, without a very substantial reason.

Lastly, attend to the manner in which every thing is performed, even the most trivial matters. It is not sufficient that a chapter be read, or a copy written, without a fault, but they should be done well, with all the skill you are master of, if you mean to excel. Whenever you speak, though it be to your own companions, be sure to express yourself in the most proper and expressive words, paying a strict attention to their grammatical construction. Whenever you read, though it be but a sign-post in the street, read with propriety and emphasis. And, whenever you have occasion to write, though it be only your name, let it be done as correct as possible. These directions, constantly observed, will render you an accomplished character, in whatever department of life you may be engaged. I hope the foregoing hints will be sufficient. Let them be well attended to, and I shall, not doubt, see the effect of them in your future acquisitions. Your mother and sister join with me in their love to you.

I am,

Dear Boy,

London,

Your loving Father,

March 20, 1808.

THOMAS * * *.

*The Son's Answer.**C—d—n School, March 25, 1808.*

Honoured Father,

I received your letter, with your kind present of the watch; for which I return you many thanks. I hope it will always have the effect you desire, of reminding me of the shortness and importance of time. I never look at it, but I recollect your alarming letter. I shall always set a just value on time—it is impossible to over-value it. I shall be particularly attentive to all the commands in your letter. I often decline the diversions of my schoolfellows, and apply to my book, conformably to your desire. Yesterday I entered into Decimal Fractions; which having gone through, my master informs me, I shall begin Algebra. When I have the pleasure of coming home, next Christmas holidays, I have no doubt, but you will be well satisfied that I have made a good use of my time. Pray give my duty to my mother, and my kind love to my sister. I am,

Honoured Father,

Your most obedient and dutiful Son,

GEORGE ***.

From a young Shopkeeper to a Wholesale Dealer.

Sir,

By the recommendation of Mr. Chapman, who is my neighbour, I think proper to apply to you for the following articles:—Two dozen pieces of yard-wide calico, ten pieces of the best Jaconet muslin, thirty pieces of ell-wide town-printed couons, &c. &c. I hope you will let them be of the best quality of their sort, and at a reasonable price; as I intend all our dealings shall be for ready money.

Hull,

March 20, 1808.

I am, Sir,

Yours, &c.

The

The Answer.

Sir,

London, March 24, 1803.

I have, in answer to your favour of the 20th instant, sent you the articles you ordered. Should you think proper to favour me with your orders, I have no doubt but I shall give you ample satisfaction. I am,

Sir,

Your obliged Servant, &c.

From Wholesale Dealers to a Retail one.

Sir,

As you have lately been very backward in your remittances, we are under the necessity of informing you, that unless you send us an immediate draft, or order, for 40*l.* which has so long been due, we must use such means as will prove very disagreeable both to you and ourselves.

We remain,

Liverpool,

Sir,

March 25, 1803.

Yours, &c.

The Answer.

Gentlemen,

Several severe and unexpected losses have been the sole cause of my late remissness. I am very sensible of the lenity you have always shewn me. I only request your acceptance of my bill, payable in two months; money being at this time very scarce, and my debtors very backward in their payments.—Should this be agreeable to you, I shall always be very punctual for the future. I am,

Berkshire,

Gentlemen,

March 30, 1803.

Your much obliged Servant, &c.

The

The Reply.

Sir,

Liverpool, April 3, 1808.

As we should be sorry to distress you, we are willing to accede to your proposals; and hope, that, by the time you mention, you will be able to make good your promise. We remain,

Sir,

Your humble Servants, &c.

From a Tradesman to his Friend.

Sir,

Relying upon the sincerity of your friendship, I have taken the liberty to solicit your assistance at this critical juncture. My affairs are greatly embarrassed; and, unless I can procure the sum of 200*l.* I shall inevitably be ruined. If, therefore, you can oblige me with the above sum, to be paid by instalments, at three, six, nine, and twelve months, at 5*o**l.* each payment, you will save me from impending bankruptcy, and infinitely oblige,

Sir,

March 27, 1808.

Your distressed Friend, &c.

P.S. You know the value of my stock, and also my expectations.

The Answer.

My dear Friend,

I am very sorry to hear of your present difficulties. I desire you will call upon me to-morrow, when I hope I shall be able to accommodate you with the sum you want; or, if I cannot furnish you with the whole, I can let you have the greater part, and the remainder in a few days.

I am,

*Westminster,**March 30, 1808.*

My dear Friend,

Yours sincerely, &c.

5

It

It is as necessary to know how to direct, or subscribe, as to write a letter. Persons in high rank, and peculiar stations in life, are to be addressed in a peculiar manner.

For the Directions or Supercriptions of Letters.

To the King—To the King's Most Excellent Majesty.

To the Queen—To the Queen's Most Excellent Majesty.

To the Prince of Wales—To His Royal Highness, George Prince of Wales.

To the Princess—To Her Royal Highness, &c.

To the Nobility.

To a Duke—To [His Grace] the Duke of, &c.*

To a Marquis—To [the Most Noble] the Marquis of, &c.

To an Earl—To [the Right Honourable] the Earl of, &c.

To a Viscount—To [the Right Hon.] Lord Viscount, &c.

To a Lord—To [the Right Honourable] Lord, &c.

The sons of dukes, marquesses, and the eldest sons of earls, have the title of Lord, and Right Honourable; and the title of Lady belongs to their daughters, with the addition of their Christian and surnames: as, Lady Caroline Russel, &c.

The eldest sons of dukes, marquesses, and earls enjoy the second title of their father, by the courtesy of England.

The children of viscounts and barons are also styled Honourable.

To a Baronet—To [the Honourable] Sir J. H. Bart, &c.

To a Knight—To [the Right Worshipful] Sir J. H. Knt.

To a Privy Counsellor—To [the Right Honourable] J. H. one of His Majesty's Most Honourable Privy Council.

To an Ambassador—To His Excellency, &c.

Also plenipotentiaries, foreign governors, the lord lieutenant of Ireland, and lords justices of Ireland, and the captain general of His Majesty's forces, have all the title of Excellency added to their other titles.

* The words included in crotchets are sometimes omitted.

The title of Right Honourable is given to no commoner, except members of the Privy Council, the three lord-mayors of London, York, and Dublin, and the lord-provost of Edinburgh, during their office only.

To the Clergy.

To an Archbishop—To the Most Reverend Father in God, &c. ; or, To His Grace the Lord Archbishop of York, or Canterbury.

To a Bishop—To the Right Reverend Father in God, &c. ; or, To the Lord Bishop of, &c.

To Deans, Archdeacons. &c.—To the Rev. A. B. Dean of, &c. ; or, To the Rev. A. B. Archdeacon of, &c.

To the inferior Clergy—To the Rev. Mr. A. &c. ; or, To the Rev. Dr. &c.

To the Military and Navy.

All colonels are styled Honourable ; and all inferior officers should have the name of their rank set first : as, To Major G. C. ; To Captain I. H. &c.

All admirals in the navy are styled Honourable ; the inferior officers are styled by their rank, as in the army.

To the Judges and Lawyers.

To the Lord High Chancellor of Great Britain—To the Right Honourable J. Lord E. Lord High Chancellor of Great Britain—Lord President of the Council—Lord Privy Seal—One of His Majesty's Principal Secretaries of State, &c.

To the Master of the Rolls—To the Right Honourable Sir W. G. Knt. Master of the Rolls.

To a Judge—To the Right Honourable E. Lord E. Lord Chief Justice of the King's Bench, or of the Common Pleas.

To the other Judges—To the Right Honourable A. B. Esq. one of the Justices of, &c. ; or, To Judge A.

To the Chief Baron—To the Hon. Sir A. M. Lord Chief Baron.

To His Majesty's Attorney, Solicitor, or Advocate General—

To Sir A. B. His Majesty's Attorney, Solicitor, or Advocate General.

Every judge has the title of Right Honourable, if a Privy Counsellor; otherwise only Honourable. All others in the law are styled according to the office they bear; and all barristers have the title of Esquire.

To the Lieutenantcy and Magistracy.

To a Sheriff—To A. B. Esq. High Sheriff of the County (of York).

To a Recorder—To the Right Worshipful A. B. Recorder of the City of, &c.

To an Alderman—To the Right Worshipful A. B. Esq. Alderman of Tower Ward.

The aldermen and recorder of London are styled Right Worshipful, and Esquire; as are also all sheriffs, and mayors of corporations, during their office.

Every gentleman in the commission of the peace has the title of Esquire, and Worshipful.

To the Governors of the Crown.

To the Lord Lieutenant of Ireland—To his Excellency A. Lord B. Lord Lieutenant of Ireland.

To the Governor of Dover Castle—To the Right Honourable A. Earl of D. Governor of Dover Castle, &c.

The second governors of foreign settlements, appointed by the king, are called Lieutenant Governors. Those appointed by the proprietors of the settlement, as the East India Company, &c. are called Deputy Governors.

To either House of Parliament, to Commissioners, Bodies Corporate, &c.

To the House of Lords—To the Right Honourable the Lords Spiritual and Temporal, in Parliament assembled.

To the House of Commons—To the Honourable the Knights, Citizens, and Burgeses, in Parliament assembled.

To the Right Honourable the
 Lords Commissioners of the Treasury; or Admiralty.

To the Commissioners of His Majesty's Customs;
At the Office of the Commissioners of His Majesty's Customs;
London.

City of London—To the Master,
Magistrates and Court of Aldermen of the Worshipful
Commons of the City of London

Address of the Right Worshipful—To the Right Wor-
shipful Governors of Christ's Hospital.

~~Sheweth~~ To the Honourable Court of
~~the~~ The United Company of Merchants trading
~~in~~ in India

... To the Honourable Society of the
... of the ... of the ...

... .. of hospitals, colleges, &c.

or of any other of the said Orders, or of any of those titles.

... to know how to begin a letter to any of
 my friends

The Language of Letters.

1. Learn or Discover your Mission.

Yours at War to serve your Majesty.

... - - - - - our Rev'd Highness.

I am, Sir, your most obedient servant.

Yours in Christ, and in the love of your Grace.

— **Adieu, et May it please your Grace.**

My Lord, or May

— I am, Sir, at your Ladyship's service.

... **and the House.**

... and the ... Worship.

Yours most Obedient Son, &c.

THE UNIVERSITY OF CHICAGO

And I beg to assure your Lordship,

To

*To their Consorts—*Madam, or May it please your Ladyship.

*To all the other Clergy—*Reverend Sir, or Sir.

*To an Ambassador—*Sir, or Your Excellency.

*To the Lord Mayor of London, York, or Dublin, and Lord Provost of Edinburgh, during their office—*My Lord, or May it please your Lordship.

*To an Alderman, Recorder, or Sheriff—*Sir, or May it please your Worship.

*To the Lord Lieutenant of Ireland—*My Lord, or May it please your Lordship.

*To the House of Lords—*My Lords, or May it please your Lordships.

*To the House of Commons—*May it please your Honours.

*To the Lords of the Treasury, or Admiralty—*My Lords, or May it please your Lordships.

*To the Commissioners of the Customs, or Excise—*Gentlemen, or May it please your Honours.

*To the Benchers of an Inn of Court—*Gentlemen, or May it please your Honours.

*To the Directors of the East India Company—*Gentlemen, or May it please your Honours.

*To the Governors of Christ's Hospital—*Gentlemen, or May it please your Worships.

*To a Company of the City of London—*Gentlemen, or May it please your Worships.

It may not be amiss, in this place, to insert a table of precedence, by which the reader will see the rank which each individual bears in England, according to his office and occupation. Taken from the Heralds' books.

Table of Precedence.

The King.

Queen.

The King's Children, and
Grandchildren.

The King's Brethren.

Uncles.

Nephews.

Archbishop of Canterbury.

Lord Chancellor, or Keeper, if a Baron.		Lords Commissioners of the Great Seal.
Archbishop of York.		Viscounts' eldest Sons.
Lord Treasurer,	} if barons	Earls' younger Sons.
Lord President of the Council,		Barons' eldest Sons.
Lord Privy Seal,		Knights of the Garter.
Lord Great Chamberlain,	} above all Peers of their own degree.	Privy Counsellors.
Lord High Constable,		Chancellor of the Exchequer.
Lord Marshal,		Chancellor of the Duchy.
Lord Admiral,		Chief Justice of the King's Bench.
Lord Steward of the Household,		Master of the Rolls.
Lord Chamber- lain of the Household,		Chief Justice of the Common Pleas.
Dukes.		Chief Baron of the Exchequer.
Marquisses.		Judges and Barons of the Coif.
Dukes' eldest sons.		Knights Bannerets, royal.
Earls.		Viscounts' younger Sons.
Marquisses' eldest Sons.		Barons' younger Sons.
Dukes' younger Sons.		Baronets.
Viscounts.		Knights Bannerets.
Earls' eldest Sons.		Knights of the Bath.
Marquisses' younger Sons.		Knights Bachelors.
Secretary of State, if a Bishop.		Baronets' eldest Sons.
Bishop of London.		Knights' eldest Sons.
———— Durham.		Baronets' younger Sons.
———— Winchester.		Knights' younger Sons.
Bishops.		Colonels.
Secretary of State, if a Baron.		Serjeants at Law.
Barons.		Doctors.
Speaker of the House of Commons.		Esquires.
		Gentlemen.
		Yeomen.
		Tradesmen.
		Artificers.
		Labourers.

Married

Married women and widows are entitled to the same rank among each other as their husbands would have been entitled to among themselves; except such rank be merely professional, or official:—and unmarried women to the same rank as their eldest brothers would bear during the lives of their fathers.

SECT. IV.

THE ART OF STENOGRAPHY, OR SHORT HAND.

STENOGRAPHY, or Short Hand, is the art of writing in a more expeditious manner than by the common mode, for the purpose of taking down a speech, or discourse, as delivered by the speaker.

For this purpose, the writer of short hand is permitted to avail himself of several advantages which no other writer is allowed. He has the liberty of inventing an alphabet of his own, consisting of certain arbitrary marks, or characters, which, for the sake of expedition, are of a less complex form than those of the other alphabets. He may reject any of the letters in the common alphabet, which are not absolutely necessary for the writer to recollect the sense; or substitute one and the same character to serve for two distinct letters; as is done here in the short-hand alphabet, where *f* and *v* are used indiscriminately the one for the other, and represented by the same character; as are also the *k* and hard *c*, the *s* and the soft *a*. He also omits the vowels, except where they are indispensably necessary to discover the sense. Consequently, he is not to follow the customary mode of spelling; but is to insert no more letters in a word than are precisely necessary to express the sound.

The

The short-hand writer, besides these advantages, sometimes makes use of single characters to express whole words, and even whole sentences; but of this hereafter.

The first and principal rule in short hand is, *to make use of no more letters than are necessary to give the reader an idea of the sound of the word.* For if the writer use all the letters that are necessary to express the *sound itself*, he will gain but little advantage by it; as he will be obliged to use too many letters to be very expeditious.

But it must be here observed, that this rule must not be followed too strictly at first, and during the learner's exercises; lest it render his writing too imperfect, and unintelligible to himself. He should, therefore, at first, content himself with using every character for every consonant, till he be perfectly acquainted with them all, and can very readily form them; marking the points for the vowels, as is hereafter directed.

Of the Short-hand Alphabet.

The short-hand alphabet consists of the following consonants: *b, d, f* or *v, g* or *j, k* or hard *c, l, m, n, p, q, r, s* or soft *c*, and *x, t, w, x*: each of which has its proper character, as seen in the plate. The characters standing for these letters are to be neatly joined together, to form words; and written in the most expeditious manner possible, without taking the pen off the paper till the word be finished; and the vowels are to be noted afterwards (if necessary) by points.

Though the omission of the vowels may at first somewhat puzzle the reader, to read even his own writing, yet a little practice will render it perfectly familiar to him; as the chief difficulty, both in writing and reading short hand, arises from the novelty of the characters, and from the want of a familiar acquaintance with them.

In order to perfect the learner in the construction of all the characters in the short-hand alphabet, it is absolutely necessary that he frequently copy them. He should, daily,
write

write a copy of the short-hand alphabet; ruling his lines according to the dotted lines in the first line of the first lesson in the plate; and writing the signification of each character at the end of each line, and the letter it represents at the beginning of the line; using three or four lines of a quarto copy for each letter.

In the alphabet given, I doubt not but the characters will be found as convenient for practice as any extant. I have endeavoured to make it consist of the most simple, and, at the same time, the most distinct characters. I shall only observe, that by the help of this alphabet, and the abbreviations that follow, I have been able to follow the most rapid speakers.

This alphabet, though consisting of the characters for fifteen consonants only, will be found quite sufficient. The *c* being supplied by either *k* or *s*, according as it is pronounced, hard or soft. The *k* may very well be omitted in short hand, as it is only an aspiration of the breath; and the sound of the word (which is all that is minded in this species of writing) may be discovered without it. The *j* is supplied by the *g* soft, which has the same sound. *V* is also supplied by *f*, being only a coarser sound than that letter. *Y* is represented, if necessary, by the vowel *i*, having exactly the same sound; and *z* by *s*, from which it differs in sound but a little, being of a coarser nature.

The vowels are seldom used in short hand; but when it is necessary to insert any of them, they are represented by points, as will be seen hereafter.

Three of the characters in this alphabet are horizontal figures: viz. *d*, *k*, and *m*; as are also the three characters which represent the double letters, *ch*, *th*, *sh*. Seven of them are vertical or lineal, running directly from the top line to the bottom; viz. *f*, *g*, *l*, *n*, *q*, *r*, and *x*. The other five, *b*, *p*, *t*, *s*, and *w*, are inclined characters.

In forming the short-hand characters singly, the learner may observe (if he wishes to have his writing look exact),

that the horizontal characters are only half the height of the lineal, and inclined ones. For the two latter ones generally occupy the whole space between the two lines, as seen in the plate. But this direction is to be strictly adhered to only while the learner is forming the characters singly: when he comes to join the characters together, to form words, it will be often necessary to make them less or greater than their usual size; sometimes for convenience, and at other times from necessity: as will be shewn in its proper place.

Of the B.

This letter, in short hand, is formed by an inclined curve line, or small segment of a circle; it should always be formed from the top, which is on the left hand; and universally I would advise, that all inclined and horizontal letters be formed from the left hand part, as it is the most natural, and will be found the most expeditious. This letter may also be used to express the words, *be*, *by*, *but*, and *black*; and, with a short lateral stroke joined to the fore part, it represents the word *before*; and the same stroke, when placed in the hinder part, signifies the word *behind**; as shewn in the plate. This letter is also used to denote the inseparable prepositions *be*, *re*, and *an*, when belonging to a word: as, *be-have*, *re-quire*, &c. It also forms the terminations *ble*, or *ble*, both singular and plural, and *by*: as, remark-*able*, move-*ables*, and *re-by*.

C.

This letter is not distinguished by a separate character; for its sound (except before *c*, which is provided for) is either hard, like *k*; or soft, like *s*. It is, therefore, represented by the one or other of these characters, according to its sound.

* When the stroke is drawn straight the character, it signifies the words *before* and *behind*; and when the stroke is set at the left hand point, it stands for the word *beginning*.

D.

This letter is the segment of a small circle, with its two points downwards; it is begun at the left hand point, as the last. When used alone, it represents the word *and*, *hand*, and *end*. It also stands for the prepositions *de*, *di*, *dis*: as, *de-bate*, *di-vide*, *dis-pute*. And is used for the termination *ed*: as, *turn-ed*.

F, or V.

This letter is formed by a straight, downright or upright stroke of the pen (for it may be made either upwards or downwards, as is found most convenient to the writer). When it follows a letter that is finished at the bottom, it is formed upwards; but when it succeeds a letter that is concluded at the top, it is formed downwards, that the pen be taken off the paper as seldom as possible. It also stands for the *v*, which is a letter of nearly the same sound, being only a little coarser, and is, therefore, not distinguished in short hand by a separate character: but if it be necessary to distinguish it at any time from the *f*, it is only making the character a little blacker, or fuller. This letter also represents the words *for*, *if*, *of*, *off*, and *few*; and the inseparable preposition *for*: as, *for-get*. And the terminations *fy*, *ify*, *ful*, and *fulness*: as, *de-fy*, *sanct-ify*, *forget-ful*, *forget-fulness*.

G, or J.

This letter consists of a segment of a circle, the same size as the *d*, but formed in a different direction from that letter; this being in a vertical direction, running from the top line to the bottom one. It is also used for the *j*, which has partly the same sound as the soft *g*. But, when used for the *j*, it should be written fainter, particularly towards the bottom. This letter may also be begun either at the top or the bottom, as is found most convenient to make it join with the foregoing letter. When it stands alone, and as a *g*, it represents the words *again*, *against*, *great*; and as a *j*, it signifies the words *judge*, *just*, *join*. Joined to other characters, it repre-

from the word *grand* : as, *grand-father* ; and the terminations
grace and *grace* : as, *grace*, *grace*.

H.

There is no character to distinguish this letter ; for as it is only an aspiration of the breath, its insertion is not necessary to discover the sound of the word ; it is, therefore, wholly omitted.

I.

This is represented by *g*, as hath been shewn.

K, or C hard.

This letter is formed in the same shape and size as the *d*, but in the quite opposite direction ; it is always begun at the left hand. When it stands alone, it represents the words *car*, *cald* ; and with a short stroke drawn under the middle, it signifies *canon*. Used as a preposition, it signifies *con*, *con*, and *contra* : as, *con-pound*, *con-ceal*, and *contra-ry* ; as a termination, it represents *con*, *con*, and *con* : as, *spect-acle*, *period-ical*, and *art-icle*.

L.

This letter is formed of a straight lineal stroke, like the *f*, but turned a little at the top, towards the left hand. It is mostly begun at the top ; but, if found necessary, may be made from the bottom. By itself, it represents the words *all*, *always*, *altogether* ; joined to other characters, it stands for *low*, *latter*, and *late* : as, *low-ly* ; *latter-ly* ; *late-ly*, and, as a termination, it signifies *ly* : as in *low-ly*, *latter-ly*, and *late-ly* ; thus, two of those characters represent each of these words ; and when thus used, should have its vowel point to the first syllable.

M.

This letter is represented by a straight lateral stroke ; and is always begun at the left hand. Alone, it represents the words *am*, *amongst*, *my*, *mine*, *me*, *whom* ; also, *much* or *many* ; when made larger than common, it signifies the word *more* ; with a short stroke across the middle, it stands for the words
twice

twice as much, or twice the number, or double the quantity; with two such strokes, it signifies three times as much, &c.; and with a dot placed under the middle, it stands for half as much, &c.; and with two such dots placed under it, it signifies the word most. With the negative stroke under it, it signifies not me, not so much, &c. Joined with other characters, it represents the prepositions magni, and mis: as, magni-fy, mis-fortune; and as a termination, it stands for ment or ments: as, testa-ment.

N.

This letter is formed of a straight lineal stroke, with the top a little turned towards the right hand. It is mostly begun at the bottom, but it may be formed from the top, if requisite. When used alone, it represents the words *an, in, under*; as a preposition, it stands for *ante, anti, in, inter, under*: as, *ante-cedent, anti-podes, in-ter, inter-rogation, under-mine*. As a termination, it stands for *ant* and *ness*: as, *garm-ent, har-ness*.

P.

This letter is formed of an inclined curve line, leaning towards the right hand. It is always begun at the bottom. When used alone, it stands for the words *apart* and *poor*. As a preposition, it represents *per, pre, and pro*: as, *per-plex, pre-sage, and pro-long*. As a termination, it is seldom or never used in English.

Q.

This letter is formed of a straight stroke in a lineal direction, with the top curled quite round towards the left hand. It may be begun either at the top or the bottom, as is found most convenient. When it stands alone, it signifies the words *question* and *quantity*; with a short stroke across the middle, it represents the phrase *double the quantity*; and with a dot placed under it, it stands for *half the quantity*; with the negative mark under it, it stands for *not the quantity*. When joined to other characters, it represents the words *a quarter, and a quart*: as, *a quarter of an hour, a quart of water*. As a termination, it stands for *quire*: as *re-quire*.

N 2

R. This

R.

This letter is formed like the *g*, but in an opposite direction. It may be formed either upwards or downwards, as most convenient. When used alone, it stands for *er* and *ether*; when joined to other characters, it represents *re*: as, *re-solve*. As a termination, it stands for *ary*, *ing*, *ings*, and *er*: as, *di ary*, *exceed-ing*, *turn-er*.

S, soft C and Z.

This character is formed of an inclined curve line, and is mostly begun at the bottom. It represents also the soft *c*, and the *z*: but as the *z* has a coarser sound, it may be represented by a blacker and fuller character. When used alone it signifies *as*, *is*, *us*, and *yes*: when it is joined to other characters, it stands for *satis*, *circum*, *super*, and *sub*: as, *satis-fy*, *circum-vent*, *super-fine*, *sub-tract*. As a termination, it stands for *tion*, *self*, and *soever*: as, *ora-tion*, *my-self*, *whom-soever*.

T.

This letter is formed of an inclined but straight line, and is mostly begun at the top. Alone, it stands for *the* and *to*. As a preposition, it stands for *trans*: as, *trans-fer*. As a termination, it signifies *ty* or *ties*: as, *beau-ty*, *beau-ties*.

W.

This letter is formed of a straight but inclined stroke, like the former, but in the opposite direction. It is mostly begun at the bottom. By itself, it stands for *will*, *wilt*, *would*, and with a short stroke drawn under the middle it represents *will not*, or *would not*. Joined to a following character, as a preposition, it signifies *with*: as, *with-out*. As a termination, it stands for *ward* and *well*: as, *out-ward*, and *holy-well*.

X.

This letter is formed of a straight lineal stroke, a little turned at the bottom towards the right hand. It is generally formed downwards, but it may also be formed upwards, if found more convenient. Alone it stands for *except*, *exercise*, and *excellent*. Joined to other characters, it represents *ex*,
extra,

extra, and *exceed*: as, *ex-pose*, *extra-ordinary*, *exceed-ingly*. As a termination, it represents *lex*: as, *comp-lex*.

Y.

There is no character appropriated to this letter, being represented by the vowel mark for *i*.

Z.

This letter is represented by the character which stands in common for the soft *c*, and *s*; as hath been seen.

Besides the characters appropriated to express the consonants singly, there are three other characters for three double consonants, viz. *ch*, *sh*, *th*.

CH.

The character used to express these two letters, is a straight lateral stroke, turned upwards a little, at that end towards the left hand. It is always begun at the left hand point. When it stands alone, it signifies *each*, *which*, and *such*. As a preposition, it stands for *arch*: as, *arch-itect*.

SH.

The character appropriated to these letters is like the foregoing; but turned downwards at the left hand point, instead of upwards, and like that is always begun at the left hand. When it stands alone, it represents the words *shall*, *shalt*, and *should*; and with a short stroke under the middle, *shall*, *should*, or *would not*. As a termination, it stands for *ship*: as, *ward-ship*.

TH.

The character used to denote these two letters, is also a straight lateral stroke, with the end turned a little upwards, at the right hand point. This is also begun at the left hand. When it stands alone, it represents the words *they*, *them*, *that*, *this*, and *than*.

There are also other characters appropriated to express several treble consonants, which frequently occur in words of most common use.

CHR.

CHR.

The character used for these three letters, is two perpendicular lines joined at the top by a curve:—it is mostly begun at the left hand point; and is used in the words *Chr*-ist; and, with the vowel *y* for *a*, in the word *chr*-mer.

LCH.

This character is formed like the two sides of a triangle, with the point downwards; and is mostly begun at the left hand: as the foregoing.

LTR.

This character is formed like the two sides of a square, with the conjunction of the line, at the bottom, and towards the left hand. It is generally begun, like most of the others, at the left hand.

LTH.

This character is formed like that of *Lh*, but with the point upwards; and is begun at the left hand point.

MPL.

This character is a double curve, and inclined towards the left hand, at the top; at which part it is begun.

NCH.

This character is just the reverse of that for *chr*, and begun at the left hand.

RCH.

This character is composed of the two sides of a square, joined at the top, towards the right hand.

RTL.

This character is an inclined double curve, leaning towards the right hand, being in the opposite direction to that for *mpl*.

RTH.

This character is an inclined line, leaning towards the left hand, where it is begun, and curled quite round at the top.

SHR.

This character is also an inclined line, but leaning towards the right hand, and curled round at the bottom.

SKR.

This character is also an inclined line, leaning towards the right hand, and curled round at the top.

STR.

This character is formed of two straight lateral lines, joined together at the left hand end by a curve. This letter must be begun at the top, and at the right hand,

SPL.

This character consists of the two sides of a square, joined together at the bottom of the right hand side. It is always begun on the left hand.

SPR.

This character is formed of a double curve, and inclined towards the left hand, like that for *mpl*; but the curves are formed in an opposite direction.

THR.

This character is an inclined line, leaning towards the left hand, and curled round at the bottom.

Here it must be noted, that the characters for those treble consonants are to be used, though the consonants which they represent do not follow each other immediately; as in the words *altar*, and *arithmetic*.

When the learner is perfectly acquainted with the construction and full signification of each of the foregoing characters, he may then proceed to join these characters together, in order to form words; observing, that each character be neatly joined with the foregoing one; for which purpose, it is often requisite, that many of the characters be formed less than their real size; as may be seen in the plate.

Of the Vowels.

The vowels, in short hand, are represented by dots. In the perpendicular and inclined letters, *a* is represented by a dot, placed on the left hand side of the letter, and near the top of it: *e*, by a similar dot, near the bottom, on the left hand side;

... above the letter, like the tittle
 marked by a dot placed near the top, but
 of the letter, just opposite the place of
 by a dot near the bottom of the right
 the place of *e*. But, in horizontal cha-
 is placed under the letter, and at the left
 for *e*, under the letter also, but at the
 the *i*, over the letter, as before; the *o*, over
 end of the letter, and opposite the place of *a*;
 the right hand point, opposite the place of *e*.

The order of placing the vowels about the character for
 consonant, is to be observed only when the vowel imme-
 diately follows such consonant; if the vowel precede the con-
 sonant, just the reverse takes place, in placing the vowels.
 When this is the case, the dots for the *a* and *e* are placed on
 the right hand side of perpendicular and inclined letters, in
 the place of *o*, and *u*, respectively; and *o* and *u* are placed on
 the left hand where *a* and *e* are placed in the former case;
 that is, when the vowel follows the consonant. And *i*, or *y*,
 is always noted by a dot, placed above the consonant it
 follows. In horizontal letters the same rule obtains, whether
 the vowel follows or precedes the consonant.

But the former method of marking the vowels, viz. by
 setting the mark about the consonant which immediately
 precedes the vowel, is more natural and perspicuous; and
 should always be followed, if possible.

To mark the double or treble vowels, when necessary, a
 dot should be set in the place of that vowel which is of the
 nearest sound to such double or treble vowel: as for example,
 to express the *ea* in the word *great*, we use the vowel point
 for *a*: as, *grat*; and to express the triphthong *eau*, in *beau*,
 we use the vowel point for *e*: as, *bo*: but for the vowels in
 the word *beauty*, we use *u*: as, *bu:y*.

It is necessary that the learner, in his first exercises in this
 art, make the character for every letter of which each word
 is composed, marking all the points for the vowels. This is
 necessary

necessary in order to render every word intelligible at first view; for many learners, who can tell the signification of every character, when it stands alone, are often at a loss to distinguish the meaning of their own writing, when the characters are joined together; partly owing to the strange appearance they make to the eye, and partly to the omission of some of the letters. I have, therefore, in the first lesson in the plate, inserted the character for every consonant, and also all the points for the vowels; which method the learner must follow, till the compound characters are become quite familiar to him.

When the learner can readily, and at a future time (or at least several days after it is written), read those compound characters easily, he may proceed to avail himself of the next advantage this art allows, namely, that of inserting no more characters in a word than are absolutely necessary to discover its sound. Thus, for the word *universe*, *uv*, with the vowel mark for *u*, will be sufficient to convey an idea of the sound, as it shews the letters *uvu*. Using also the simple characters of the alphabet for the words, prepositions, and terminations, they there signify.—Upon this principle, the second lesson is formed.

But as the chief excellency of short-hand writing consists in expedition, and which the foregoing rules will not sufficiently favour, there are also other methods of abbreviation, by which much time is saved. It is not sufficient that words be expressed by an alphabet of very simple characters, but they should also be expressed by as few of those characters as possible; or the chief end of writing short hand, which is that of following a speaker, would be frustrated. I shall, therefore, lay down a few simple rules, which, if strictly adhered to, will enable the learner to write as expeditiously as is necessary.

RULE I.

In writing the consonants of the same word, the end of the foregoing character is always joined to the beginning of the

writing, the *i* may be written as *iv* and *iv* as *i*. In the case of *iv* the *i* is the same as the *i* in *iv* and *iv* as *i* in *iv*. The *i* and *iv* are the same. The *i* and *iv* are the same. The *i* and *iv* are the same.

RULE II.

When a word is used which requires a point in writing, as a substantive, adjective, verb, or adverb, the point is to be placed after the word, as in the following examples: *substantive*, *adjective*, *verb*, *adverb*. If the word is a substantive, a point is to be placed after the word, as in the following examples: *substantive*, *adjective*, *verb*, *adverb*. If the word is an adjective, a point is to be placed after the word, as in the following examples: *substantive*, *adjective*, *verb*, *adverb*. If the word is a verb, a point is to be placed after the word, as in the following examples: *substantive*, *adjective*, *verb*, *adverb*. If the word is an adverb, a point is to be placed after the word, as in the following examples: *substantive*, *adjective*, *verb*, *adverb*.

RULE III.

When a word is used which requires a point in writing, as a substantive, adjective, verb, or adverb, the point is to be placed after the word, as in the following examples: *substantive*, *adjective*, *verb*, *adverb*. If the word is a substantive, a point is to be placed after the word, as in the following examples: *substantive*, *adjective*, *verb*, *adverb*. If the word is an adjective, a point is to be placed after the word, as in the following examples: *substantive*, *adjective*, *verb*, *adverb*. If the word is a verb, a point is to be placed after the word, as in the following examples: *substantive*, *adjective*, *verb*, *adverb*. If the word is an adverb, a point is to be placed after the word, as in the following examples: *substantive*, *adjective*, *verb*, *adverb*. The point where the word is to be placed should be marked with a straight line, which line shall be at right angles to the line of the character: as *iv* in the plate: for distinguishing names. When the substantive thus expressed is a noun, the substantive point is to be made a little longer than usual. The word *iv* has no point to express it.

RULE IV.

When the same word is required to be repeated with a point, as a substantive, adjective, verb, or adverb, it is only placing another point after its substantive, adjective, or adverb point: as in fig. 5, *time after time*.

RULE

RULE V.

When two words are expressed by their first consonants, and a preposition should be between them, which is omitted, they are to be joined together in the common manner, and a point placed at their point of conjunction : as in fig. 7 and 8, *the love of money, the thirst for gain.*

RULE VI.

When any word begins with any of the prepositions, it is to be begun with the mark for such preposition ; and, in general, the next consonant will be sufficient. And when they end with any of the terminations, the first, or first and second consonant of the word with such termination, will generally suffice to discover the sound. But when prepositions and terminations are used, as they are joined to the other characters like single letters, the word should always have at least one of its vowel points to prevent confusion.

RULE VII.

Most of those phrases which consist of a word with a preposition, conjunction, or article, before and after it, may be written as one word.

RULE VIII.

When a pronoun follows a preposition, they are joined together, as the foregoing. And pronouns may always be expressed by their first, or first and second consonants, having the vowel point if requisite.

RULE IX.

To express the definite article *the*, the consonant mark for *t* is used ; and to express the article *a*, a small point is placed at the beginning of the following consonant.

The foregoing rules are all that can be safely given ; and if strictly attended to, will answer every purpose that can be

expected from this art. I have avoided arbitrary characters for certain long words and phrases, as I am convinced they are not adapted to every one's memory; and from the great number of words with which our language abounds, it is impossible that a few hundred characters to express so many peculiar words can be of much utility to the learner, and particularly as the chief use of short hand is to follow a public speaker, many of whom possess a very extensive flow of language. Besides this, the method of substituting a set of characters for many words or phrases is very burdensome to the memory, difficult to learn, and is easily and soon forgotten.

I have thought proper to mark all the vowels by points, according to many of my predecessors in this art, though I have differed from all others in the order of placing them. Dr. Mavor, Mr. Angel, Mr. Mason, and Mr. Gurney, also express the vowels by points; but the method of the three latter is to denote the vowels by points placed on the same side of the letter: thus *a* and *e* are represented by points near the top of the character, or the beginning of it; *i* by a point in the middle of the character; and *o* and *u* by points near the bottom or latter end of the character: whereas, in this system, the points for *a* and *e* are placed on one side the character, and those for *o* and *u* on the other side of the character, and the point for *i*, at the top; which must give this method a great advantage over the other. By the former method the place of the *a* and *e* cannot always be distinguished from each other, nor that of the *o* and *u*: by this method, I will venture to assert, they will never be confounded.

Though some writers on this subject have given different characters for all the vowels, I have given the preference to points: they give the writer infinitely the advantage over the other method, in point of expedition; and by an attentive perusal of his work after he has written it, and while the subject is still fresh in his memory, he may mark all the vowels; which will render it intelligible at any future time.

With regard to the characters for the consonants, I have
endeavoured

endeavoured to render them as simple as possible in their construction, yet perfectly different from each other. Most writers on this art either make their characters too complex, or, if they make them simple, form them so much alike, that they are easily confounded with each other. In some modern systems, the difference between different characters consists only in one being a fine and the other a black stroke; which distinction, writers of short hand have seldom time to make; not to mention the other similarities between the characters, such as, different characters being of the same form, but of different sizes; with many other absurdities. While, on the other hand, those who have rendered their characters sufficiently different, have formed them so complex, as to give the learner no advantage, in point of facility, above the common round hand. It must be allowed by every one, that the more simple a character is, the more easily it is formed. When a character consists of two lines, whether straight or curve, it requires almost twice as much time to form it as to form a character consisting of only one of such lines: and characters which consist of two lines joined together at one end, so as to form an angle, require still more time to form them. Mr. Willis, who published a system of short hand in the year 1628, has no less than thirteen characters in his alphabet, which are formed with angles in them, besides two which are complete ovals, or circles. And Mr. Mawd, who published another system in 1635, has also eleven angled characters, and those for the most part taken from Mr. Willis. Mr. Skelton, also, who improved upon the two former, has, in his alphabet, published in 1659, fourteen very complex characters, namely, ten angled ones, three curled at one end, and one complete circle. But Mr. Rich, who published his system in 1669, has only seven angled characters, three curled ones, and one entire circle. His system was in great repute for many years, and Mr. Locke recommends it in his treatise on Education (because there was at that time none better). Yet Mr. Nicholas, who published his system thirty years after Mr. Rich, and evidently took

short hand, in which the characters are all simple, straight, or curve lines, except one curled character, one circular one, and a cross one. His alphabet is, therefore, more simple and fit for practice than that of any of his predecessors, and the letters are sufficiently different from each other to prevent mistakes.

Since Mr. Blanchard published his system, several others have appeared, which I have not room to mention; suffice it to say, that from that period to the present, most, if not all the writers on the art have run into the opposite extreme to the writers before his time, which is, that of rendering their alphabetical characters *too simple*. In many modern alphabets some of the characters resemble each other so nearly, that the writer, some time after he has written his piece, is often at a loss to discover the sense of the greater part of it. I have met with several writers who have learned from Mr. Taplin's system, and have experienced this difficulty. Mr. Taplin's alphabet has not one angled or compound character, and only one small circular one for the letter *s*; but then he has one character for two or more letters: thus, *b*, *f*, *p*, and *v* are all represented by a straight stroke in the same position, and distinguished from each other only by the length or boldness of the character: the same may be observed of the characters for *g*, *j*, *k*, and *x*, which are also straight lines in the same position: and though the similarity between those and other characters in that alphabet may not be so apparent when viewed singly, yet when such characters come in contact with others, it seldom fails to puzzle its writer, except he be very conversant in the system.

Mr. Taplin's method of marking the vowels is sometimes by dots, and sometimes by the situation of one of the consonants.

I have noticed the alphabetical characters only of the foregoing writers, without adverting to their other characters for double and treble letters, words, and phrases, as those characters form the standard of each writer's system.

Concerning the alphabet I have here the honour to propose, I must just observe, that I have avoided all angular, compound,

and

and circular characters, and used only one curled one, which is for the letter *q*; and I trust I have made them sufficiently distinct in their forms. There are some angled and compound characters, used for the treble letters; but then it should be remembered that each of these characters with a vowel point, will very often serve for a whole word, and I was afraid, by indulging too great a simplicity in these characters, I might degenerate into the modern uniformity.

The method of marking the vowels by points, as I have here advised, will be found a very great saving of time. It will also afford those who can so far depend upon their memory, an opportunity of entirely omitting them, till the writing be finished; when, by an attentive perusal, the piece may have all its vowels marked; which will render it intelligible to the writer at a future time, when he has, perhaps, forgotten the subject matter.

A PRAXIS,

OF THREE LESSONS.

On Modesty.

Lesson I.—I know no two words, that have been more abused by the different and wrong interpretations which are put upon them, than these two, Modesty and Assurance. To say, such a one is a modest man, sometimes indeed passes for a good character; but, at present, is very often used to signify a sheepish, awkward fellow, who has neither good breeding, politeness, nor any knowledge of the world.

Again, a man of assurance, though, at first, it only denoted a person of a free and open carriage, is now, very usually, applied to a profligate wretch, who can break through all the rules of decency and morality without a blush.

I shall endeavour, therefore, in this essay, to restore these words to their true meaning; to prevent the idea of Modesty from being confounded with that of Sheepishness, and to hinder Impudence from passing for Assurance.

... I would call
 ... when a man
 ... himself, or
 ... of others.

... much so when he
 ... to a bias in his
 ... upon him.

... any instance of mo-
 ... that celebrated one
 ... a tributary king to
 ... against him before the
 ... subjects. The prince
 ... coming into the se-
 ... proved upon him,
 ... to speak, that he

... The story tells us, that the
 ... instance of modesty and in-
 ... the most pathetic ora-
 ... father for this early

... a man's self,
 ... any uneasi-
 ... generally gives a
 ... of the world, but
 ... to do nothing
 ... An open and af-
 ... such a resolu-
 ... are at any
 ... from a con-
 ... enough to de-
 ...

... and encourage in
 ... have here mentioned.

A man without modesty is liable to be made uneasy by the
 ... of every one he converses with. A man
 ... is lost to all sense of honour and virtue.

It is more than probable, that the prince above mentioned possessed both these qualifications in a very eminent degree. Without assurance he would never have undertaken to speak before the most august assembly in the world; without modesty he would have pleaded the cause he had taken upon him, though it had appeared ever so scandalous.

From what has been said, it is plain, that modesty and assurance are both amiable, and may very well meet in the same person. When they are thus mixed and blended together, they compose what we endeavour to express when we say a modest assurance: by which we understand the just mean between bashfulness and impudence.

I shall conclude with observing, that as the same man may be both modest and assured, so it is also possible for the same person to be both impudent and bashful.

We have frequent instances of this odd kind of mixture in people of depraved minds and mean education; who, though they are not able to meet a man's eyes, or pronounce a sentence without confusion, can voluntarily commit the greatest villanies or most indecent actions.

Such a person seems to have made a resolution to do ill even in spite of himself, and in defiance of all those checks and restraints his temper and complexion seem to have laid in his way.

Upon the whole, I would endeavour to establish this maxim, that the practice of virtue is the most proper method to give a man a becoming assurance in his words and actions. Guilt always seeks to shelter itself in one of the extremes, and is sometimes attended with both.

I have, for the ease of the learner, distributed all the words of each of the foregoing lessons, into a regular order; and prefixed to each word, or phrase, its number, as it is numbered in the plate; by which the characters that compose it may be seen at one view.

In the first lesson is inserted every consonant belonging to each word, with the point for every vowel ; which practice, as was before observed, the learner must pursue, till he be very ready in both writing and reading all his exercises. The first ten words in this lesson have their characters formed separately from each other.

In the second lesson no more letters in a word are inserted than are absolutely necessary to discover the sound. The single consonant marks are also used for the words, prepositions, and terminations they represent ; and the characters for the double and treble letters.

The third lesson is written upon the most finished principles of this art. Above all the advantages which it possesses, in common with the second lesson, it has, moreover, all the abbreviations delivered in the foregoing rules. And which rules the writer should have very perfectly in his memory.

LESSON I. (*See the Plate.*)

1 I know	24 Modestly	47 to
2 no	25 and	48 signify
3 two	26 Assurance.	49 a sheepish,
4 words	27 To	50 awkward
5 that	28 say,	51 fellow,
6 have	29 such	52 who
7 been	30 a one	53 has
8 more	31 is	54 neither
9 abused	32 a modest	55 good
10 by	33 man,	56 breeding,
11 the	34 sometimes	57 politeness,
12 different	35 indeed	58 nor
13 and	36 passes	59 any
14 wrong	37 for	60 knowledge
15 interpretations	38 a good	61 of
16 which	39 character ;	62 the
17 are	40 but	63 world.
18 put	41 at	64 Again,
19 upon	42 present	65 a man
20 them,	43 is	66 of
21 than	44 very	67 assurance,
22 these	45 often	68 though,
23 two,	46 used	69 at

70 first,	92 the	114 to
71 it only	93 rules	115 prevent
72 denoted	94 of	116 the
73 a person	95 decency	117 idea
74 of	96 and	118 of
75 a free	97 morality	119 modesty
76 and	98 without	120 from
77 open.	99 a blush.	121 being
78 carriage,	100 I shall	122 confounded
79 is	101 endeavour,	123 with
80 now,	102 therefore,	124 that
81 very	103 in	125 of
82 usually,	104 this	126 sheepishness,
83 applied	105 essay,	127 and
84 to	106 to	128 to
85 a profligate	107 restore	129 hinder
86 wretch,	108 these	130 impudence
87 who	109 words	131 from
88 can	110 to	132 passing
89 break	111 their	133 for
90 through	112 true	134 assurance.
91 all	113 meaning ;	

LESSON II.

1 If	21 for	41 is
2 I were	22 which	42 as
3 put	23 he	43 much
4 to	24 censures	44 so
5 define	25 himself,	45 when
6 modesty,	26 or	46 he
7 I would	27 fancies	47 is
8 call	28 that	48 alone
9 it,	29 he	49 as
10 the	30 is	50 in
11 reflection	31 exposed	51 company,
12 of	32 to	52 and.
13 an ingenuous	33 the censure	53 as
14 mind,	34 of others.	54 subject
15 either	35 For	55 to
16 when	36 this	56 a blush
17 a man	37 reason	57 in
18 has	38 a man	58 his
19 committed	39 truly	59 closet,
20 an action	40 modest	60 as

61 when	106 a tyrant	155 that
62 the	109 and	156 the fathers
63 eyes	110 oppressor	157 were
64 of	111 of	158 more
65 multitudes	112 his	159 moved
66 are	113 subjects.	160 at
67 upon	114 The prince	161 this
68 him.	115 went	162 instance
69 I do not	116 to	163 of
70 remember	117 Rome	164 modesty
71 to	118 to	165 and
72 have	119 defend	166 ingenuity,
73 met	120 his	167 than
74 with	121 father ;	168 they
75 any	122 but	169 could
76 instance	123 coming	170 have
77 of	124 into	171 been
78 modesty	125 the senate,	172 by
79 with	126 and	173 the most
80 which	127 bearing	174 pathetic
81 I am	128 a multitude	175 oration ;
82 so	129 of	176 and,
83 well	130 crimes	177 in
84 pleased,	131 proved	178 short,
85 as	132 upon	179 pardoned
86 that	133 him,	180 the guilty
87 celebrated	134 was	181 father
88 one	135 so	182 for
89 of	136 oppressed	183 this
90 the young	137 when	184 early
91 prince,	138 it	185 promise
92 whose	139 came	186 of
93 father,	140 to	187 virtue
94 being	141 his	188 in
95 a tributary	142 turn	189 the son.
96 king	143 to	190 I take
97 to	144 speak,	191 assurance
98 the Romans,	145 that	192 to
99 had	146 he	193 be
100 several	147 was	194 the faculty
101 complaints	148 unable	195 of
102 laid	149 to	196 possessing
103 against	150 utter	197 a man's
104 him	151 a word.	198 self,
105 before	152 The story	199 or
106 the senate,	153 tells	200 of
107 as	154 us,	201 saying

202 and	230 determined	258 or
203 doing	231 in	259 actions
204 indifferent	232 itself	260 are
205 things	233 to	261 at
206 without	234 do	262 any
207 any	235 nothing	263 time
208 uneasiness	236 against	264 misinterpreted
209 or	237 the rules	265 retires
210 emotion	238 of	266 within
211 in	239 honour	267 himself
212 the mind.	240 and	268 and
213 That	241 decency.	269 from
214 which	242 An open	270 a consciousness
215 generally	243 and	271 of
216 gives	244 assured	272 his
217 a man	245 behaviour	273 own
218 assurance	246 is	274 integrity
219 is	247 the natural	275 assumes
220 a moderate	248 consequence	276 force
221 knowledge	249 of	277 enough
222 of	250 such	278 to
223 the world,	251 a resolution.	279 despise
224 but	252 A man	280 the little
225 above	253 thus	281 censures
226 all,	254 armed,	282 of
227 a mind	255 if	283 ignorance
228 fixed	256 his	284 and
229 and	257 words	285 malice

LESSON III.

1 Every	15 to be
2 one	16 made
3 ought	17 uneasy
4 to cherish	18 by the folly or
5 and encourage	19 ill-nature
6 in himself	20 of every
7 the modesty and	21 one
assurance	22 he
8 I have	23 converses with.
9 here	24 A man without modesty
10 mentioned.	25 is lost
11 A man	26 to all
12 without assurance	27 sense
13 is	28 of honour
14 liable	29 and virtue.

30 It is	77 thus
31 more	78 mixed
32 than probable,	79 and blended
33 that the prince	80 together,
34 above mentioned	81 they
35 possessed both	82 compose
36 these qualifications in	83 what
37 a very	84 we endeavour
38 eminent	85 to express
39 degree.	86 when
40 Without assurance	87 we say
41 he	88 a modest assurance:
42 would never	89 by which
43 have undertaken	90 we understand
44 to speak	91 the just mean
45 before the most	92 between
46 august	93 bashfulness & impudence.
47 assembly	94 I shall conclude
48 in the world;	95 with observing,
49 without modesty	96 that
50 he	97 as the same
51 would have	98 man
52 pleaded	99 may be
53 the cause	100 both modest and assured;
54 he	101 so it is also
55 had taken	102 possible
56 upon him,	103 for the same
57 though it had	104 person
58 appeared	105 to be
59 ever	106 both impudent and bash-
60 so scandalous.	ful.
61 From what	107 We have
62 has been	108 frequent
63 said,	109 instances
64 it is	110 of this
65 plain,	111 odd
66 that modesty and assurance	112 kind
67 are	113 of mixture
68 both	114 in people of
69 amiable,	115 depraved
70 and may	116 minds
71 very well	117 and mean
72 meet	118 education;
73 in the same	119 who,
74 person.	120 though they
75 When	121 are not able
76 they are	122 to meet

123 a man's	151 to have laid
124 eyes,	152 in his way.
125 or pronounce	153 Upon the whole,
126 a sentence	154 I would endeavour
127 without confusion,	155 to establish
128 can	156 this maxim,
129 voluntarily	157 that the practice
130 commit	158 of virtue
131 the greatest	159 is the most
132 villanies,	160 proper
133 or most	161 method
134 indecent	162 to give
135 actions.	163 a man
136 Such a person	164 a becoming assurance
137 seems	165 in his words
138 to have made	166 and actions.
139 a resolution	167 Guilt
140 to do ill,	168 always
141 even	169 seems
142 in spite	170 to shelter
143 of himself,	171 itself
144 and in defiance	172 in one
145 of all	173 of the extremes,
146 those checks	174 and is
147 and restraints	175 sometimes
148 his temper	176 attended
149 and complexion	177 with both.
150 seem	

CHAP. III.

OF TENSAR ARITHMETIC.

SECT. I.

REVIEW OF NUMBERS.

ARITHMETIC is the most necessary of all the sciences. It is the basis of all the sciences, and the perfection to which this science is brought is sought above any other branch of knowledge.

ARITHMETIC is that part of arithmetic which treats of the use of numbers as in proper characters, and consequently in their due places; also of the value of each number, when it stands alone.

NUMBERS are the various combinations of them, are one, two, three, four, five, six, seven, eight, nine, ten, or nothing.

THESE NUMBERS are used alone, stands for no value, or stands for one, two, three, four, five, six, seven, eight, nine, ten, or nothing. But when any of them are joined together, they stand for more than one, two, three, four, five, six, seven, eight, nine, ten, or nothing. Thus, one, two, three, four, five, six, seven, eight, nine, ten, or nothing, joined together, stands for one, two, three, four, five, six, seven, eight, nine, ten, or nothing.

In

In order to discover the value of any compounded number it must be observed, that a number placed in the first place towards the right hand, stands for no more than its real intrinsic value, but increases in value in a tenfold proportion by every remove towards the left hand: thus, in the number 1799, the first figure 9 stands for 9 only; the second figure being in the second place towards the left hand, its value is increased tenfold; thus it represents ninety, or ten times nine; which, with the foregoing 9, stands for ninety-nine. Again, the figure 7, which stands in the third place towards the left hand, is increased to ten times as much as it would be if it stood in the next inferior place, viz. where the last-mentioned 9 stands: thus it represents seven hundred; which, with the two fore-mentioned figures, stand for seven hundred and ninety-nine. The figure 1, which stands in the fourth place, towards the left hand, is also increased ten times in value to what it would be if it stood in the next inferior place, where the 7 is placed; in which case it would represent one hundred; whereas, in the present instance, it stands for one thousand; and with the other figures represents one thousand, seven hundred, and ninety-nine.

This description of the four foregoing figures may serve to give the uninformed an idea of the value of figures, according to the different places they occupy in a compounded number. For every remove of a figure towards the left hand increases its value to ten times as much as before; as will more fully appear by the following table, called the Numeration Table:—

The Numeration Table.

<i>Mill.</i> <i>millions</i>			<i>Thous.</i> <i>millions</i>			<i>Millions</i>			<i>Thous.</i>		
Hundreds of millions of millions.			Hundreds of thousands of millions.			Hundreds of millions.			Hundreds of thousands.		
Tens of millions of millions.			Tens of thousands of millions.			Tens of millions.			Tens of thousands.		
Millions of Millions.			Thousands of millions.			Millions.			Thousands.		
1	2	3	4	5	6	7	8	9	0	9	8
4	3	2	1	0	1	2	3	4	5	6	7
			9	8	7	6	5	4	3	2	1
			4	5	6	7	8	9	0	9	8
			4	3	2	1	0	1	2	3	4
						7	8	9	0	9	8
									3	2	1
									6	7	8
									0	9	8
										1	2
											3
											4
											5
											6
											7
											8
											9
											0
											1

In order to read any number with facility and ease, it is necessary that the learner have all the names of the numbers at the head of the table perfectly in his memory, that he may apply them to any other number he may have occasion for; calling the first figure, on the right hand, units; the second, tens; the third, hundreds; the fourth, thousands; &c. as in the table.

Thus, the bottom figure in the table, standing under the place of units, is itself an unit, or a single one. The next higher

higher number in the table consists of two figures, 1 and 0; the first whereof, standing in the place of tens, stands for ten, being only a figure one: the other figure, being a cypher, and in the place of units, stands for nothing, or no *units*: these two figures, therefore, express only ten. The next number consists of the figures 432; the four being in the place of hundreds, signifies so many hundreds: the three, as many tens; and the two, as many units; and is thus expressed: four hundred and thirty-two. The fourth number in the table consists of four figures, the first whereof stands in the place of thousands; this number, therefore, is thus expressed: eight thousand, seven hundred, and sixty-five. The fifth number has its highest figure in the place of tens of thousands, and is thus expressed: seventy-eight thousand, nine hundred, and nine; it having a cypher in the place of tens, which stands for nothing. The sixth number consists of hundreds of thousands, and is thus expressed: one hundred and twenty-three thousand, four hundred, and fifty-six. The highest place of the seventh number is that of millions; it is expressed thus: six million, five hundred and forty-three thousand, two hundred, and ten. The eighth number consists of tens of millions, and is thus expressed: sixty-seven million, eight hundred and ninety thousand, nine hundred, and eighty-seven. The ninth number has its highest figure in hundreds of millions; it is expressed thus: three hundred and twenty-one million, twelve thousand, three hundred, and forty-five.

The six other numbers are expressed as follows:—The tenth number: seven thousand, eight hundred and ninety millions, nine hundred and eighty-seven thousand, six hundred, and fifty-four. The eleventh number: forty-three thousand, two hundred and ten million, one hundred and twenty-three thousand, four hundred, and fifty-six. The twelfth number: four hundred and fifty-six thousand, seven hundred and eighty-nine million, ninety-eight thousand, seven hundred, and sixty-five. The thirteenth number:

nine

nine million of millions, eight hundred and seventy-six thousand, five hundred and forty-three million, two hundred and ten thousand, one hundred, and twenty-three. The fourteenth number: forty-three million of millions, two hundred and ten thousand, one hundred and twenty-three millions, four hundred and fifty-six thousand, seven hundred, and eighty-nine. The fifteenth number: one hundred and twenty-three million of millions, four hundred and fifty-six thousand, seven hundred and eighty-nine millions, ninety-eight thousand, seven hundred, and sixty-five.

I have, in the table, distinguished every three figures by a point, or comma, beginning at the right hand, as is generally done in public offices, and by men of extensive business.

This method also affords an easier way of enumerating numbers, than by the foregoing table, as every three figures may have a common surname appropriated to them (inserted in italics at the head of the table), besides their names of units, tens, and hundreds: thus, when the learner can enumerate the first three figures in a number, and knows the proper surname to apply to each three figures, he may enumerate any number, however large. The first three figures on the right hand have no surname, as they stand simply for units, tens, and hundreds; but the next three figures have the surname of *thousands*; the next three have the surname of *millions*; the next three, *thousands of millions*; and the other three figures have the surname of *millions of millions*. Thus, to repeat the highest number in the table, beginning at the left hand, I say, one hundred and twenty-three (to which I add its surname of) million of millions; four hundred and fifty-six (with its surname) thousands of millions; seven hundred and eighty-nine (with its surname) millions; ninety-eight (surname) thousands; seven hundred, and sixty-five.

I have been more particular in the description of the Numeration Table, as it is generally found the most difficult of all the tables in Arithmetic to a learner; and several persons who have arrived to a tolerable proficiency in this science,

are,

are, nevertheless, very imperfectly acquainted with this most essential part.

Besides the foregoing ten characters, used to express numbers, there are also letters employed for the same purpose, called Numerical Letters. This was the ancient method of expressing numbers, and is still made use of frequently, in the title-pages of books, and in funeral monuments in Roman history, to express the date of the year.

I, stands for one.

V, five.

X, ten.

L, fifty.

C, an hundred.

D, or IO, five hundred.

M, or CIO, a thousand.

IOO, five thousand.

CCIOO, ten thousand.

IOOO, fifty thousand.

CCCCIOOOO, a hundred thousand.

IOOOOO, five hundred thousand.

CCCCCIOOOO, ten hundred thousand, or, a million.

The letters MDCCCVIII express the number 1808, the date of the present year—M standing for one thousand, D for five hundred, CCC three hundred more, which is eight hundred, and VIII eight; together one thousand, eight hundred, and eight.

If a letter or letters of inferior value follow one of superior value, they are to be added thereto: thus, VI signify six, VII seven, VIII eight, and DCC seven hundred. But when a letter of inferior value is placed before one of superior value, it is then to be deducted therefrom: thus, IV signify four, IX nine, XL forty, CD four hundred, &c.

the same column, adding the other figure or figures to the next column, proceeding in this manner till the whole be finished, and setting down the total amount of the last column under the same.

Thus, in the first example, I say 2 and 4 is 6, and 9 is 15, and 7 is 22, and 3 is 25, and 2 is 27, and 1 is 28; this being the amount of the first column, I set down 8 (which is the figure in the place of units) under the same column, and carry the remaining figure 2 to be added to the next column, saying, 2 and 2 is 4; and 5 is 9, and 4 is 13, and 1 is 14, and 3 is 17, and 1 is 18, and 2 is 20, the whole amount of the last column; wherefore I set it down under the column, and the total is thus found to be 208.

Proceeding in the same manner, in the second example, I say 5 and 4 is 9 (for the 0 stands for nothing), and 3 is 12, and 9 is 21, and 1 is 22; wherefore, I set down the 2 under the column, and carry the remaining 2 to be added to the next column; saying, 2 and 3 is 5, and 3 is 8, and 1 is 9, and 3 is 12, and 7 is 19, the amount of the second column; wherefore I set down the 9 under the column, and carry the remaining figure 1 to be added to the next column, saying, 1 and 1 is 2, and 1 is 3, and 8 is 11, and 6 is 17, and 7 is 24, and 2 is 26, and 1 is 27, the amount of the last column, and to be set under the same; wherefore the total is 2792 (two thousand seven hundred and ninety-two).

In the same manner the third example is wrought; as also the three following; in which the operation is purposely omitted, for the practice of the learner.

Gallons.	Yards.	£.
17964017	12794368	16729420
43706123	64368901	12090736
94218740	20107036	67342617
29991815	45736298	74298064
58997252	12469927	29020721
55122753	72970617	41739402
Total 300000700	Total 228447147	Total 241220900
282036683	215652779	224491540
Proof 300000700	Proof 228447147	Proof 241220960
Vol. I.	K	Here

Here it will be seen, that when the amount of any column is 12, that has a cypher in the place of units, such cypher is to be placed under the column; as in the first of the above examples.

The above two examples are proved—which is done in this manner—after the total is found according to the foregoing rules, and placed in the first of the three bottom lines, the top line of the sum is to be separated by a line drawn under it, the remaining part of the sum cut up, and the amount of it placed under the original total; this last amount, and the top line of the sum, are then to be added together; and if the amount of these two lines be equal to the total in the first bottom line, the sum is rightly cut up; otherwise not.

All other sums in addition are to be proved in a similar manner, whether they be of one denomination or of divers denominations.

Addition of divers denominations consists in adding together numbers of different denominations, whether they be money, weights, or measures.

Before the learner proceeds to addition of money, it is necessary that he have the following tables by heart; called the *Pence Table*, and the *Shilling Table*.

The Pence Table.

		s.	d.
20	pence is	1	8
30	—	2	6
40	—	3	4
50	—	4	2
60	—	5	0
70	—	5	10
80	—	6	8
90	—	7	6
100	—	8	4
110	—	9	2
120	—	10	0

Shilling Table.

		£.	s.
20	shillings is	1	0
30	—	1	10
40	—	2	0
50	—	2	10
60	—	3	0
70	—	3	10
80	—	4	0
90	—	4	10
100	—	5	0
110	—	5	10
120	—	6	0

In addition of divers denominations, those numbers which are of the same denomination are always placed in the same column, and their name or mark generally at the top: thus, in addition of money, the pounds are placed in one column, with an £ at the top for *libra*, the Latin word for *pounds*; the shillings in another column, with an s at the top for *solidi*, the Latin word for *shillings*; the pence also by themselves, in another column, with a d at the top for *denarii*, the Latin for *pence*; and sometimes the farthings are placed in a separate column, with a qr. at the top for *quadrans*; but the most general rule is, to place them immediately after the pence, as a fraction of a penny: thus, $\frac{1}{4}$ for one fourth part of a penny, or one farthing; $\frac{1}{2}$ for one half, or a halfpenny; $\frac{3}{4}$ for three fourths of a penny, or three farthings: as in the following examples:—

£.	s.	d.
5	7	9
12	0	2
1	6	10
0	7	9
20	10	4
3	15	11
2	7	2
6	10	11
52	15	10
47	8	1
52	15	10

£.	s.	d.
27	9	4 $\frac{1}{2}$
130	4	0
124	11	3
60	7	9 $\frac{1}{2}$
166	19	10 $\frac{1}{2}$
101	7	9 $\frac{1}{2}$
24	0	0 $\frac{1}{2}$
107	15	6
778	15	7 $\frac{1}{2}$
751	6	3
778	15	7 $\frac{1}{2}$

£.	s.	d.
172	9	2
225	8	7 $\frac{1}{2}$
144	19	11
176	12	10 $\frac{1}{2}$
238	17	11
734	14	7 $\frac{1}{2}$
814	17	10
037	0	0
3445	0	11 $\frac{1}{2}$
3272	11	9 $\frac{1}{2}$
3445	0	11 $\frac{1}{2}$

In addition of numbers of divers denominations, this is the general rule:—To begin at the right hand column, or the column of the least denomination: adding all the numbers in the said column together, and observing how many units there be in it of the next higher denomination, and how much overplus: the overplus is to be set down under the line, and the units of the next denomination are to be added to the next column. The next column being cast up, it must

also be noted how many units there are in it of the next higher denomination, and how much overplus: the overplus is to be placed under its own column, and the units of the next denomination added to the next column, as before; and so on to every new column.

Thus, in the first of the three foregoing examples, I begin with the unit figures of the lowest denomination, which is pence, saying, 1 and 2 is 3, and 1 is 4, and 4 is 8, and 9 is 17, and 2 is 19, and 9 is 28; to which I add the tens in the same column, saying, 28 and 10 is 38, and 10 is 48, and 10 is 58; now, by the pence table, I know that 58 pence is 4 shillings and 2 pence, therefore 58 pence is 4 shillings and 10 pence: thus, I must set down the 10 pence, which is the overplus, under the same column; and the 4 shillings, which are 4 units of the next higher denomination, I carry to be added to the next denomination, saying, 4 and 9 is 13, and 7 is 20, and 5 is 25, and 7 is 32, and 6 is 38, and 7 is 45; to which I add the tens, saying, 45 and 10 is 55, and 10 is 65, and 10 is 75: now, by the shilling table, I find that 75 shillings is 3 pounds 15 shillings; the 15 shillings are to be set under the column of shillings, and the 3 pounds, being units of the next denomination, viz. pounds, are to be added thereto: and the pounds, being the last and highest denomination, are to be added together as simple numbers of one denomination, and the total placed under the same: thus the total is found to be 52*l.* 15*s.* 10*d.*

In the next example I begin with the farthings, being of the lowest denomination, saying, 2 farthings and 2 is 4, and 3 is 7, and 1 is 8, and 2 is 10; 10 farthings is 2 pence and 2 farthings; the 2 farthings is to be set under the line as overplus, and the 2 pence is to be added to the column of pence, which, added up, amounts to 43 pence, which is 3 shillings and 7 pence; the 7 pence is to be set under the column of pence, and the 3 shillings is to be added to the column of shillings; in which column, with the 3 added, there is 75 shillings, or 3 pounds 15 shillings; the 15 shillings

lings must be placed under the shillings, and the 3 pounds added to the column of pounds, and the pounds added together as before, and the total placed underneath: thus the amount of this sum is 778*l.* 15*s.* 7*d.*

In the same manner the third, and every other sum in this part of addition, is to be wrought. The sums are all ready proved, for the ease of the learner. The line under the total shews the amount of the sum without the top line: the bottom line shews the amount of the line just over it added to the top line of the sum, and proves the work right, by being equal to the first total.

More Examples for the Learner's Practice.

£.	s.	d.
472	7	4½
19	2	0
145	19	10½
7	0	1
0	10	0½
27	8	2
30	7	9½
145	0	4
301	10	10
29	7	2½
6	6	0
1185	0	2
712	12	9½
1185	0	2

£.	s.	d.
1762	9	6½
1000	3	2
10	10	6½
12	9	3
17	4	0½
123	18	0½
129	0	0
71	6	9½
41	2	11
7	4	2½
8	8	0
3183	16	5½
1421	6	11½
3183	16	5½

£.	s.	d.
179	9	9½
184	19	0½
209	7	9
301	8	10½
401	6	4½
21	8	7½
33	0	0½
12	12	0½
19	4	4
8	8	0½
6	12	10½
1368	17	9
1189	7	11½
1368	17	9

Addition of Avoirdupois Weights.

By this weight are weighed all kinds of goods whatever, except bread, gold, silver, and precious stones.

The Table of Avoirdupois Weight.

4 quarters	make	1 dram,	—	marked <i>dr.</i>
16 drams	—	1 ounce,	—	<i>oz.</i>
16 ounces	—	1 pound,	—	<i>lb.</i>
28 pounds,	—	1 quarter of a hundred weight,	—	<i>qr.</i>
4 quarters	—	1 hundred weight,	—	<i>C.</i>
20 hundred weight	1 ton,	—	—	<i>T.</i>

Avoirdupois weight is divided into two parts, called the greater and the less. Avoirdupois the greater consists of hundreds, quarters, and pounds: avoirdupois the less is divided into pounds, ounces, and drams.

	4	28	
<i>C.</i>	<i>qrs.</i>	<i>lb.</i>	
7	3	12	
12	0	9	
13	2	7	
37	1	0	
8	2	7	
40	0	20	
1	3	0	
2	0	7	
<hr/>			
123	1	6	

	4	28	
<i>C.</i>	<i>qrs.</i>	<i>lb.</i>	
24	2	4	
10	0	9	
0	1	20	
24	2	0	
36	3	21	
40	2	0	
50	1	22	
40	0	10	
<hr/>			
227	2	11	

	16	16	
<i>lb.</i>	<i>oz.</i>	<i>dr.</i>	
9	9	10	
20	13	14	
37	12	0	
49	4	7	
62	0	1	
10	7	0	
11	6	7	
14	0	12	
<hr/>			
215	6	3	

These examples are wrought in the same manner as the former; but having a regard to the avoirdupois table, instead of the pence table. And the number of units in each column, which compose an unit in the next higher column, is placed at the head of the column: thus, in the first example, as 28 is the number of pounds which form one quarter of an hundred, it is placed over the column of pounds; and 4, which is the number of quarters contained in an hundred, is placed over the column of quarters. And in the last example, 16 being the number of drams contained in an ounce, it is placed over the column of drams; and, as 16 ounces form a pound, 16 is placed over the column of ounces;

ounces ; and the like is observed in addition of other weights and measures.

In the first of these examples the number of pounds in the first column is 62, which is 2 quarters, and 6 pounds over ; the 6 is to be set under the pounds, and the 2 quarters added to the column of quarters, which, with the amount of the whole column, is 13 quarters, which is 3 hundreds and 1 quarter ; the 1 quarter is to be set under the quarters, and the 3 hundreds added to the next column, and the whole amount is found to be 123c. 1qr. 6lb.

Addition of Troy Weight.

By this weight are weighed bread, gold, silver, and precious stones. The usual denominations are pounds, ounces, pennyweights, and grains.

The Troy Weight Table.

24 grains	make	1 pennyweight.
20 pennyweights		1 ounce.
12 ounces		1 pound troy *.

Examples of Troy Weight.

lb.	oz.	pw.	gr.
4	9	1	17
2	7	19	7
7	0	17	8
1	2	12	9
0	10	14	20
4	11	10	11
7	4	3	2
<hr/>			
28	10	19	2
<hr/>			

lb.	oz.	pw.	gr.
12	7	11	21
14	8	12	19
19	2	19	8
20	10	7	7
7	11	6	12
14	10	0	15
12	0	10	12
<hr/>			
102	3	8	22
<hr/>			

lb.	oz.	pw.	gr.
32	9	2	20
154	8	1	0
47	0	19	21
39	2	12	12
249	4	10	10
347	7	0	9
558	9	19	1
<hr/>			
1429	8	6	1
<hr/>			

* The goldsmiths divide the pound troy into 12 marks, instead of ounces ; and each mark into 24 carats ; and eachcarat into 4 grains.

These

These examples are wrought according to the general rule given in the former part of this section, and in the same manner as addition of avoirdupois; but paying a strict regard to the table, or the figures at the head of each column.

Note. That the pound avoirdupois is heavier than the pound troy; but the troy ounce is heavier than the avoirdupois ounce, for 1*lb.* 2*oz.* 12*grs.* troy is equal to a pound avoirdupois; and a pound troy is about 1*3oz.* 2½*grs.* avoirdupois.

It is unnecessary to give examples of the other parts of addition, concerning apothecaries' weight, liquid, dry, and long measures, &c. as the same method serves for all of them; having respect to the table belonging to each.

The Table of Apothecaries' Weight.

20 grains	make	1 scruple,	marked ℞.
3 scruples	—	1 dram,	℥.
8 drams	—	1 ounce,	℥.
12 ounces	—	1 pound,	℔.

By this weight apothecaries mix their drugs; but they are bought and sold by avoirdupois weight.

The Table of Wool Weight.

7 pounds	make	1 clove.
2 cloves, or 14 <i>lb.</i>		1 stone.
2 stone, or 28 <i>lb.</i>		1 tod.
6 tod and a half		1 wey, or 182 <i>lb.</i>
2 weys, or 364 <i>lb.</i>		1 sack.
12 sacks	—	1 last, or 4368 <i>lb.</i>
240 <i>lb.</i>	—	1 pack of wool.

Note. That a stone is generally 14*lb.* in most parts of England; but among the London butchers it is only 8*lb.*

The

The Table of Cloth Measure.

4 nails, or 9 inches	make	1 quarter of a yard.
4 quarters, or 36 inches		1 yard.
5 quarters, or 45 inches		1 ell English.
6 quarters, or 54 inches		1 ell French.
3 quarters, or 27 inches		1 ell Flemish.

The Table of Wine Measure.

8 pints	make	1 gallon.
42 gallons	—	1 tierce.
63 gallons	—	1 hogshead.
84 gallons	—	1 puncheon.
2 hogsheads	—	1 pipe, or butt.
2 pipes, or butts		1 tun, or 252 gallons.

Note. That sweet oil hath only 236 gallons to the tun. All liquids are measured by wine measure, except beer and ale.

Ale or Beer Measure.

8 pints	make	1 gallon.
8 gallons	—	1 firkin.
2 firkins	—	1 kilderkin.
2 kilderkins	—	1 barrel.
1½ barrel	—	1 hogshead.

The ale and beer gallons are the same; but the beer firkin contains 9 gallons; consequently, the kilderkin 18, the barrel 36, &c.

In a Tun of Wine there are,

2 pipes, or butts.
6 tierces.
252 gallons.
504 pottles.
1008 quarts.
2016 pints.

In a Pipe, or Butt, there are,

2 hogsheads.
3 tierces.
126 gallons.
252 pottles.
504 quarts.
1008 pints.

<i>In a Tun there are,</i>	<i>In a Hogshead there are,</i>
2 gallons	63 gallons.
48 pottles	126 pottles.
192 quarts	252 quarts.
384 pints	504 pints.

<i>In a Barrel there are,</i>	<i>In a Butt there are,</i>
2 kilderkins	2 kilderkins.
4 firkins	4 firkins.
12 gallons	12 gallons.
24 pottles	24 pottles.
96 quarts	96 quarts.
192 pints	192 pints.

The Table of Dry Measure.

4 bushels	1 quarter	1 quarter
8 bushels	—	2 quarters.
12 bushels	—	3 quarters.
16 bushels	—	4 quarters.
20 bushels	—	5 quarters, land measure.
24 bushels	—	5 quarters, water measure.
28 bushels	—	6 quarters.
32 bushels	—	8 quarters, or half quarter.
36 bushels	—	9 quarters.
40 bushels	—	10 quarters.
44 bushels	—	11 quarters.
48 bushels	—	12 quarters.
52 bushels	—	13 quarters.
56 bushels	—	14 quarters.
60 bushels	—	15 quarters.
64 bushels	—	16 quarters.
68 bushels	—	17 quarters.
72 bushels	—	18 quarters.
76 bushels	—	19 quarters.
80 bushels	—	20 quarters.
84 bushels	—	21 quarters.
88 bushels	—	22 quarters.
92 bushels	—	23 quarters.
96 bushels	—	24 quarters.
100 bushels	—	25 quarters.
104 bushels	—	26 quarters.
108 bushels	—	27 quarters.
112 bushels	—	28 quarters.
116 bushels	—	29 quarters.
120 bushels	—	30 quarters.
124 bushels	—	31 quarters.
128 bushels	—	32 quarters.
132 bushels	—	33 quarters.
136 bushels	—	34 quarters.
140 bushels	—	35 quarters.
144 bushels	—	36 quarters.
148 bushels	—	37 quarters.
152 bushels	—	38 quarters.
156 bushels	—	39 quarters.
160 bushels	—	40 quarters.
164 bushels	—	41 quarters.
168 bushels	—	42 quarters.
172 bushels	—	43 quarters.
176 bushels	—	44 quarters.
180 bushels	—	45 quarters.
184 bushels	—	46 quarters.
188 bushels	—	47 quarters.
192 bushels	—	48 quarters.
196 bushels	—	49 quarters.
200 bushels	—	50 quarters.

Note. That in wheat four there are always five bushels to the hick: and malt and iron-ore are heaped, or else there are five pecks to the bushel.

In a Last there are,

2 weys.
 10 quarters.
 80 bushels.
 320 pecks.
 1280 pottles.
 2560 quarts.
 5120 pints.

In a Wey there are,

5 quarters.
 40 bushels.
 160 pecks.
 320 gallons.
 640 pottles.
 1280 quarts.
 2560 pints.

Long Measure.

3 barley-corns	make	1 inch.
12 inches	—	1 foot.
3 feet	—	1 yard.
3 feet 9 inches	—	1 ell English.
5 feet	—	a geometrical pace.
6 feet	—	1 fathom.
5½ yards	—	1 pole, perch, or rod.
40 poles, or perches	—	1 furlong.
8 furlongs	—	1 English mile.
3 miles	—	1 league.

In a Mile there are,

8 furlongs.	5280 feet.
320 poles.	63360 inches.
1760 yards.	190080 barley-corns.

Land Measure.

5½ yards	make	1 pole, perch, or rod.
40 square poles	—	1 rood, or quarter of
160 square poles	—	1 acre. [an acre.
80 poles in length, and 2 in breadth		1 acre.
40 poles in length, and 4 in breadth		1 acre.
4 poles in length	—	1 chain.
10 chains in length, and 1 in breadth		1 acre

The Table of Time.

60 seconds	make	1 minute.
60 minutes	—	1 hour.
24 hours	—	1 day, natural.
7 days	—	1 week.
12 months	—	1 month, lunar.
12 months, 1 day, and near 6 hours	—	1 solar year.

In 1 Year there are,

365 hours, 1 day, 12 hours.	8,760 hours.
52 weeks, 1 day, 12 hours.	325,920 minutes.
365 days, 12 hours.	31,536,000 seconds.

Note. The solar year is divided into 12 unequal months, which are not equal months according to the ancient verse, which may serve to impress the number of days each month contains, on the memory :

Thirty days hath September,
April, June, and November;
February hath twenty-eight alone,
And all the rest have thirty-one.

SECT. III.

OF SUBTRACTION.

SUBTRACTION, vulgarly called *Subtraction*, teaches how to take a less number from a greater; and sheweth the remainder, excess, or difference. Thus, if I take 7 from 9, there will remain 2.

Rule. Place the less number under the greater; observing, that the figures of each denomination in the less number stand directly

directly under the figures of the same denomination in the greater number; that is, the units under units, tens under tens, and pounds, shillings, pence, ounces, drams, &c. &c. directly under the same, as in addition. Then, beginning at the right hand, or the least denomination, take the value of each figure in the less number from that in the greater number, which stands directly over it; setting down the remainder underneath. Proceed in this manner till the work be finished. But if, as it frequently happens, any single figure in the less number be greater than that in the greater number, from which it is to be taken; an unit is to be borrowed from the next figure towards the left hand of the greater number, and added to the uppermost figure, that the bottom figure may be taken therefrom; which borrowed unit must be paid, or added to the next figure of the less number on the left hand.

Examples.

From 6954	1729886	370290370 greater numbers.
Take 5443	1098907	128403492 less numbers.
Rem. 1511	630979	241886878
Proof 6954	1729886	370290370 /

To shew the use of this rule, I say—Suppose a merchant owed 6954*l.* whereof he has paid 5443*l.* : to know what remains to be paid, the sums are to be set orderly one under the other, according to the foregoing rule, and as seen in the examples: then, beginning with the unit figures, in the first example, I say, take 3 from 4 and there remains 1, which I set under the line; next take 4 from 5 and there remains 1, which is also set under the line; again, take 4 from 9 and there remains 5, which must be set down as before; and, lastly, take 5 from 6 and there remains 1: thus there remains due 1511*l.*

Subtraction is proved by adding the remainder to the less of the two given numbers, and if the total of these two numbers amount to the exact sum of the greater number, the work

were 3 right, otherwise not: thus, in this example, I add the remainder 15112 to the less number 54451, and the amount is 69563; the same as the greater number.

In the second example, I begin with the units, as before: saying, 2 from 2 I cannot, but by borrowing 1 from the next figure 3, and which added to the 2 makes 12 (the 3 being the next superior number), I say, 2 from 12 and there remains 10: then for the 1 that I borrowed, I carry 1 in return to the next figure of the less number, saying, 1 that I borrowed and 2 is but 3, therefore, 1 from 3 and there remains 2: again, 3 from 3 I cannot, but borrowing 1 as before from the next figure, I say, 3 from 13 and there remains 10: then for 1 that I borrowed, I must add 1 to the next figure 3, saying, 3 from 4 and there remains 1 (which may always be done when the two figures are the same); again, 4 from 4 I cannot, but 4 from 14 (borrowing 1 from 7) and there remains 10: then 1 that I borrowed, added to the 1, and taken from the 7, there remains 6: thus the work is finished. The proof demonstrates it right.

Proceeding in the same manner in the third example, I say, 2 from 2 I cannot, but 2 from 12 (borrowing one from the 7), there remains 10: then 1 that I borrowed and 3 is 13, 10 from 13 I cannot, but 10 from 17 and there remains 7; again, 1 that I borrowed and 4 is 5, 5 from 5 I cannot, but 5 from 15 (borrowing 1) and there remains 10; then, 1 from 0 (or nothing) I cannot, but 1 from 10 and there remains 9; again, 1, that I borrowed, from 9 and there remains 8; and 4 from 18 and there remains 14: and 1 that I borrowed and 5 is 9, from 14, and there remains 5; and 1 that I borrowed and 2 is 3 from 7 and there remains 4; lastly, 1 from 5 and there remains 4.

More Examples.

From 178927
Take 12533
Rem. 15359
Proof 178927

22572300
1127543
2049687
22572300

13942683 greater numb.
754008 less numbers.
13208685
13942683

3 The

The number of years since any event happened, may be discovered by subtracting the date of the year the event happened from that of the present year. Thus :—

The present year	1808	Spanish	1808	Gunpowder	1808
The fire of London	1666	Invasion	1588	Treason	1605
Years since	<u>142</u>		<u>220</u>		<u>203</u>
Proof	<u>1808</u>		<u>1808</u>		<u>1808</u>

Subtraction of divers denominations is performed upon the same principle as subtraction of numbers of one denomination. Observing, that when an unit is borrowed of the next higher denomination, it must be considered according to its true intrinsic value, and must be repaid to the lower figure of the same denomination; as will be seen in the following examples :—

	20	12	4		20	12	4
	£.	s.	d.		£.	s.	d.
Money owing	17	14	9½		123	14	7½
paid	10	10	6½		111	19	9½
Remains	7	4	2½		11	14	10½
Proof	17	14	9½		123	14	7½

In the first of these examples, beginning with the farthings, I say, 3 farthings from 2 I cannot, but borrowing an unit from the next denomination, or 1 penny from the 9 pence (and which added to the 2 farthings makes 6 farthings), I say, 3 farthings from 6 farthings and there remains 3 farthings, which I set under the farthings; then for the unit I borrowed of the pence, I add 1 to the 6, saying 6 and 1 is 7; now 7 from 9 and there remains 2; then 10 from 14 and there remains 4; lastly, 10 from 17 and there remains 7; thus the remains is 7*l.* 4*s.* 2½*d.* which is proved in the example.

In the second example, I say, 1 farthing from 3 and there remains 2 farthings, or an halfpenny; which I set down in its proper place, viz. under the denomination of farthings; then 9 from 7 I cannot, wherefore borrowing 1 shilling from the 14, and adding it to the 7 pence, which makes 19 pence, I say, 9 from 19 and there remains 10; then 1 that I borrowed

PROOF OF THEOREM 1.

Let n be a positive integer and let a_1, a_2, \dots, a_n be a sequence of positive integers such that $a_1 + a_2 + \dots + a_n = 2n$. We shall prove that there is a subsequence of a_1, a_2, \dots, a_n whose sum is a multiple of n .

Consider the sequence of partial sums s_1, s_2, \dots, s_n where $s_k = a_1 + a_2 + \dots + a_k$. Since $a_1 + a_2 + \dots + a_n = 2n$, we have $s_n = 2n$. If $s_k \equiv 0 \pmod{n}$ for some k , then the subsequence a_1, a_2, \dots, a_k has sum s_k which is a multiple of n . If not, then s_1, s_2, \dots, s_n are n numbers between 1 and $2n$ inclusive. By the pigeonhole principle, two of these numbers must be congruent modulo n . Suppose $s_i \equiv s_j \pmod{n}$ for $i < j$. Then $s_j - s_i = a_{i+1} + a_{i+2} + \dots + a_j$ is a multiple of n . This completes the proof.

EXAMPLE 1.

	124
124	124
124	124
124	124
124	124
124	124
124	124
124	124
124	124
124	124

The sum of the terms is 124. The sum of the terms is 124.

	124
124	124
124	124
124	124
124	124
124	124
124	124
124	124
124	124
124	124

EXAMPLE 2.

	15	15
15	15	15
15	15	15
15	15	15
15	15	15
15	15	15
15	15	15
15	15	15
15	15	15
15	15	15

Example 2.

Examples in Troy Weight.

	lb.	oz.	part.	gr.		oz.	part.	gr.
From	336	10	18	20	3184	16	21	
Take	247	11	19	22	320	19	22	
Remains	88	10	15	22	28	16	23	
Proof	336	10	18	20	3184	16	21	

These examples, and all others of other denominations, are wrought in the same manner as subtraction of money; having respect to the table belonging to the denomination: thus, in the first example of avoirdupois weight, I say, take 21 pounds from 20 I cannot, wherefore I borrow 1 quarter of an hundred from the 3 quarters, and add it to the 20, which makes 48 pounds; then I say, 21 from 48 and there remains 27; next, 1 that I borrowed and 2 is 3, 3 from 3 I cannot, wherefore I set down 0, as before hinted; lastly, 10 from 12, and there remains 2.

The figures at the head of the columns shew the number of units which each unit of the next higher denomination contains.

SECT. IV.**OF MULTIPLICATION.**

MULTIPLICATION is, perhaps, the most necessary rule in arithmetic for business, on account of its dispatch in resolving several long questions.

Multiplication teaches how, from two given numbers, to find a third, that shall contain either of the two given numbers as often as the other contains units: thus, 3 times 4 is 12; here 3 and 4 are the two given numbers, and 12 is the third number, or product, which contains 3 as often as 4 contains units, viz. 4 times; or it contains 4 as often as 3 contains units, 3 times.

There are three ends principally to be answered by multiplication :—

First. It serves to bring the greater denominations of money, weights, or measures, into small ones ; as, pounds into shillings, pence, and farthings ; hundred weights into pounds, ounces, or drams ; miles into yards, feet, barley-corns, &c.

Secondly. Having the length and breadth of a plain surface, we find its contents.

Thirdly. Having the rate or value of any one thing, we know the rate or value of any number of such things, however great.

But before the learner can begin this rule, it is absolutely necessary that he have the following table perfectly by heart :

The Multiplication Table.

2 times	2 is 4	4 times	4 is 16	7 times	7 is 49
	3 — 6		5 — 20		8 — 56
	4 — 8		6 — 24		9 — 63
	5 — 10		7 — 28		10 — 70
	6 — 12		8 — 32		11 — 77
	7 — 14		9 — 36		12 — 84
	8 — 16		10 — 40	8 times	8 — 64
	9 — 18		11 — 44		9 — 72
	10 — 20		12 — 48		10 — 80
	11 — 22	5 times	5 — 25		11 — 88
	12 — 24		6 — 30		12 — 96
			7 — 35	9 times	9 — 81
3 times	3 — 9		8 — 40		10 — 90
	4 — 12		9 — 45		11 — 99
	5 — 15		10 — 50		12 — 108
	6 — 18		11 — 55	10 times	10 — 100
	7 — 21		12 — 60		11 — 110
	8 — 24	6 times	6 — 36		12 — 120
	9 — 27		7 — 42	11 times	11 — 121
	10 — 30		8 — 48		12 — 132
	11 — 33		9 — 54		
	12 — 36		10 — 60		
			11 — 66		
			12 — 72		

The

The use of this table is, to find the product of any two numbers: thus, to find the product of 6 times 7, I look in the brace which has 6 at the point, and in the line which has 7 at the beginning, opposite to which is the product, which is 42.

The table is to be thus read:—

Beginning with the first brace, I say—2 times 2 is 4, 2 times 3 is 6, 2 times 4 is 8, 2 times 5 is 10, 2 times 6 is 12, 2 times 7 is 14, 2 times 8 is 16, 2 times 9 is 18, 2 times 10 is 20, 2 times 11 is 22, 2 times 12 is 24.

Then, proceeding in the same manner, I begin with the second brace, saying, 3 times 3 is 9, 3 times 4 is 12, 3 times 5 is 15, 3 times 6 is 18, 3 times 7 is 21, 3 times 8 is 24, 3 times 9 is 27, 3 times 10 is 30, 3 times 11 is 33, 3 times 12 is 36.

Again, I begin with the third brace, saying, 4 times 4 is 16, 4 times 5 is 20, 4 times 6 is 24, 4 times 7 is 28, 4 times 8 is 32, 4 times 9 is 36, 4 times 10 is 40, 4 times 11 is 44, 4 times 12 is 48.

Then, I say, 5 times 5 is 25, 5 times 6 is 30, 5 times 7 is 35, 5 times 8 is 40, 5 times 9 is 45, 5 times 10 is 50, 5 times 11 is 55, 5 times 12 is 60.

Then, 6 times 6 is 36, 6 times 7 is 42, 6 times 8 is 48, 6 times 9 is 54, 6 times 10 is 60, 6 times 11 is 66, 6 times 12 is 72.

Then, 7 times 7 is 49, 7 times 8 is 56, 7 times 9 is 63, 7 times 10 is 70, 7 times 11 is 77, 7 times 12 is 84.

Then, 8 times 8 is 64, 8 times 9 is 72, 8 times 10 is 80, 8 times 11 is 88, 8 times 12 is 96.

Again, 9 times 9 is 81, 9 times 10 is 90, 9 times 11 is 99, 9 times 12 is 108.

Then, 10 times 10 is 100, 10 times 11 is 110, 10 times 12 is 120.

And 11 times 11 is 121, 11 times 12 is 132.

Lastly, 12 times 12 is 144.

In multiplication there are three terms : viz.

The *Multiplicand*, 12 or sum to be multiplied;

" *Multiplier*, 10

Product, 120 or result of the whole.

Multiplication consists of two parts: multiplication of numbers of one denomination, and multiplication of divers denominations.

Multiplication of numbers of one denomination is either single or compound.

Single multiplication is, when the multiplicand and multiplier consist, each of them, of only 12, or less than 12; and may be performed by one operation, and with one product. Hence the greatest product that can arise by single multiplication is 144, as that is the product of 12 times 12.

Compound multiplication is, when the multiplicand or multiplier, or both, consist of more than 12, and requires more than one operation.

Rule. Place the multiplier under the multiplicand, in the natural order of figures, viz. units under units, &c.; then, if the multiplier consists of 12 or less, multiply every figure of the multiplicand by the multiplier; beginning at the place of units, and placing the units of the product right under the units of the multiplier; carrying the tens, or tens and hundreds, if there be any, to be added to the next product. Proceed in this manner through the whole, and set down the whole product of the last figure, as in the annexed example; observing, for every ten that is carried to the next product, an unit is to be added thereto; and for every hundred, tens to be added.

Multiplicand	1200
Multiplier	<u>12</u>
Product	<u>14400</u>

In this example, the multiplier being 12, I say, 12 times 9 is 108, wherefore I set down 8, the units of this product, and

carry

carry 10 to be added to the next product, saying, 12 times 0, or 12 times nothing, is nothing; thus I have no product from this figure, wherefore I set down the unit figure of the 10 I carried to be added to this product, which is 0, and carry the remaining 1 to be added to the next product; saying, 12 times 1 (as the two next figures in the multiplicand is 12) is 144, and the 1 I carried makes 145; wherefore I set it down, being the product of the last figures. And the whole product of 1209, multiplied by 12, is 14508.

But, if the multiplier consist of several places, after having multiplied the multiplicand by the first, or two first figures, as before directed, multiply it in like manner by the next, or two next figures, if they be 12 or less, and in like manner by all the other figures; placing the products below each other, strictly observing that the product of each new multiplier is to have its unit placed exactly under the unit in such new multiplier, and the subsequent figures in their regular order towards the left hand; and then the different products added together in the order in which they stand; as in the following examples:—

<i>Example.</i>		<i>Proof.</i>	
Multiplicand	1097012	First multiplicand	1097012
Multiplier	1294	Half the multiplier	647
	4388048		7679084
	9873108		4388048
	13162144		6572072
	141953328	First product, or	709706764
		2d multiplicand	2
		Proof, or 2d	141953328
		product	

In this example, I begin with the first figure in the multiplier (as the two first are more than 12), saying, 4 times 2 is 2 times 4, that is 8, which I place in the first line of the product, and under 4 the multiplier; then 4 times 1 is 4, which I also set down in the next place; then 4 times 0 is 0, or nothing, wherefore I set down 0; next, 4 times 7 is 28, therefore I say, 8 to be set down, and carry 2; 4 times 9 is

36, and 2 that I carried is 38, 8 and carry 3; 4 times 0 is nothing, wherefore I have nothing to set down but the 3 I carried; again, 4 times 1 is 4: thus I have done with the first figure in the multiplier. Beginning with the next, I say, 9 times 2, or 2 times 9, is 18, 8 and carry 1; the 8 is to be set exactly under the multiplier 9; then 9 times 1 is 9, and 1 I carried is 10, 0 and carry 1; 9 times 0 is 0, wherefore I set down the 1 I carried; again, 9 times 7 is 63, 3 and carry 6; 9 times 9 is 81, and 6 is 87, 7 and carry 8; 9 times 0 is 0, but 8 was carried; 9 times 1 is 9: thus I have done with the second multiplier. The next and last two figures in the multiplier being 12, I multiply by them as by one figure, placing the unit figure of the first product under that of the multiplier, which is the 2; saying, 12 times 2, or 2 times 12, is 24, 4 and carry 2; 12 times 1 is 12, and 2 is 14, 4 and carry 1; 12 times 0 is 0, but 1 was carried; 12 times 7 is 84, 4 and carry 8; 12 times 9 is 108, and 8 is 116, 6 and carry 11; 12 times 0 is 0, but 11 was carried, therefore I set down 1, and carry 1; 12 times 1 is 12, and 1 is 13; which being the last, I set it down. The work being finished, all the several products are added together, and the total is the real product.

The work is proved by dividing the multiplier in half, and multiplying the original multiplicand by one half, and the product thence arising by the number 2, as seen in the foregoing example. But if the multiplier cannot be divided exactly in half, as is the case when it consists of an odd number, then a number that is an unit less than the multiplier is to be divided in half; and the multiplicand multiplied by the one half, and that product by 2, as before, and the original multiplicand added to the last product, as in the following examples;—

Example.

$$\begin{array}{r}
 379643052 \\
 \times 4321 \\
 \hline
 379643052 \\
 759286104 \\
 1138929156 \\
 1518574208 \\
 \hline
 1640437627692
 \end{array}$$

Proof.

$$\begin{array}{r}
 379643052 \\
 \times 2160 \\
 \hline
 82778583120 \\
 379643052 \\
 759286104 \\
 820028992320 \\
 \hline
 1640057982640 \\
 + 1640437627692 \\
 \hline
 1640437627692
 \end{array}$$

In proving this example, I cannot divide the multiplier 4321 exactly in half, because it contains an odd number in the place of units; I therefore take a number that is an unit less, viz. 4320, and divide it in half thus: saying, the half of four thousand is two thousand, and the half of three hundred and twenty is one hundred and sixty; which, together, is 2160 for the first multiplier in the proof; the product of which is again multiplied by 2, and that product added to the first multiplicand, which gives a product equal to the product in the example: which proves the work right.

When the multiplier has cyphers Intermixed with the other figures, the significant figures only are to be regarded as multipliers, observing the directions before given, to place the unit figure of each product under that of the multiplier; as in the following examples:—

Example.

$$\begin{array}{r}
 279638249 \\
 \times 2020 \\
 \hline
 1118529960 \\
 5592764980 \\
 5704624000 \\
 \hline
 5704624000
 \end{array}$$

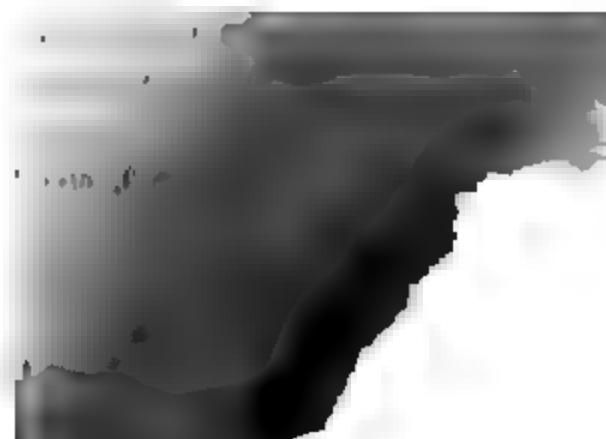
Proof.

$$\begin{array}{r}
 279638249 \\
 \times 1020 \\
 \hline
 5592764980 \\
 2796382490 \\
 2865211013980 \\
 \hline
 5704624000
 \end{array}$$

When the multiplier, or multiplicand, or both, consist of cyphers

when I come to the 3, I say, 3 times 0 is nothing, therefore I have nothing to set down but the 3, I carried again, 3 times 1 is 3, thus I have done with the first figure of the multiplier. Beginning with the next, I say, 3 times 2 is 6, thus 6 is 6, 6 and carry 1; the 6 is to be set under the multiplier 3; then 3 times 1 is 3, and 1 I carried, 3 is 4, and carry 1; 1 times 0 is 0, wherefore I set down the 1 I carried. Again, 3 times 7 is 21, 3 and carry 6; 6 times 1 is 6, and 6 is 12, and carry 5; 9 times 0 is 0, but 5 was carried; 9 times 1 is 9; thus I have done with the second multiplier. The next and last two figures in the multiplier being 12, I multiply by them as by one figure, placing the unit figure of the first product under that of the multiplier, which is the 2; saying, 12 times 2, or 2 times 12, is 24, 4 and carry 2; 12 times 1 is 12, and 2 is 14, 4 and carry 1; 12 times 0 is 0, but 1 was carried; 12 times 7 is 84, 4 and carry 8; 12 times 9 is 108, and 8 is 116, 6 and carry 11; 12 times 0 is 0, but 11 was carried, therefore I set down 1, and carry 1; 12 times 1 is 12, and 1 is 13; which being the last, I set it down. The work being finished, all the several products are added together, and the total is the real product.

The work is proved by dividing the multiplier in half, and multiplying the original multiplicand by one half, and the product thence arising by the number 2, as seen in the foregoing example. But if the multiplier cannot be divided exactly in half, as is the case when it consists of an odd number, then a number that is an unit less than the multiplier is to be divided in half: and the multiplicand multiplied by the one half, and that product by 2, as before, and ~~the~~ ^{the} ~~last~~ ^{last} ~~product~~ ^{product} added to the last product, as in the following examples:—



Example.

$$\begin{array}{r}
 379643052 \\
 \times 4321 \\
 \hline
 379643052 \\
 759286104 \\
 1138929156 \\
 1518572208 \\
 \hline
 1640437627692
 \end{array}$$

Proof.

$$\begin{array}{r}
 379643052 \\
 \times 2160 \\
 \hline
 22778583120 \\
 379643052 \\
 759286104 \\
 \hline
 820028992520 \\
 \times 2 \\
 \hline
 1640057981640 \\
 \hline
 1640437627692
 \end{array}$$

In proving this example, I cannot divide the multiplier 4321 exactly in half, because it contains an odd number in the place of units; I therefore take a number that is an unit less, viz. 4320, and divide it in half thus: saying, the half of four thousand is two thousand, and the half of three hundred and twenty is one hundred and sixty; which, together, is 2160 for the first multiplier in the proof; the product of which is again multiplied by 2, and that product added to the first multiplicand, which gives a product equal to the product in the example: which proves the work right.

When the multiplier has cyphers intermixed with the other figures, the significant figures only are to be regarded as multipliers, observing the directions before given, to place the unit figure of each product under that of the multiplier; as in the following examples:—

Example.

$$\begin{array}{r}
 2746123456 \\
 \times 123456789 \\
 \hline
 2746123456 \\
 5492246912 \\
 8238370368 \\
 1098123456 \\
 1372654320 \\
 1647165432 \\
 1921676543 \\
 2196187654 \\
 2470698765 \\
 \hline
 338901234567890
 \end{array}$$

Proof.

$$\begin{array}{r}
 2746123456 \\
 \times 123456789 \\
 \hline
 2746123456 \\
 5492246912 \\
 8238370368 \\
 1098123456 \\
 1372654320 \\
 1647165432 \\
 1921676543 \\
 2196187654 \\
 2470698765 \\
 \hline
 338901234567890
 \end{array}$$

or both, consist of
cyphers

cyphers towards the right hand, and significant figures towards the left, such cyphers are omitted in the operation, and placed on the right hand of the product; as follows:—

Example.

1	3	7	6	4	3	0	7	2	0										
0	1	5	0	6	3	0	3	7	2	0									
0	3	3	0	4	5	5	5	0	0										
0	3	0	0	1	0	1	5	5	0	0	0	0	0	0	0	0	0	0	0

Proof.

1	3	7	6	4	3	0	7	2	0										
3	7	5	2	0	5	1	4	4	0										
4	6	9	1	0	2	6	8	0	0	0	0	0	0	0	0	0	0	0	2
9	3	8	2	4	5	3	7	0	0	0	0	0	0	0	0	0	0	0	0
9	3	7	0	1	0	2	6	7	2	0	0	0	0	0	0	0	0	0	0

In the proof of the last example, the multiplicand is added to the product as before directed, when the original multiplier cannot be divided exactly in half.

Multiplication of divers Denominations.

Multiplication of divers denominations is performed by multiplying each denomination by the multiplier; beginning with the least denomination, and carrying the units of the next denomination to be added thereto, as in addition of money.

Multiplication of divers denominations is either single or compound: single, when the multiplier consists of 12 or less, and is performed by one multiplier; compound, when the multiplier consists of more than 12, and requires more than one multiplier.

Example.

£.	s.	d.
9	10	8
		8
70	5	4

Proof.

£.	s.	d.
9	10	8
		4
32	2	8
		2
76	5	4

In

In this example I begin with the pence, saying 8 times 8 is 64, which is 5 shillings and 4 pence; the 4 pence I set down under the pence, and carry the 5 shillings to be added to the product of shillings; saying, 8 times 10 is 80 and 5 is 85, or 4*l.* 5*s.* the 5 shillings I set under the shillings, and carry the 4 pounds to the next product: then 8 times 9 is 72 and 4 is 76.

The work is proved as the former examples, viz. by dividing the multiplier in half, and multiplying by one half, and that product by 2.

But in compound multiplication of divers denominations, or when the multiplier is more than 12, the multiplier is to be resolved into its two commensurable parts, if it be a commensurable number, and the multiplicand multiplied by one of those parts, and that product by the other part: thus, if the multiplier be 36, I multiply by 6, and also the product thence arising by 6, which are the two commensurable parts: for 6 times 6 is 36.

But if the multiplier be not a commensurable number, or one which cannot be resolved into parts exactly, then the next lower number which is a commensurable one, is to be taken, and the multiplicand multiplied by the two parts as before, and the multiplicand multiplied by the overplus, and that product, added to the last product: as in the number 46. This number cannot be resolved exactly into two, or three commensurable parts, there being no such number in the multiplication table, wherefore I take the next lower commensurable number 45, because 5 times 9 is 45, and multiply by these two numbers, and the multiplicand by 1, which must be added to the product.

1 Query. What is the amount of 36 pieces of cloth, at 7*l.* 1*s.* 6*d.* each piece?—Here I multiply by 6, and that product by 6.

Example.

£.	s.	d.
7	14	6
		6
<hr/>		
46	7	0
		6
<hr/>		
278	2	0
<hr/>		

Proof.

£.	s.	d.
7	14	6
		3
<hr/>		
23	3	6
		6
<hr/>		
139	1	0
		2
<hr/>		
278	2	0
<hr/>		

2 Qu. What is the amount of 46 score of sea-coal, at 38s. 18s. od. per score?

Example.

£.	s.	d.
38	18	0
		5
<hr/>		
194	10	9
		9
<hr/>		
1720	10	0
<hr/>		
1780	8	0
<hr/>		

Proof.

£.	s.	d.
38	18	0
		3
<hr/>		
110	14	0
		7
<hr/>		
816	18	0
		2
<hr/>		
77	16	0
<hr/>		
894	14	0
		2
<hr/>		
1780	8	0
<hr/>		

In the first of these examples the work is proved by multiplying by 3, and that product by 6 (which is equal to multiplying by 18, half the original multiplier), and the last product by 2, as directed before.

In the second example, I multiply by 5, and that product by 9, which is equal to multiplying by 45, and for the other one I add the multiplicand to the last product. In the proof of this example, I multiply by 3, and that product by 7, which being equal to 21, wants 2 of half the multiplier 46; I therefore multiply the original multiplicand by 2, and add the product to the last product; the whole of which I lastly multiply by 2.

Examples

Examples in Avoirdupois Weight.

3 Qu. What is the weight of 107 chests of tea; each chest weighing four hundred, three quarters, and twenty-two pounds?

Example.

C. qrs. lb.		
4	3	22
		10
49	1	24
		10
494	2	16
		7
34	2	14
529	1	2

Proof.

C. qrs. lb.		
4	3	22
		5
24	2	26
		10
247	1	8
		3
14	3	10
262	0	18
		2
524	1	8
529	1	2

In this example 106 is the nearest commensurable number, but the multiplier is 107; I therefore multiply by 10 and 10, and the original multiplicand by 74 which last product is added to the other.

In the proof, because 107 is an odd number, I take the half of 106, which is 53; multiplying by 5 and 10, for 50, and the original multiplicand by 3; then I multiply the whole product by 2, which doubles it to 106; and lastly, add the multiplicand to the product, for the 1 that was wanting.

By these examples it may be seen, that there is no occasion to have always a commensurable number, or to come very near one; for, if it want ever so much, it may be worked in this manner; always multiplying the original multiplicand by the overplus, and adding such product to the other.

It must be observed, that if the multiplier, in this part of multiplication, consist of more than 144, it must be resolved into three multipliers at least; and if it consist of more than 1728, it must be resolved into more than three.

4 Q^u. What is the superficial content of a piece of ground, whose breadth is 102 74 feet, and length 24640?

Here I multiply the length by the breadth (which is the general rule in superficial measure), and the product is 253151360 square feet; for the answer.

$$\begin{array}{r}
 24640 \\
 10274 \\
 \hline
 98560 \\
 172480 \\
 49280 \\
 246400 \\
 \hline
 253151360
 \end{array}$$

5 Q^u. How many solid feet does a piece of timber contain, that is forty feet in length, four in breadth, and three in depth?

Here the general rule is, to multiply the length by the breadth, and that product by the depth. And the product is 480 solid feet, for the answer.

$$\begin{array}{r}
 \text{length} \quad 40 \\
 \text{breadth} \quad 4 \\
 \hline
 160 \\
 \text{depth} \quad 3 \\
 \hline
 480
 \end{array}$$

Multiplication teaches also how to multiply different denominations of measure by different denominations, called *cross multiplication*; of which I shall speak in Mensuration.

Multiplication also serves to bring great denominations of money, weights, or measures, into small ones: but this more properly belongs to the rule of Reduction.

SECT. V.

OF DIVISION.

DIVISION teacheth how, from two given numbers, to find a third; that shall be contained in the largest of the two given numbers, as often as the smallest contains units: or the third number contains units, as often as the smallest of the two given numbers is contained in the other. Thus, if 15 were to be divided by 3, the answer would be 5: for 3 is contained in 15, 5 times: and 5 is contained in 15, 3 times.

As multiplication teacheth how to bring great denominations

utions into small ones; and having the rate or value of one thing, to know the rate or value of many; and from the length and breadth of a superficies, to know its contents, &c.; so, on the contrary, division teacheth how to bring small denominations into great ones; and from the rate or value of many things, and the number of them given, to know the rate or value of one; and from the contents of a superficies, and the length to find the breadth; or from the superficies, and the breadth, to find the length; and from the contents of a solid, and one dimension thereof, to find the other two.

In every sum in division, there are three parts which are to be particularly remembered, viz.

The *Dividend*, or number to be divided.

Divisor, or number by which we divide.

Quotient, or answer to the work, which shews how often the divisor is contained in the dividend. Thus, in the before-mentioned instance, 15 is the dividend, 3 the divisor, and 5 the quotient.

Besides these three parts, which are in every sum, there is sometimes a remainder, when the work is finished; which will always happen when the dividend does not exactly contain the divisor a certain number of times: as, if I were to divide 13 by 4; here 4 the divisor is contained 3 times in the dividend 13, and there is a remainder of 1; and in division of divers denominations, it must be noted, that the remainder is always of the same denomination with the dividend.

Division is either single, or compound: single division is when the divisor does not consist of more than 12, and the dividend of not more than 144. Any questions of this sort may be answered at once by the multiplication table, without setting them down: thus, if it were required to divide 110 by 10, by that table I know that 10 times 11 is 110; thus 10 is contained 11 times in 110, and 11 is the quotient.

Compound division is when the divisor contains more than 12, or dividend more than 144, or both.

In division the dividend is to have a crooked line placed at

each end of it; and before the line on the left hand, the divisor is to be placed, and behind that on the right hand, the quotient, or the answer to the work, is placed.

Rule. Mark off from the dividend, by a point placed under the figure, as many figures as there are in the divisor; but if the divisor consist of a greater number than the figures so pointed off from the dividend, then another figure is to be pointed off from the dividend; then seek how often the divisor, or (if the divisor consist of many figures) the first, or first and second figures of the divisor, is contained in the same number of figures pointed off from the dividend; place the answer in the quotient, and multiply the divisor thereby; then place the product of such multiplication under the pointed-off figures, and subtract the said product therefrom; but if the product amounts to more than the figures pointed off, the divisor is to be multiplied by a number that is an unit less than the former answer, and this answer placed in the quotient, instead of the former, and this product subtracted from the aforesaid figures in the dividend (if this product be still too large, the dividend must be multiplied by an unit less); to the remainder, the next figure in the dividend is to be brought down, and placed on the right hand thereof, which number is to be taken for a new dividend: seek how often the divisor is contained in such new dividend, place the answer in the quotient, multiply the divisor thereby, subtract the product from the last dividend, and bring down the next figure in the dividend, as before. Proceed in this manner till the work be finished; as in the following examples:—

Let 4876 be divided into 7 parts.

In this example, I place the numbers as before directed: and because 7 the divisor is more than 4, the first figure in the dividend, I make a point under the second figure 8 in the dividend, which makes 48 for the first dividend; then I ask how often 7 the divisor is contained in 48, the first dividend, which I find is 6 times.

$$\begin{array}{r}
 7 \overline{)4876} \quad (696 \\
 \underline{42} \\
 67 \\
 \underline{63} \\
 40 \\
 \underline{42} \\
 4
 \end{array}$$

I therefore

I therefore place 6 in the quotient, and multiply the divisor 7 thereby, and the product 42 I set under the dividend 48, to be subtracted therefrom; and to the remainder 6, I bring down the next figure 7, of the original dividend; thus I have 67 for a new dividend. Then I ask how often 7 is contained in 67, which I find is 9 times: I therefore put 9 in the quotient, and multiply the divisor thereby, and the product 63 I place under, and subtract from, the last dividend; and to the remainder 4 I bring down the next and last figure 6 in the original dividend, which makes 46 for a new dividend. Then I ask how often 7 is contained in 46, which I find is 6 times: I place 6 in the quotient, and multiply the divisor thereby: the product 42 I subtract from the last dividend 46, and there being no more figures in the original dividend to bring down, there is a remainder of 4 after the work is finished, and which remainder is always less than the divisor, if the work be right.

The remainder may be set over the divisor as a fraction of an unit; thus in the answer to this question: if 4876*l.* were to be divided equally among 7 men, what would each man's share amount to? *Ans.* 696*l.* and 4-7ths of a pound.

Division is proved by multiplication: thus the foregoing example is proved by multiplying the quotient by the divisor, adding thereto the remainder, and if the product be equal to the dividend the work is right; otherwise not.

$$\begin{array}{r}
 8 \overline{) 61280} (7660 \\
 \underline{56 \dots 8} \\
 52 61280 \text{ proof.} \\
 \underline{48} \\
 48 \\
 \underline{48} \\
 00 \\
 \hline
 \hline
 \end{array}$$

$$\begin{array}{r}
 11 \overline{) 18727} (1702 \\
 \underline{11 \dots 11} \\
 77 18727 \text{ proof.} \\
 \underline{77} \\
 02 \\
 \underline{0} \\
 27 \\
 \underline{22} \\
 5 \\
 \hline
 \hline
 \end{array}$$

In the first of these examples, I say, 8 is contained in 27, 3 times, and there remains 3; the 3 I place under the 7 in the dividend, and the 3 which remains I imagine to be placed before the 9 in the dividend, which makes 39 for a new dividend. Then 8 in 39 is 4 times, and remains 7; the 4 I place in the quotient, and the 7 placed before the 0 (the next figure in the dividend) gives 70 for a new dividend. Then 8 in 70, 8 times, and remains 6, which, with the next figure 8, make 68. 8 in 68, 8 times, and remains 4, which, with the last figure, gives 43 for the last dividend. And 8 in 43, 5 times, and there remains 3 at the last.

Proceeding in the same manner in the next example, I say, 11 in 74, 6 times and there remains 8. Then 11 in 83, 7 times, and remains 7. 11 in 60, 5 times, and remains 5. 11 in 50, 4 times, and remains 6. 11 in 62, 5 times, and remains 7.

This is the most expeditious manner of dividing by a figure under 12; and is generally done in public offices, shop-books, &c.

But when the divisor consists of more than 12, the work must be performed according to the regular method first laid down, as the divisor is to be multiplied by each answer placed in the quotient.

$$\begin{array}{r}
 375)743201(1981 \\
 \underline{375} \dots \\
 3682 \\
 \underline{3375} \\
 3070 \\
 \underline{3000} \\
 701 \\
 \underline{375} \\
 326 \\
 \hline
 \hline
 \end{array}$$

$$\begin{array}{r}
 570)2487021(4363 \\
 \underline{2280} \dots \\
 2070 \\
 \underline{1710} \\
 3602 \\
 \underline{3420} \\
 1821 \\
 \underline{1710} \\
 111 \\
 \hline
 \hline
 \end{array}$$

In the first of these examples, I mark off by a point the three first figures on the left hand, because there are three figures in the divisor; then I ask how often the two first figures in the divisor are contained in the two first figures of

The use of this table is, to find how often the divisor is contained in any particular dividend; thus, in this example, after having pointed off the first nine figures in the dividend 243098249 (because the first eight figures are less than the divisor), I cast my eye on the table, and find that the next less number is 238994464, and which I find by the table contains the divisor 8 times; I therefore place 8 in the quotient, and the multiple of 8, taken from the table, I place under the dividend, and subtract it therefrom, and to the remainder bring down the next figure 3 for a new dividend: then, by the table, I find the divisor is contained in this dividend but once, I place 1 in the quotient, and subtract the multiple of 1 (or the divisor) from such dividend as before, and to the remainder I bring down the next figure 7: in this dividend (by the table) I find the divisor is contained 3 times, which I place in the quotient, and take the multiple of 3 from the table to be subtracted from the last dividend, to which remainder I bring down the last figure 2: and, by the table, I find the divisor is contained in the last dividend 7 times; and the multiple of 7 being subtracted from this last dividend, there remains 11005176 after the work is ended.

When the divisor has a cypher or cyphers towards the right hand, the dividend may be divided by the significant figures only, and the cyphers separated from the divisor by a stroke of the pen; in which case there must be as many figures separated from the right hand of the dividend, as there are separated cyphers, and such separated figures are to be set down at the last as a remainder: and if there be any other remainder, the separated figures are to be set on the right hand thereof. When the divisor has an unit only on the left hand, with nothing but cyphers on the right hand thereof, the division is performed at once, by cutting off as many figures from the right hand of the dividend as there are cyphers in the divisor: the remainder of the dividend is the quotient. See the following examples:—

X 2

12,0)

THE RULES OF ARITHMETIC

$$\begin{array}{r}
 123456789 \\
 \times 123456789 \\
 \hline
 123456789 \\
 246913578 \\
 369470367 \\
 487135954 \\
 608594943 \\
 731357932 \\
 854120921 \\
 976883910 \\
 \hline
 15490317084857
 \end{array}$$

THE RULES OF ARITHMETIC

The first rule of arithmetic is contained by dividing the dividend by the divisor, placing the quotient in the quotient place, and the remainder in the remainder place. The second rule is contained by multiplying the quotient by the divisor, and subtracting the product from the dividend. The third rule is contained by adding the quotient to the dividend, and subtracting the product from the dividend.

The first rule of arithmetic is contained by dividing the dividend by the divisor, placing the quotient in the quotient place, and the remainder in the remainder place.

$$\begin{array}{r}
 123456789 \\
 \times 123456789 \\
 \hline
 123456789 \\
 246913578 \\
 369470367 \\
 487135954 \\
 608594943 \\
 731357932 \\
 854120921 \\
 976883910 \\
 \hline
 15490317084857
 \end{array}$$

The first rule of arithmetic is contained by dividing the dividend by the divisor, placing the quotient in the quotient place, and the remainder in the remainder place. The second rule is contained by multiplying the quotient by the divisor, and subtracting the product from the dividend. The third rule is contained by adding the quotient to the dividend, and subtracting the product from the dividend.

THE FIRST RULE OF ARITHMETIC IS CONTAINED BY DIVIDING THE DIVIDEND BY THE DIVISOR, PLACING THE QUOTIENT IN THE QUOTIENT PLACE, AND THE REMAINDER IN THE REMAINDER PLACE.

Qu. 1. If the expense of a country feast be 26*l.* 10*s.* 6*d.* to be paid by 28 stewards, what must each steward pay?—In this example I divide by 4, and the quotient thence arising by 7, which is equal to dividing by 28, and the answer is found to be 9*l.* 8*s.* 2*d.* for each steward to pay.

Qu. 2. If a gentleman spend 347*l.* 15*s.* 9*d.* in the space of one year and eight weeks, it is desired to know how much it is per week on an average?—Here I divide the whole sum by 5, and the quotient thence arising by 6, and that quotient by 2; which is equal to dividing by 60 (the number of weeks in one year and eight weeks), and the answer is 5*l.* 15*s.* 11*d.* per week, as in the example.

Qu. 3. If the capital stock of a tontine amount to 4372*l.* 14*s.* 0*d.* and there be in it 160 shares, what will be the amount of each share?—In this example I divide by 4, and that quotient by 10, and the quotient thence arising by 4, which is equal to dividing by 160; as 4 times 10 is 40, and 4 times 40 is 160, and the answer is 27*l.* 6*s.* 7*d.* for each share.

Question 1.

$$\begin{array}{r} \text{£. s. d.} \\ 4)263 \ 10 \ 6 \\ \hline 7) \ 65 \ 17 \ 7 \ 2 \\ \hline \quad 9 \ 8 \ 2 \ 5 \\ \hline \end{array}$$

Question 2.

$$\begin{array}{r} \text{£. s. d.} \\ 5)347 \ 15 \ 9 \\ \hline 6) \ 69 \ 11 \ 1 \ 4 \\ \hline 2) \ 11 \ 11 \ 10 \ 1 \\ \hline \quad 5 \ 15 \ 11 \\ \hline \end{array}$$

Question 3.

$$\begin{array}{r} \text{£. s. d.} \\ 4)4372 \ 14 \ 0 \\ \hline 10)1093 \ 3 \ 6 \\ \hline 4) \ 109 \ 6 \ 4 \ 2 \\ \hline \quad 27 \ 6 \ 7 \\ \hline \end{array}$$

To find the exact remainder in sums where there are two or more divisors, as in the foregoing ones, the rule is to multiply the first divisor by the last remainder, adding thereto the first remainder, if any, and the product will be the true remainder; as if it had been divided by the long method: thus in the first of the foregoing examples, I multiply 4, the first divisor, by 5, the last remainder, which produces 20, to which adding 2, the first remainder, the true remainder is found to be 22, which may be proved at leisure.

Qu. 4. There is a piece of land, having 4 sides, containing 1398 acres, 3 roods, 35 perches, and 240 feet in breadth, it is desired to know how many feet it is in length?

Qu. 5. There is a piece of timber, the solid contents of which is 600 feet, its length is 40 feet, and its depth 3 feet; it is required to know its superficial contents?

Question 4.

	A.	R.	P.
4)	1398	3	35
6)	349	2	38 3
10)	58	1	6 2
	5	3	12 6

Question 5.

3)	600
	200

In the fourth question I divide the contents of the land, first by 4, and that quotient by 6, and the next quotient by 10, which is the same as dividing at once by 240; and the answer is found to be 5 acres, 3 roods, and 12 perches, for the length of the piece of land.

In the last question I divide the solid contents of the piece of timber by 3, the depth, and the quotient 200 feet, is the superficial contents, which if divided again by 40 feet, the length, would give 5 feet for the breadth.

In the same manner as the foregoing examples are wrought division of other denominations may be performed, having respect to the table of quantity belonging to the same.

But in this species of division, if the divisor be not a commensurable number, or one which cannot be divided into parts exactly, the division must then be performed by one divisor.

Division also teacheth how to bring small denominations into great ones; but as this part more properly belongs to Reduction, I have deferred treating of it till I come to that rule.

SECT. VI.

OF REDUCTION.

REDUCTION is only the application of the rules of multiplication and division, and teacheth how to bring numbers of one denomination into another denomination without altering their value.

Reduction is either *descending* or *ascending*. Reduction descending is performed by multiplication, and serves to bring great denominations into small ones; as pounds into shillings, pence, or farthings; hundred-weights into pounds or ounces, &c. Reduction ascending is performed by division, and brings small denominations into great ones; as farthings into pence, shillings, or pounds; drams or ounces into pounds, hundred-weights, &c.

Rule. In reduction descending, multiply the number by the number of units of the next lower denomination which make an unit of the next greater, and multiply such product by the number of units of the next lower denomination which make one of the next greater; and proceed in this manner till the number be reduced to the denomination required.

EXAMPLE 1. Reduce 250*l.* *or.* *od.* into farthings.

In this example, I first multiply the 250 by 20, which is the number of units of the next lower denomination which make an unit of the next higher; that is, the number of shillings contained

Shillings in 1 pound	250
	<u>20</u>
Shillings in 250 <i>l.</i>	5000
Pence in 1 shilling	<u>12</u>
Pence in 250 <i>l.</i>	60000
Farthings in 1 penny	<u>4</u>
Answer	<u><u>240000</u></u>

in a pound, and the product shews the number of shillings contained in 250*l.*; which product must be again multiplied by 12, the number of units of the next lower denomination which make one of the next greater, or the number of pence contained in one shilling, and the product gives the number

of

of pence contained in 250*l.*; which product, again multiplied by 4, the number of farthings in one penny, the product gives the number of farthings contained in 250*l.*; or the answer.

Rule. In reduction-ascending, divide the given number by the number of units of that denomination which make one of the next greater; and divide that quotient by the number of units of the same denomination which make one unit of the next higher, and proceed in this manner till the whole is finished.

Thus, as in the foregoing example I reduced 250*l.* into 240000 farthings; so inversely, here I say, in 240000 farthings, how many pounds?

Farthings in 1 penny	4) 240000
Pence in 1 shilling	12) 60000
Shillings in 1 pound	20) 5000
	<u>250</u>

In this example, I first divide the given number of farthings by 4, the number of units of that denomination which are contained in an unit of the next higher denomination; or the number of farthings contained in one penny, and the quotient gives the number of pence contained in 240000 farthings; which quotient is again divided by 12, the number of pence contained in one shilling, and that quotient gives the number of shillings contained in the given sum of farthings; and lastly, these shillings are again divided by 20, the number of shillings in one pound, and the quotient is 250*l.* for the answer.

Thus it may be seen, that reduction ascending and descending prove each other. For if the sum be performed by reduction descending, it must be proved by reduction ascending, as in the two foregoing examples; and if it be in reduction ascending, it must be proved by reduction descending.

In reduction descending, when the sum consists of several denominations, the number in each denomination, after the first, is to be added to the denomination to which it belongs; as in the following example:

Example.

Example 3. In 239*l.* 10*s.* 6*d.* how many pence?—In this example, after having reduced the pounds into shillings by multiplying by 20, I add to the product the 10 shillings which stand in the place of shillings; and having reduced the shillings into pence by multiplying by 12, I add to that product the 6 pence which stand in the place of pence; and the answer to the work is thus found to be 57,486 pence.

$$\begin{array}{r} 239 \text{ } 10 \text{ } 6 \\ \underline{20} \\ 4780 \\ \underline{10} \\ 4790 \\ \underline{12} \\ 57480 \\ \underline{6} \\ \underline{\underline{57486}} \end{array}$$

Example 4. In 24 tons, 10 hundreds, and 3 quarters, how many pounds?—This is performed as the foregoing, but having respect to the table of avoirdupois weight. I therefore multiply the tons by 20 to reduce them to hundred weights, as 20 hundred is 1 ton, and to the product I add the 10 cwt. I then multiply by 4 to bring the hundreds into quarters, to which I add the 3 quarters: and, lastly, multiply the quarters by 28, the number of pounds in a quarter: the product is 54964 for the answer.

$$\begin{array}{r} 24 \text{ } 10 \text{ } 3 \\ \underline{20} \\ 480 \\ \underline{10} \\ 490 \\ \underline{4} \\ 1960 \\ \underline{3} \\ 1963 \\ \underline{28} \\ 15704 \\ \underline{3926} \\ \underline{\underline{54964}} \end{array}$$

Examples in Reduction Ascending.

Example 5. In 24,649,721 minutes, how many days, hours, and minutes?

Example 6. In 47,398 grains of troy weight, how many ounces, pennyweights, and grains?

Example 7. In 29,472,986 square perches of land, how many acres, roods, and perches?

Example 5.

$$\begin{array}{r} 6,0)2464972,1 \\ \underline{2) \quad 410828,41} \\ 12) \quad 205414 \\ \underline{17117,10} \\ \underline{\underline{2}} \\ \underline{\underline{20}} \\ \underline{\underline{2}} \end{array}$$

Example 6.

$$\begin{array}{r} 2)47398 \\ \underline{12)23699} \\ 2,0) \quad 197,422 \\ \underline{98 \text{ } 14} \end{array}$$

Example 7.

$$\begin{array}{r} 4,0)2947298,6 \\ \underline{4) \quad 736824,26} \\ \underline{\underline{154206}} \end{array}$$

In the first of these examples I divide the minutes by 60, to bring them into hours, by cutting off the 0 from the 6, and dividing by 6 only, as taught in Division, and the quotient is 410828 hours, and there remains 4 minutes, which, placed before the 1 cut off from the dividend, makes 41 minutes for a remainder. This quotient I again divide by 2, and that quotient by 12, which is equal to dividing by 24, the number of hours in 1 day, and the quotient is 17117 days, and there remains 10; but to find the true remainder, I multiply the 10, the last remainder, by 2, the first divisor of the hours, and the product is 20 for the true remainder: thus the answer to the question is, 17117 days, 20 hours, 41 minutes.

In the second example, I divide the grains by 2 and 12, to bring them into pennyweights, as 24 grains make 1 pennyweight; the quotient is 1974 pennyweights, and 22 grains remain. The pennyweights I divide by 20, to bring them into ounces, and the quotient is 98 ounces, and 1 pennyweight remains, to which I bring down the 4 I cut off from the dividend, and the last remainder is 14 pennyweights: thus the answer is 98 ounces, 14 pennyweights, and 22 grains.

In the last example, I divide the perches by 40, as 40 perches make 1 rood, and the quotient is 736824 roods, and 26 perches; the roods I divide by 4, to bring them into acres; and the answer is found to be 184,206 acres, and 26 perches.

Reduction, both ascending and descending, may be performed by one divisor or multiplier: thus, to bring farthings into pounds, the pounds may be divided by 960, the number of farthings in a pound, and the quotient will give the number of pounds, and the remainder (if any) must be resolved into the inferior denominations. And to reduce hundredweights into single pounds, they may be multiplied by 112, the pounds in an hundred-weight, and the product is the answer. But the method before laid down is the more regular, and at the same time the more expeditious way of performing this rule.

Examples

Examples of both Kinds for Practice.

Example 8. If 420 pieces of cloth contain 8420 ells Flemish, it is required to know how many ells English they contain? — *Ans.* 5052 ells English.

Example 9. In 220 puncheons of rum how many hog-
heads?—*Ans.* 293 hogheads, and 21 gallons remain.

Example 10. A silversmith hath 1000 ounces of silver to be made into spoons, salts, and tankards; each spoon to weigh 2 oz. 12 pwt. each salt 3 oz. and each tankard 30 oz. and to make an equal quantity of each, it is desired to know how many he can make of each?—*Ans.* 28, and 64 pwt. remain.

Example 10.

$$\begin{array}{r} 1000 \\ 20 \\ \hline 712 \overline{) 20000} (28 \\ 1424 \\ \hline 5760 \\ 5696 \\ \hline 64 \\ \hline \hline \end{array}$$

In the first of these examples the 8420 ells Flemish are reduced into quarters of a yard, by multiplying by 3 (as there are 3 quarters in an ell Flemish), and then brought into English ells, by dividing these quarters by 5, the number of quarters in an ell English.

The 9th example is wrought in the same manner: viz. by reducing the 220 puncheons into gallons, by multiplying by 84, and bringing these gallons into hogsheds, and by dividing them by 7 and 9, which is equal to 63, the gallons in a hogshed.

**This method of reduction always takes place when the less
denomination**

denomination is not contained any certain number of times exactly in the greater.

In the 10th example I reduce the weight of 1 spoon, 1 salt, and 1 tankard, into pennyweights, by multiplying by 20, and then add them together; and by the total 712, I divide the 1000 ounces of silver, which is also reduced into pwts. and it quotes 28 of each, and 64 pwts. remain.

By reduction we are enabled to reduce the coin of one country into that of another, without having recourse to the rule of three, or exchange.

Example 11. What is the value, in English coin, of 350 ducats, at 4s. 2d. per ducat?

Example 12. In 246l. 18s. 6d. Flemish money, how much English, the course of exchange being 30s. 6d. per pound sterling?

Example 13. How much money English is there in 4420 pieces of eight, the course of exchange being at 49½d. sterling?

<p><i>Example 11.</i></p> $ \begin{array}{r} 350 \\ 50 \\ 12 \overline{) 17500} \\ 2,0 \overline{) 145,84} \\ \text{Ans. } \underline{\underline{£72 \ 18 \ 4}} \end{array} $	<p><i>Example 12.</i></p> $ \begin{array}{r} 30 \ 6 \quad 246 \ 18 \ 6 \\ 12 \quad 20 \\ \hline 366 \quad 4938 \quad \dots \\ 12 \\ 366 \overline{) 59262} \{ 161 \\ 366 \dots \\ 2266 \\ 2196 \\ 702 \\ 366 \\ 336 \\ 20 \\ 365 \overline{) 6720} \{ 18 \\ 366 \dots \\ 3060 \\ 2918 \\ 132 \\ 12 \\ 365 \overline{) 1584} \{ 4 \\ 1464 \\ 120 \\ 4 \\ 366 \overline{) 450} \{ 1 \\ 366 \\ 114 \\ \hline \hline \end{array} $	<p><i>Example 13.</i></p> $ \begin{array}{r} 4420 \quad 4420 \\ 5 \quad 49 \\ 8 \overline{) 22100} \quad 39780 \\ 2762 \quad 17680 \\ 216580 \\ 2762 \\ 12 \overline{) 219340} \\ 2,0 \overline{) 1827,84} \\ \text{Ans. } \underline{\underline{£713 \ 18 \ 4}} \end{array} $
---	---	--

Ans. 161l. 18s. 4½d.

In the 11th example, I multiply the 350 ducats by 50, to bring them into pence, and then divide by 12, and it quotes 1458 shillings and 4 pence: I then divide the shillings by 20, and the quotient is 72*l.* 18*s.* 4*d.* for the answer.

In the 12th example the 246*l.* 18*s.* 6*d.* Flemish money is reduced into pence, and then divided by 366, the course of exchange; and the answer is 161*l.* 18*s.* 4½*d.* and 114 farthings remain.

The 13th example is wrought in the same manner as the 11th: viz. by reducing the pieces of eight into pence; and for the fraction $\frac{1}{4}$ of a penny, I multiply the given number of pieces of eight by 5, the numerator, or upper figure of the fraction, and divide the product by 8, the denominator, or lower figure of the fraction, and to the quotient I add the pence contained in the pieces of eight, and then reduce the whole into shillings and pounds, by dividing by 12 and 20, as before, and the answer is 913*l.* 18*s.* 4*d.*

This method of reducing foreign coin into English may serve for those who are unacquainted with the rule of practice; for practice performs this much more expeditiously, as will be shewn in its proper place.

Those sums in reduction, in which both division and multiplication are used, must be proved by multiplication and division; as, for example, that part which is performed by multiplication must be proved by division, and that part performed by division must be proved by multiplication.

The foregoing examples, perfectly understood, will be sufficient to give the learner a complete knowledge of this rule, and the various uses to which it may be applied.

It is absolutely necessary that the learner be perfectly acquainted with what has been delivered in the foregoing part of this chapter, as all the following rules in arithmetic are performed by one or more of the foregoing rules; I have therefore been more explicit in the former part; being the basis of the whole.

SECT. VII.

THE GOLDEN RULE; OR, SINGLE RULE OF THREE
DIRECT.

THIS rule, which, for its universal use in most parts of the mathematics, is called the *golden rule*, is also called a *rule of proportion*, because the number sought bears a certain proportion to one of the numbers given.

It is called the *rule of three*, because it consists of three given numbers, from which a fourth number is to be found, which, in the direct rule, bears the same proportion to the second number as the third does to the first.

Rule. Multiply the second and third numbers together, and divide the product by the first number, and the quotient is the answer sought, or the fourth number.

Example 1. ^{1st number.} If 3 yds. of ^{2^d number.} muslin cost 12s. ^{3^d number.} what will 9 yds. cost at that rate?

$$\begin{array}{r} 9 \\ 3 \overline{) 108} \\ \underline{36s.} \end{array}$$

Here the fourth number or answer, 36, bears the same proportion to the second number 12, as 9 the third number, bears to the first number 3, that is, it contains it three times; or it bears the same proportion to the third number 9 as the second number 12 does to the first number 3, viz. contains it four times. This proportion is called *direct* proportion; from whence this rule is called the rule of three *direct*, and is always performed as above.

But when the fourth number bears the same proportion to
the

the second as the first does to the third, it is then called *indirect* proportion. Questions of this nature belong to the next rule, called the rule of three *inverse*, of which hereafter.

In order to know which is the second and third number, it must be noted, that of the three numbers which are in every question in this rule, that number which asks the question must occupy the third place, and is called the third number; and that number which is of the same nature with the fourth number or answer, must be the second number, and consequently the other number must be the first. The second and fourth numbers are therefore always of the same nature; as are the first and third. Thus, in the foregoing example, the number 9 asks the question, for the question is, how much will 9 yards cost? 9 is therefore the third number. 12 is of the same nature with the fourth number, being money; it must therefore possess the second place: and 3, the other number, must be the first, which is of the same nature with the third, viz. yards.

When either the first or third numbers consist of different denominations, they must both be reduced to the same denomination; and when the second number consists of divers denominations, it must also be reduced to the lowest:—this reduction must be performed before the work can be wrought; and it must be observed, that the fourth number or answer to the work, is always of the same denomination with the second number so reduced.

The numbers being so reduced, the second and third numbers are to be multiplied together, and the product divided by the first, as before directed; and the fourth number or answer must be brought into the proper denominations required by reduction, and if any thing remain after the products of the second and third numbers are divided by the first, such remainder must be reduced into the next lower denomination, and then divided by the first number, as before; and if any thing still remain, it must be reduced into the next lower denomination (if there be any lower), and divided by the first number;

In the proof of this example, I reduce the first number 45 $\frac{1}{2}$ into shillings, because the third number 4 $\frac{1}{2}$ 10s. must be reduced into shillings, consisting of pounds and shillings; and then multiplying the second and third numbers together, and dividing by the first, the answer is 12 gallons, as in the example; it therefore proves the work right.

Example 3. If the income of a person be 3 farthings a minute, how much is it per annum?

1st number. *2d number.* *3d number.*
Say, if 1 min. produce 3 farthings, what will 365 days 6 hours produce?

$$\begin{array}{r}
 24 \\
 \hline
 1466 \\
 730 \\
 \hline
 8766 \text{ hours} \\
 60 \\
 \hline
 525960 \text{ minutes} \\
 3 \\
 \hline
 4) 1577880 \text{ farthings} \\
 12) 394470 \text{ pence} \\
 2,0) 3287,2-6 \\
 \hline
 1643 \\
 12 \\
 \hline
 \end{array}$$

Answer 1643 $\frac{1}{2}$ 12s. 6d.

Here the 365 days 6 hours are reduced into minutes by multiplying first by 24 and then by 60, the product is then multiplied by 3, the second number, and the last product is the answer in farthings, which is brought into pounds and shillings by division, and the *answer* is 1643 $\frac{1}{2}$ 12s. 6d.

In the foregoing example the first number is an unit; when this is the case, the work is performed by multiplication, and when the third number is an unit, the work is wrought by division, for 1 neither multiplies nor divides; questions of this sort, therefore, properly belong to reduction.

Example 4. If the effects of a bankrupt amount to 2796 $\frac{1}{2}$ 10s. and his debts be 9990 $\frac{1}{2}$ 12s. it is requested to know how much he can pay in the pound?

VOL. I.

Z

Say,

number; proceed in this manner till the remainder be brought to the lowest denomination.

Example 2. If 12 gallons of brandy cost 4*l.* 10*s.* what will 120 gallons cost at that rate?

<i>1st number.</i>	<i>2^d number.</i>	<i>3^d number.</i>
If 12 gallons cost	4 <i>l.</i> 10 <i>s.</i>	what will 120 gallons cost?
	20	
	<u>90</u>	
		90
		12) 10800
		<u>2,0) 90,0</u>
		Ans. £ 45

In this example (the numbers being placed as before directed); the second consisting of two denominations, viz. pounds and shillings, it must be reduced to the lowest denomination (shillings), and the product is 90 shillings: the question will then be, if 12 gallons cost 90*s.* what will 120 cost? I therefore multiply 120 the third number, by 90 the second number, and divide the product by 12 the first number, and the quotient 900 is the fourth number, or answer to the question, which, because the second number is reduced to shillings, is shillings also, and is divided by 20 to bring them into pounds, and the quotient is 45 pounds, the true answer, or price of 120 gallons at that rate.

The Proof.

There are several methods of proving questions in the rule of three, but the truest and most improving to the learner is, to back state the question: thus, to prove the last example, I state the question backwards, making that number which was the fourth number in the question the first number in the proof, and that which was the third number here I make the second, and the second I make the third.

<i>1st number.</i>	<i>2^d number.</i>	<i>3^d number.</i>
Proof. If 45 <i>l.</i> purchases	120 gallons,	what will 4 <i>l.</i> 10 <i>s.</i> purchase?
	20	
	<u>900</u>	
		90
		9,00) 108,00
		<u>12</u>

There

In the proof of this example, I reduce the first number 45^l. into shillings, because the third number 4^l. 10s. must be reduced into shillings, consisting of pounds and shillings; and then multiplying the second and third numbers together, and dividing by the first, the answer is 12 gallons, as in the example; it therefore proves the work right.

Example 3. If the income of a person be 3 farthings a minute, how much is it per annum?

^{1st number.} Say, if 1 min. produce 3 farthings, what will ^{2^d number.} 365 day: 6 hour: ^{3^d number.} produce?

$$\begin{array}{r}
 24 \\
 \hline
 1440 \\
 730 \\
 \hline
 6720 \text{ hours} \\
 6720 \\
 \hline
 322560 \text{ minutes} \\
 322560 \\
 \hline
 1027680 \text{ farthings} \\
 1027680 \\
 \hline
 256920 \text{ pence} \\
 256920 \\
 \hline
 21410 \text{ shillings} \\
 21410 \\
 \hline
 2141 \text{ pounds}
 \end{array}$$

Answer 2141^l. 10s. 6d.

Here the 1st number 1 min. is reduced into minutes, by multiplying it by 60, and then by 24, the 2^d number 365 day: 6 hour: is reduced into 3, the 3^d number 3 farthings, and the 4th number 1027680 farthings is reduced into pounds, shillings, and pence, by dividing it by 4, then by 12, and then by 20, the answer is 2141^l. 10s. 6d.

In the foregoing example the 1st number 1 is the unit, the 2^d number 365 day: 6 hour: is the number of minutes, and the 3^d number 3 farthings is the number of farthings, and the 4th number 1027680 farthings is the number of farthings, and the answer is 2141^l. 10s. 6d.

Example 4. If the price of a barrel of wheat be 3^l. 10s. 6d. what will 1200 barrels cost?

number; proceed in this manner till the remainder be brought to the lowest denomination.

Example 2. If 12 gallons of brandy cost 4*l.* 10*s.* what will 120 gallons cost at that rate?

<i>1st number.</i>	<i>2^d number.</i>	<i>3^d number.</i>
If 12 gallons cost 4 <i>l.</i> 10 <i>s.</i>	what will	120 gallons cost?
	20	90
	<u>90</u>	12) 10800
		<u>2,0) 90,0</u>
		<u>Ans. £ 45</u>

In this example (the numbers being placed as before directed); the second consisting of two denominations, viz. pounds and shillings, it must be reduced to the lowest denomination (shillings), and the product is 90 shillings: the question will then be, if 12 gallons cost 90*s.* what will 120 cost? I therefore multiply 120 the third number, by 90 the second number, and divide the product by 12 the first number, and the quotient 900 is the fourth number, or answer to the question, which, because the second number is reduced to shillings, is shillings also, and is divided by 20 to bring them into pounds, and the quotient is 45 pounds, the true answer, or price of 120 gallons at that rate.

The Proof.

There are several methods of proving questions in the rule of three, but the truest and most improving to the learner is, to back state the question: thus, to prove the last example, I state the question backwards, making that number which was the fourth number in the question the first number in the proof, and that which was the third number here I make the second, and the second I make the third.

<i>1st number.</i>	<i>2^d number.</i>	<i>3^d number.</i>
<i>Proof.</i> If 45 <i>l.</i> purchases	120 gallons,	what will 4 <i>l.</i> 10 <i>s.</i>
purchase?	20	90
	<u>900</u>	9,00) 108,00
		<u>12</u>

There

In the proof of this example, I reduce the first number 45*l.* into shillings, because the third number 4*l.* 10*s.* must be reduced into shillings, consisting of pounds and shillings; and then multiplying the second and third numbers together, and dividing by the first, the answer is 12 gallons, as in the example; it therefore proves the work right.

Example 3. If the income of a person be 3 farthings a minute, how much is it per annum?

1st number. *2d number.* *3d number.*
Say, if 1 min. produce 3 farthings, what will 365 days 6 hours produce?

$$\begin{array}{r}
 24 \\
 \hline
 1466 \\
 730 \\
 \hline
 8766 \text{ hours} \\
 60 \\
 \hline
 525960 \text{ minutes} \\
 3 \\
 \hline
 4) 1577880 \text{ farthings} \\
 12) 394470 \text{ pence} \\
 2,0) 3287,2-6 \\
 \hline
 1643 \\
 12 \\
 \hline
 \end{array}$$

Answer 1643*l.* 12*s.* 6*d.*

Here the 365 days 6 hours are reduced into minutes by multiplying first by 24 and then by 60, the product is then multiplied by 3, the second number, and the last product is the answer in farthings, which is brought into pounds and shillings by division, and the *answer* is 1643*l.* 12*s.* 6*d.*

In the foregoing example the first number is an unit; when this is the case, the work is performed by multiplication, and when the third number is an unit, the work is wrought by division, for 1 neither multiplies nor divides; questions of this sort, therefore, properly belong to reduction.

Example 4. If the effects of a bankrupt amount to 2796*l.* 10*s.* and his debts be 9990*l.* 12*s.* it is requested to know how much he can pay in the pound?

VOL. I.

Z

Say,

Say, if 9990*l.* 12*s.* pay 20*s.* in the pound, what will 2796*l.* 10*s.* pay? $\frac{20}{199812}$

In stating this question, I say, if 9990*l.* 12*s.* the whole amount of the bankrupt's debts, will pay 20*s.* in the pound, as it certainly will, what will 2796*l.* 10*s.* the net value of the bankrupt's property, pay? —And the answer is 5*s.* 7*d.* in the pound.

$$\begin{array}{r}
 20 \\
 \hline
 55930 \\
 20 \\
 \hline
 199812 \overline{) 1118600} (5 \\
 \underline{999060} \\
 119540 \\
 12 \\
 \hline
 199812 \overline{) 1434480} (7 \\
 \underline{1398684} \\
 35796 \\
 4 \\
 \hline
 \underline{\underline{143184}}
 \end{array}$$

In performing this example, the first and third numbers are reduced into shillings, and multiplying the second and third numbers together, and dividing by the first, the quotient is 5, which is of the same denomination with the second number, viz. shillings; and there is a remainder of 119540 shillings, which is reduced into pence, and then divided by the first number as before, and it quotes 7 pence, and there yet remains 35796 pence, which, reduced into farthings, does not contain the divisor once; these pence, therefore, remain over and above the 5*s.* and 7*d.* in the pound which the bankrupt pays.

The foregoing examples will be found sufficient to instruct the learner in the nature and method of working this rule; I shall therefore give a few examples for practice, leaving the operation to be performed by the learner.

Example 5. If 56*lb.* of indigo cost 11*l.* 4*s.* what will 1008*lb.* cost at that rate?

Say, if 56*lb.* cost 224*s.* what will 1008*lb.* cost? —Answer 4032*s.* or 201*l.* 12*s.*

Example 6. If a debtor owes his creditors 593*l.* 12*s.* and compounds at 7*s.* 6*d.* in the pound, what will pay his creditors at that rate?

Say,

Say, if 20s. can be paid by 90d. what will pay 11872s.?
—*Answer* 222l. 12s.

Example 7. If 100l. gain 6l. interest in 12 months, how much will 340l. gain in the same time at that rate of interest?

If 100l. gain 6l. what will 340l. gain?—*Answer* 20l. 8s.

Example 8. A draper bought 6 packs of cloth, each pack containing 12 pieces, for which he paid 1080l. being 8s. 4d. per ell Flemish: how many yards were there in each piece?

If 100d. purchases 3 qrs. what will 259200d. purchase?—*Answer* 7776 qrs. which divided by 72, the number of pieces in the whole, it quotes 108 qrs. or 27 yards in each piece.

The general use of this rule is, from having the rate, value, proportion, produce, interest, gain or loss of one or any other number of things, to find the rate, value, proportion, produce, interest, gain or loss of one or any other number of the same things in a direct proportion.

SECT. VIII.

OF THE SINGLE RULE OF THREE INVERSE.

THE rule of three inverse is that which teaches how from three given numbers to find a fourth, which shall bear the same rate or proportion to the second number as the first does to the third; or the third number bears the same proportion to the second as the first does to the fourth.

Rule. Multiply the first and second numbers together, and divide the product by the third.

The method of placing the numbers in this rule is the same as that of the rule of three direct, and therefore need not be repeated.

In order to discover whether a question belong to the direct or inverse rule, it must be remembered that the first and

third numbers are called the extremes; if, therefore, the fourth number be greater than the second, the less extreme must be the divisor; but if it be less, the greater extreme must be the divisor.

And when the first number is the divisor, the work belongs to the rule of three direct; but when the third number is the divisor, it belongs to the rule of three inverse; as in the following examples.

Example 1. If 8 men perform any certain piece of work in 6 days, how many men will perform the same work in 3 days?

If 6 days require 8 men, what will 3 days require?

$$\begin{array}{r} 6 \\ 3 \overline{)48} 16 \end{array} \quad \text{Answer 16 men.}$$

In this example I consider, that if to do the work in 6 days it requires the labour of 8 men, then, to do the same work in 3 days, it will require more men, consequently the fourth number will be greater than the second, therefore I divide the product of the first and second numbers multiplied together by the third number (the least extreme), and the quotient is 16 (men) for the answer.

The proof of questions in this rule, as well as the foregoing rule, is performed by back stating the question; but it must be observed, in all questions in the direct rule, the proof is wrought by the rule of three direct; and in questions of the inverse rule the proof is wrought by the rule of three inverse.

Thus, to prove the foregoing example, I say, if 16 men require 3 days to perform the work, how many days will 8 men require to perform it?

If 16 men require 3 days, what will 8 men require?

$$\begin{array}{r} 3 \\ 8 \overline{)24} 6 \end{array} \quad \text{Answer 6 days.}$$

Example 2. If a loaf at a certain price weighs $4\frac{1}{2}$ lb. when wheat is 6s. per bushel, what should it weigh when wheat is 4s. 6d. per bushel?

If

If 72d. per bushel give 9 half lbs. what will 54d. give ?

$$\begin{array}{r} 9 \\ 54 \overline{) 648} (12 \\ \underline{54} \\ 108 \\ \underline{108} \end{array}$$

Ans. 12 half lbs. or 6lb.

In this example, I multiply 72 pence, the price of the bushel of wheat, by 9, the number of the half pounds in the loaf, and dividing by 54 pence, the price of the bushel of wheat at the other price, the answer is 12 half pounds or 6 pounds for the weight of the loaf at that price of the wheat.

In London the price of the loaf is varied, and the weight continues the same. Questions which concern the price are wrought by making the price the second number.

Example 3. If a board be 8 inches broad, how much in length will make a square foot ?

If 12 in. in breadth require 12 in. in length, what will 8 in. in breadth require ?

$$\begin{array}{r} 12 \\ 8 \overline{) 144} (18 \end{array} \quad \text{Ans. 18.}$$

Example 4. How many yards of shalloon at 3qrs. wide are sufficient to line throughout the garments made with 1000 yards of cloth at 7qrs. wide ?

If 7qrs. wide require 1000yds. what will 3qrs wide require ?

$$\begin{array}{r} 7 \\ 3 \overline{) 7000} (2333\frac{1}{3} \end{array} \quad \text{Ans. } 2333\frac{1}{3} \text{ yards.}$$

Qu. 5. If 150l. be lent for nine months, how long should 90l. be lent to gain the same interest at the same rate ?

Answer 15 months.

Qu. 6. If a colonel be besieged in a town with 1000 men, having provisions for only 2 months, how many must he dismiss, that the provision may serve the remainder 5 months ?

—*Answer* 600, and retain 400.

Qu. 7. If a person perform a journey (travelling at an uniform rate) in 24 days, by travelling 12 hours per day, how

how long will it take to perform the same journey by travelling 16 hours per day at the same rate?—*Ans.* 18 days.

Qu. 8. If a carrier carry 12 cwt. 72 miles for 5*l.* how many miles will he carry 18 cwt. for the same money?—*Answer* 48 miles.

This rule serves to find a fourth number to the given three, in an inverted proportion. And it particularly answers seven sorts of questions: viz. 1. Having the value of two different sorts of coin, it shews how many pieces of the one are equal in value to a given number of the other. 2. From two different values of one commodity, and the value of an article made from the same commodity at one value thereof, to find the weight, measure, &c. of the same article; or, on the contrary, from the values of the commodity and the weight, measure, &c. of the article, to find the value thereof (the 2d example is of this nature). 3. From the breadths of two equal rectangular figures, and the length of one of them, to find the length of the other; or, from the two lengths and one breadth, to find the other breadth (of this nature are the third and fourth examples). 4. From the given weight, expense of carriage, and number of miles carriage of any goods, to find the number of miles any other weight could be carried for the same price: from a given weight and price, and two distances, to find the weight answerable to the other distance (of this nature is the eighth question). 5. From two sums of money lent, and the time for which one of them is lent, to find the time for which the other should be lent; or, from the two different times, and the sum which is lent for one of them, to find the sum which should be lent for the other time (of this nature is the fifth example). 6. From the quantity of work which a given number of men can perform in a given time, to find the number of men that can perform it in any other given time; or, from the number of men, to find the time any other given number would require (of this nature is the first example). 7. From the quantity of provisions, or money, and the

the number of men, or other creatures, it would serve a certain time, to find the number of men, or other creatures, it would serve any other time; or, from the quantity of provision and the number of consumers, to find the time it would serve any other number of consumers (of this nature is the sixth example).

SECT. IX.

OF THE DOUBLE RULE OF THREE DIRECT*.

In this rule there are five given numbers to find a sixth, which shall bear the same proportion to the product of the fourth and fifth numbers, as the third number bears to the product of the first and second.

Questions in this rule are resolved either by two operations in the single rule of three direct, or the *rule of three composed of five given numbers*.

Each question in this rule consists of two parts, the *supposition* and the *demand*.

Rule 1. By two operations. Place that number which is of the same nature with the sixth number, or answer, in the second place in the first operation; and the two other numbers in the supposition in the first place, the one over the other; and the two numbers in the demand in the third place, one over the other, in like manner as the two in the first place; observing that the bottom numbers in the first and third places be of like nature, as will also the top ones.

* Some modern writers compound the two double rules of three into one; I have, however, given them distinctly, being more consonant to the true theory of science. In the next Section is, nevertheless, shewn an infallible method of working both by one rule.

The work is then performed by two operations in the single rule of three direct; the answer to the first operation forming the second number in the second operation.

: 2. *By one operation.* Multiply the two numbers which stand one over the other in the supposition, together, for the first number; the two numbers in the demand for the third number; and the second number in the first question will be also the second number in the work. Then the answer is found by one operation in the single rule of three direct, as in the following example:

Example 1. If 100*l.* principal gain 5*l.* interest in 1 year, what will 140*l.* gain in 9 months?

The Question stated.

If 100*l.* gain 5*l.*
in 12 months,

What will 140*l.*
gain in 9 months?

In this example, the numbers 100, 5, and 12, belong to the supposition, and 140 and 9 is the demand; for the meaning of the work is, *suppose* 100*l.* gain 5*l.* interest in 12 months, (then follows the demand) I *demand* to know how much 140*l.* will gain in 9 months at the same rate of interest?

Thus the questions are stated according to the foregoing directions: the 5*l.* being the interest of the money (and of the same nature with the sixth number or answer) must be the second number; and the two other numbers in the supposition 100 and 12 are placed one above the other, as are the two numbers 140 and 9 in the demand. It matters not which of these two numbers is uppermost, provided that the numbers in each, which are of the same nature, occupy the corresponding places respectively: thus, in the supposition, the pounds principal is the uppermost number, so it is in the demand, and the number of months is undermost in both.

The question being thus stated, the work is wrought by two operations of the single rule of three direct. The three uppermost numbers are the numbers for the first operation, and the fourth number, or answer to these forms, the second number

number for the second operation; the bottom number of the supposition forms the first number, and the bottom number of the demand the third number; then the answer, or fourth number of this second operation, is the true answer to the question; as in the following example, which is the foregoing one at large.

First Operation.

If 100*l.* gain 5*l.* what will 140*l.*

$$\begin{array}{r} 5 \\ 1,00 \overline{) 7,00} (7 \end{array}$$

Second Operation.

If 12 months gain 7*l.* what will 9 months?

$$\begin{array}{r} 7 \\ 12 \overline{) 63} (5 \\ 60 \\ \hline 3 \\ 20 \end{array}$$

Answer 5*l.* 5*s.*

$$\begin{array}{r} 5 \\ 12 \overline{) 60} (5 \end{array}$$

The answer would have been the same if the number of months had been the uppermost numbers instead of the pounds principal; in which case the first question would be, if 12 months give 5*l.* interest, what will 9 months give? and the answer is 3*l.* 15*s.*; then the second question would be, if 100*l.* gain 3*l.* 15*s.* what will 140*l.* gain? and the answer is as before, 5*l.* 5*s.*

By one Operation.

If 100*l.* gain 5*l.* what will 140*l.* gain?

$$\begin{array}{r} \text{months } 12 \\ 1200 \end{array}$$

$$\begin{array}{r} 9 \text{ months} \\ 1260 \end{array}$$

$$\begin{array}{r} 5 \\ 12,00 \overline{) 63,00} \end{array}$$

$$\begin{array}{r} \text{remains } 5,00 \\ 20 \end{array}$$

$$\begin{array}{r} 5 \\ 12,00 \overline{) 60,00} \end{array}$$

5 shillings.

In this example, the work is stated as in the former; the two first numbers in the supposition are multiplied together, and the two numbers in the demand are also multiplied together, then these two products are made, the one the first, and the other the third number, and the second number in the first question is also the second number. This rule is the most sure and practical method of proving the double rule of three direct, when wrought by two single ones.

The foregoing example worked both ways will be sufficient to instruct the learner. I shall, therefore, give a few questions, with their answers, omitting the operation.

Qu. 2. Suppose 468 men consume 175 quarters of wheat in 168 days, I demand how many quarters will serve 5612 men 58 days?—*Ans.* 724 quarters, and $\frac{3}{4}\frac{8}{14}$ of a quarter, or a little more than half a quarter.

Qu. 3. Suppose 80 acres of grass be mowed by 8 men in 14 days, I demand how many acres 28 men will mow in 12 days?—*Ans.* 240 acres.

Qu. 4. Suppose the wages of 12 men for 6 days amount to 7*l.* 4*s.* what are the wages of 25 men for 40 days?—*Ans.* 100*l.*

Qu. 5. If 150*l.* principal put out to interest for 9 months be increased, principal and interest, to 156*l.* 15*s.* I demand how much is that per cent. per annum?—*Ans.* 9*l.*

SECT. X.

OF THE DOUBLE RULE OF THREE INVERSE.

THE double rule of three inverse is when there are five given numbers to find a sixth, in an inverted proportion.

Rule.

Rule. Place the numbers as directed in the last section. Multiply the lower number of the first place by the upper one of the third, and make the product the first number; next multiply the upper term of the first place by the lower one of the third, for the third number: then if the inverse proportion be found in the three upper numbers, the answer is given by one operation in the rule of three direct; but if the inverse proportion be found in the lower numbers, the work is performed by the inverse rule (for of every sum in this rule one question is direct and the other inverse).

Example 1. If 100*l.* gain 5*l.* interest in 12 months, what principal will gain 5*l.* 5*s.* in 9 months?

5 <i>l.</i>	100 <i>l.</i>	5 <i>l.</i> 5 <i>s.</i>
months 12		months 9

The pounds interest being reduced to shillings, and multiplied by the number of months, the question will stand, and operation be performed as follows :

$$\begin{array}{r}
 1260 \quad 100 \quad 900 \\
 \underline{100} \\
 9,00 \overline{) 1260,00} \\
 \underline{140.} \text{ for the answer.}
 \end{array}$$

If the number of months had been made the upper terms, the upper proportion would then have been direct, and would have been required to have been worked by the direct method. It would in that case stand thus :

12	100	9
<u>5</u>		<u>5 5*</u>

Q_n.

* The rule laid down in this section will be found quite general, and sufficient for working all questions in the double rule of three both direct and inverse; and is so obvious, as to require no demonstration. Nevertheless, in consequence of receiving advice from some teachers (not the most competent) of the mathematics, that I had not given the *improved method* of working this rule, I shall state this much-approved rule, verbatim, from a well-known treatise, and shew its fallacy.

Qu. 2. If 48*l.* serve for the maintenance of 12 men 8 days, how long will 288*l.* serve for 4 men?—*Ans.* 144 days.

Qu. 3. If, when a bushel of wheat costs 6*s.* 8*d.* a penny loaf weighs 6 ounces, how much will a loaf weigh that costs 10½*d.* when the wheat is 10*s.* the bushel?—*Ans.* 42 ounces.

“*Rule 1.* Let the principal cause of loss or gain, interest or decrease, action or passion, be put in the first place.

“*2.* Let that which betokeneth time, distance, or place, and the like, be in the second place; and the remaining one in the third.

“*3.* Place the other two terms under their like in the supposition.

“*4.* If the blank falls under the third term, multiply the first and second terms for a divisor, and the other three for a dividend; but,

“*5.* If the blank falls under the first or second term, multiply the third and fourth terms for a divisor, and the other three for the dividend; and the quotient will be the answer.

“*Proof.* By two single rules of three.

“*Example 1.* If 14 horses eat 56 bushels of oats in 16 days, how many bushels will be sufficient for 20 horses for 24 days?

By two single rules:

$$\begin{array}{l} \text{hor. bu. hor. bu.} \\ 1. \text{ As } 14 : 56 :: 20 : 80 \\ \text{da. bu. da. bu.} \\ 2. \text{ As } 16 : 80 :: 24 : 120 \end{array}$$

Or, in one stating, worked thus:

$$\begin{array}{l} \text{hor. da. bu.} \\ 14 : 16 : 56 \\ 20 : 24 : — \end{array} \quad \frac{56 \times 20 \times 24}{14 \times 16} = 120."$$

WALKINGAME.

That this rule is not founded on mathematical principles is evident from inspection; for by a different statement of the question (though exactly agreeable to the rules) a different answer will arise. Thus, how easily might the learner, required to work this question by one statement, order the numbers as follows:

$$\begin{array}{l} \text{hor. bu. da.} \\ 14 : 56 : 16 \\ 20 : — : 24 \end{array} \quad \frac{56 \times 24 \times 14}{16 \times 20} = 58\frac{1}{2}$$

According to this statement the answer will be 58½ bushels; whereas, the true answer is 120 bushels. The question is, nevertheless, stated in this last case agreeable to the rules.

Qu.

Qⁿ. 4. If 96 pioneers in 24 days cast a trench 96 yards long, how many pioneers will cast a trench 536 yards long in 8 days?—*Ans*. 1008 pioneers.

Qⁿ. 5. If 10 men mow 40 acres in 8 days, how many days will it require 3 men to mow 150 acres?—*Ans*. 100 days.

This rule may be proved by two single rules.

The following rules in this chapter, except vulgar fractions and the extractions of roots, are wrought either by some of the rules of proportion, delivered in the four preceding sections, or by the rule of practice, hereafter to be spoken of.

SECT. XL.

OF FELLOWSHIP.

FELLOWSHIP is that rule whereby merchants and others, trading in company, and employing a joint capital stock, are enabled to ascertain each partner's particular loss or gain, according to his share in the same joint stock.

This rule also serves to divide a bankrupt's estate among his creditors; to pay legacies, when there is a deficiency of the testator's effects: and, in fine, to divide the loss or profit of any joint concern among the losers or proprietors.

The rule of fellowship is either single or double, that is, without regard to time, or with time.

Fellowship without time, or single fellowship, is when different persons employ their respective stocks for the same time.

Rule.

Rule. As the whole stock of all the partners is to the whole gain or loss, so is each man's particular stock to his particular share of the gain or loss.

Proof. By adding each person's gain or loss together.

Therefore, the whole stock of all the partners is to be made the first number in the rule of three, the whole gain or loss the second number, and the particular stock of any one partner the third number; then the fourth number, or answer, is that partner's loss or gain, whose stock was the third number. This operation in the rule of three must be repeated if there be more than two partners, and performed as often as there are partners concerned.

Example 1. Three persons enter into trade together : A put into the trade 100*l.* B 170*l.* and C 300*l.*; at the making up their accompts, they find they have gained 550*l.* profit : what is each person's share ?

A's stock £.100
B's stock 170
C's stock 300
Whole stock £.570

Whole Stock. 570 <i>l.</i>	Whole Gain. 550 <i>l.</i>	A's Stock. 100 <i>l.</i>
	100 <i>£.</i> <i>s.</i> <i>d.</i>	
	57,0)5500,0(96 9 9½ ¾	
	513	
	370	
	342	
	280	
	20	
	57,0)560,0(98.	
	513	
	470	
	12	
	57,6)564,0(9 <i>d.</i>	
	513	
	510	
	4	
	57,0)204,0(3 <i> far.</i>	
	171	
	30	

57ol.	55ol.	B's Stock.	17ol.
	<u>170</u>		
	38500		
	<u>550</u>	£. s. d.	
57,0)	9350,0(164	0	8½ ¾
	<u>57</u>		
	365		
	<u>342</u>		
	230		
	<u>228</u>		
	20		
	<u>20</u>		
57,0)	400(0s.		
	<u>12</u>		
57,0)	480,0(8d.		
	<u>456</u>		
	240		
	<u>4</u>		
57,0)	96,0(1 far.		
	<u>57</u>		
	<u>390</u>		

57ol.	55ol.	C's Stock.	30ol.
	<u>300</u>	£. s. d.	
57,0)	16500,0(289	9	5½ ¾
	<u>114</u>		
	510		
	<u>456</u>		
	540		
	<u>513</u>		
	270		
	<u>20</u>		
57,0)	540,0(9s.		
	<u>513</u>		
	270		
	<u>12</u>		
57,0)	324,0(5d.		
	<u>285</u>		
	390		
	<u>4</u>		
57,0)	156,0(2 far.		
	<u>114</u>		
	<u>426</u>		

The

The Proof.

	£.	s.	d.
A's share	96	9	9½ 378
B's share	164	0	8½ 390
C's share	289	9	5½ 420
Whole stock	£. 550	0	0

If there be any remainders, such remainders in the proof must be added together (if they be fractions of the same denomination), and divided by the common divisor of each question (*i. e.* the total stock); and the quotient, which is units of the same denomination, added to the particular shares: thus, in the foregoing example, I added the three remainders, 330, 390, and 420, together, and dividing the total by 570, the common divisor, the quotient is 2, which are farthings, as they are fractions of a farthing.

Qu. 2. Four persons place their money in the public funds: A put in 360*l.* B 480*l.* C 700*l.* and D 860*l.* When the capital came to be sold out, the principal and interest amounted to 4200*l.* what is each man's share of the net profit?—*Answer,* A 270*l.* B 360*l.* C 525*l.* D 645*l.*

Qu. 3. A bankrupt is indebted to four creditors in the following sums: to A 55*l.* 8*s.* to B 608*l.* 14*s.* to C 304*l.* and to D 208*l.* 12*s.* The bankrupt's estate is worth only 337*l.* 17*s.* how much will each creditor receive?—*Ans.* A 111*l.* 7*s.* 5*d.* B 122*l.* 18*s.* 10½*d.* C 61*l.* 8*s.* 0½*d.* and D 42*l.* 2*s.* 7½*d.*

Qu. 4. A ship being lost value 1730*l.*; of which A paid 346*l.* B 519*l.* C 692*l.* and D 173*l.* towards building her; she was insured to the amount of 1360*l.* what was each person's loss?—*Answer,* A 74*l.* B 111*l.* C 148*l.* D 37*l.*

In the second example, the whole stock first placed in the funds is to be found by adding each person's stock together, and that made the first number in the rule of three; the whole gain is to be the second number, the particular stock of any one person the third number, and the fourth number is his share.

In the third example, the whole amount of the bankrupt's debt is to be made the first number; the value of his effects the second number; and any creditor's real due the third number, and the fourth number is that creditor's share.

Fellowship with time, or double fellowship, is when the different stocks are employed for different times.

Rule. Multiply each man's particular stock by the time it is employed for; then say, by the rule of three, as the total of all these products is to the whole gain or loss, so is each man's particular product to his particular share of the gain or loss.

Thus the rule of three must be repeated as often as there are products or persons concerned, as in the single rule.

Example 1. A put 200*l.* in a certain trade; B five months after put 300*l.*; at the end of one year they find they have gained 350*l.* profit, what is each person's share?

	Total Product. £.	Whole Gain. £.	A's Product. £.
200	4500	350	2400
12 months.		2400	
2400 A's product		140000	
		70	A's Share.
		45,00)8400,00(186 <i>l.</i> 13 <i>s.</i> 4 <i>d.</i>	
300		45	
7		390	
2100 B's product.		360	
4500 Total of the products.		300	
		270	
		3000	
		20	
		45,00)605,00(13 <i>s.</i>	
		45	
		150	
		135	
		1500	
		12	
		45,00)180,00(4 <i>d.</i>	
		180	
		0	

B's Product.

£.
4500£.
350£.
21002100
35000

70 B's Share.

45,00)7550,00(167l. 6s. 8d.

45

285

270

150

135

1500

20

45,00)300,00(6s.

270

3000

12

45,00)360,00(8d.

360

0

=

The Proof.

A's share £.186 13 4

B's share 163 6 8

£.350 0 0

The method of proof in this rule is the same as in the single rule, as may be seen above.

Qu. 2. A, B, and C rent a pasture in common, at 40l. per annum; A puts in it 3 oxen for 8 months in the year, B puts in 7 for 4 months, and C puts in 2 for the whole year; what must each person pay towards the rent? *Answer, A 12l. 12s. 7½d. ⅔; B 14l. 14s. 8½d. ⅔; C 12l. 12s. 7½d. ⅔.*

Example 3. A, B, and C make a stock for 12 months, A put in at first 728l. at the end of 2 months he put in 20l. more; B put in at first 200l. and at the end of 5 months he took out 150l.; C put in at first 40l. and at the end of 6 months 50l. more, and at the end of 10 months 10l. more; at the year's end they find they have gained 500l.; what is each person's profit?

In

In these examples, and all others of the like nature, where the different parts of the stock of each person are employed for a different time, the different parts of each person's stock are to be multiplied by their own separate times, and these products added together to make up the product of each person; and then each person's product added together to make the total product.

Thus, in the last example, I first multiply the 72*l.* put in by A, by 2, the number of months it is employed alone, and the product is — *l.* 1456

Then A put in 20*l.* more, which, added to the 72*l.* is 92*l.* and which multiplied by 10 months, the product is — — — 7450

And which two added together gives A's product 8906

B put in 200*l.* for 5 months, which product is — 1000

At the end of 5 months he took out 150*l.* and then there remained only 50*l.* which multiplied by 7 months, the product is — — — 350

The two products of B added together are his true one 1350

C put in at first 40*l.* which continued alone 6 months, that product therefore is — — — 240

At the end of 6 months he put in 50*l.* more, which, added to the former 40*l.* is 90*l.* which multiplied by 4 months, gives — — — 360

The last 2 months he had 100*l.* in the stock, which, multiplied by 2 months, gives — — — 200

Therefore the product of C is — — — 800

The total sum of all the products 11086

Then the work is wrought as before, saying, as 11086 (the sum of the products) is to 500*l.* (the total gain), so is 8936*l.* (the product of A) to 403*l.* or 74*l.* $\frac{6826}{11086}$, the share of A of the profits; and repeat the operation twice more, to find the shares of B and C.

B b 2

Answer.

Answer.

	£.	s.	d.	
The share of A	403	0	7½	<u>8856</u>
B	60	17	9	<u>1126</u>
C	36	1	7½	<u>1104</u>
	500	0	0	

SECT. XII.

OF ALLIGATION, MEDIAL AND ALTERNATE.

THE rule of alligation teacheth how to mix different substances together, and to discover the value of any part of such mixture; or to make a mixture from known substances of any value.

Alligation is either *medial* or *alternate*.

Alligation medial is that which teacheth how to find the rate or price of any mixture or compound, from having the rates or prices, and the quantities of the several substances given.

Rule. Multiply each quantity by its rate or price, and add the products together for a dividend; add the sums of the several quantities together for a divisor; and divide the sum of the products by the sum of the quantities, and the quotient will be the rate or price of the compound.

Example 1. A mealman mixes 20 bushels of flour, worth 5s. per bushel, with 12 bushels worth 3s. 4d. per bushel; what is 1 bushel of this mixture worth?

Price

Price of a bushel 6od.	Price of a bushel 4od.	Quantities.
No. of bushels 20	No. of bushels 12	12
<u>1200</u>	<u>480</u>	<u>20</u>
	1200	32
	32) 1680 (52) d. or 4s. 4½d. per bushel.	
	<u>160</u>	
	80	
	<u>64</u>	
	16	
	<u>4</u> far.	
	32) 64 (2	
	<u>64</u>	
	0	
	<u>0</u>	

In this example the price of the bushel is reduced into pence (as it mostly should). The answer is the price of the quantity, which is of the same denomination with the divisor, which here is bushels.

Proof. Find the value of the whole mixture from the value of any part, and if it be equal to the value of the original simples, the work is right.

Thus, to prove the foregoing example, I multiply the price of one bushel of the mixture by 4 and 8, or 32, the number of bushels in the whole, and the product, I find, is 7l.; then I find the value of the several simples, by multiplying the number of bushels in each by the number of pence in a bushel (which is already done in the example), and the product 1680 brought into pounds, gives 7l. as follows:

The Proof.

The Number of Bushels multiplied by the Pence in a Bushel.

12) 1680 pence
2,0) 14,0 shillings
7 pounds

The Price of a Bushel of the Mixture multiplied by the Number of Bushels.

s. d.
4 4½
4
17 6
8
7 0 0

Qs.

Qu. 2. A grocer mixed the following teas together, viz. 15lbs. at 8s. per lb. 20lbs at 7s. 4d. per lb, 10lbs. at 6s. 8d. per lb. and 24lbs. at 4s. per lb. what is one pound of this mixture worth?—*Answer* 6s. 2½d. $\frac{4}{9}$.

Qu. 3. A vintner mixes 5 gallons of wine at 7s. per gallon, with 9 gallons at 8s. 6d. per gallon, and 14½ gallons at 5s. 10d. per gallon; what is one gallon of this mixture worth?—*Answer* 6s. 10½d. $\frac{2}{3}$.

Qu. 4. A goldsmith melts 101½ ounces of gold bullion, of 14 carats * fine, with 152½ ounces of 18 carats fine, how many carats fine is this mixture?—*Answer* 16 $\frac{3}{8}$ carats fine.

Alligation alternate is the method of finding what quantity of simples, whose rates or prices are given, will form a mixture of a certain given rate or price.

Rule 1. Write the rates or prices of the several simples under each other. 2. Connect with a curve line the rate or price of each simple that is less than the rate or price of the mixture, with one or more of these rates or prices that are greater than that of the mixture; and each greater rate with one or more that are less: and place the rate or price of the mixture on the left hand of the rates or prices. 3. Write the difference between the rate of the mixtures and the rate of each simple opposite the rate with which such simple is connected or linked.

Then these differences which stand opposite any rate is the quantity which that rate requires to form a mixture of the given rate; but if there is more than one difference opposite any simple, their sum is the true difference.

* Gold is generally mixed with copper or some other base metal, which is called the alloy; and the gold is said to be so many carats fine as it contains pure gold: thus, if an article weighs 24 carats, and contains 22 carats of gold, and 2 of alloy, it is said to be 22 carats fine. What a carat is may be seen, page 135.

Example

Example: 1. A grocer would mix teas at 4s. per lb. 7s. per lb. 9s. per lb. and 10s. per lb. in such proportion that the mixture may be worth 6s. per lb.; what quantity of each must be taken?

Price of the Teas.	Quantities of each.	The Answer.
4	- 1 3 4	8lb. at 4s. per lb.
7	- 2	2lb. at 7s. per lb.
9	- 2	2lb. at 9s. per lb.
10	- 2	2lb. at 10s. per lb.
		<u>14lb. at 6s. per lb.</u>

In this example, I first state the work as before directed, placing the prices of the teas in a column over each other, with 6, the given price of the mixture, on the left hand.

Secondly, I connect the prices with each other by curve lines; 4 the top figure, being less than the rate of the mixture, I connect with 7, 9, and 10, because they are all greater than 6, the rate of the mixture.

Thirdly, I find the difference between 6, the price of the mixture, and 4, that of the first simple, which is 2; I therefore place 2 opposite the 7, 9, and 10, as the 4 is linked to all of them. Then I find the difference between the 6 and the next figure 7, which is 1, I therefore place 1 opposite the 4, being the figure to which the 7 is linked. Then the difference between the 6 and 9 is 3, which I place also opposite the 4 (the 9 being linked thereto), and the difference between 6 and 10, which is 4, I also place opposite the 4 (as the 10 is also linked to it). These differences, so placed, contain the true proportion of each sort of tea at the price opposite to each, that should be taken to form a mixture at the desired rate.

But opposite the 4 there are three differences, viz. 1, 3, and 4, which are to be added together, as seen in the last column. Thus, there must be 8lb. of tea at 4s. per lb. 2lb. at 7s. per lb. 2lb. at 9s. per lb. and 2lb. at 10s. per lb.; and the whole quantity of the mixture is 14lb. at 6s. per lb.

These

These questions are proved in the same manner as those in allegation medial, viz. by finding the total value of all the simples in their separate state, and the total value of the mixture; and if these two values be equal, the work is right.

The Proof of the foregoing Example.

	£.	s.	d.	
8lb. of tea, at 4s. per lb. is	1	12	0	14lb. of the mixture at 6s. per lb.
<u>4</u>				84 shillings, or
32 Shillings				4l. 4s.
2lb. at 7s. per lb. is	0	14	0	
<u>7</u>				
14 Shillings				
2lb. at 9s. per lb. is	0	18	0	
<u>9</u>				
18 Shillings				
2lb. at 10s. per lb. is	1	0	0	
<u>10</u>				
20 Shillings				
	Total	<u>4</u>	<u>4</u>	<u>0</u>

Qⁿ. 2. A farmer mixed wheat at 4s. the bushel, with rye at 3s. the bushel, and barley at 18d. the bushel, how much must he mix of each to sell the whole mixture at 22d. the bushel?—*Answer*, 40 bushels at 18d. per bushel, 4 bushels at 3s. per bushel, and 4 bushels at 4s. per bushel.

Qⁿ. 3. A goldsmith has gold to melt of 24 carats fine, 21 carats fine, 19 carats fine, and 16 carats fine; how much of each must he take to form an article of gold that shall make 17 carats fine?—*Answer*, 1 of 24, 1 of 21, 1 of 19, and 13 of 16 carats fine.

When the whole mixture is limited to a certain quantity, after finding the quantity of each of the simples as before, say (by the rule of three), as the sum of the quantities is to the given quantity, so is the quantity of each simple to the required quantity of each.

Example 4. A vintner is desirous to mix 5 sorts of wine together: viz. at 11s. per gallon, 10s. per gallon, 9s. per gallon, 7s. per gallon, and 6s. per gallon, in such proportion as to make 40 gallons of wine, worth 8s. per gallon: how much of each sort must he take?

11	- 2	2
10	- 2	2
9	- 1	1
7	- 1	1
6	- 3 2	5
		<u>11</u>

In this example, after linking the prices together, as before directed, I have the quantity of each wine to form a mixture of 8s. per gallon; but the whole quantity of the mixture thus found is only 11 gallons, whereas it should be 40 gallons, therefore I say,

Gall.	Gall.	Gall.	Gall. Pinn.
As 11	to 2	so is 40	the quantity required to 7 2 ² / ₁₁
11	2	40	7 2 ² / ₁₁
11	1	40	3 5 ² / ₁₁
11	1	40	3 5 ² / ₁₁
11	5	40	18 1 ² / ₁₁
			<u>40 0</u>

The quantity required

Question 5. A grocer has sugar at 10d. 8d. 6d. and 4d. per lb. of which he would make a mixture to consist of 60lb. and worth 5d. per lb.; how much of each sort must he take?—*Answer,* 5lb. at 10d. 5lb. at 8d. 5lb. at 6d. and 45lb. at 4d. per lb.

Sometimes it is required to take a certain quantity of any one simple to mix with the others, and which generally alters the quantities of the other simples. To find what proportion of the others is requisite, I say (by the rule of

three), as the quantity of that simple whose particular quantity is given is to the given quantity, so is the found quantity of any other simple to the quantity required.

Example 6. A grocer would mix raisins at 11d. per lb. 10d. per lb. 9d. per lb. and 6d. per lb. with 120lb. at 7d. per lb.; how much of each sort must he take, that the whole may be worth 8d. per lb.?

8 {	11	-	2	1	3
	10	-	2	1	3
	9	-	2		2
	7	-	2	3	5
	6	-	1	2	3

Hence the several quantities requisite are as placed in the example; but to find the quantities which should be taken of each sort, I say (by the rule of three), as 5lb. (the quantity there found) is to 120lb. (the quantity required to be taken), so is 3lb. (the quantity at 11d. per lb.) to 72lb. (the quantity that should be taken).

lb.	lb.	lb.	lb.
As 5	is to 120	so is 3	to the quantity required 72
5	120	3	72
5	120	2	48
5	120	6	144

Which, with the 120
at 7d. per lb. is 456

Question 7. A vintner mixes wine at 12s. 10s. and 6s. per gallon, with 20 gallons at 4s. per gallon; how much of each sort must he take to make the mixture worth 8s. per gallon?—*Answer,* 20 gallons at 12s., 10 gallons at 10s. and 10 gallons at 6s.

From the foregoing examples, it is evident that there are several ways of working this rule, according to the method

of

of stating the question; as may be partly seen in the fourth and sixth examples, which, though quite different questions from each other, yet consist of the very same figures, and may be stated in the same manner; but are here varied for the information of the learner, and admit of still greater variety, as the learner may prove at his leisure.

All the caution that is necessary in linking the numbers together, is, that of every two numbers that are linked together, one must be greater and the other less than the rate or price of the mixture. Therefore, the first example in this rule, having but one number less than the rate of the mixture, admits of no other method of stating than that described.

SECT. XIII.

OF VULGAR FRACTIONS.

Reduction, Subtraction, Multiplication, Division, and the Rule of Three.

A FRACTION is any part or parts of an integer or unit, and (in vulgar fractions) is represented by two numbers placed one above the other, with a line drawn between them.

The number below the line is called the *denominator*, and shews how many parts the integer is divided into; the number above the line is called the *numerator*, and shews how many of those parts the fraction represents.

Thus the fraction to represent *three farthings* is thus written $\frac{3}{4}$ Numerator Denominator and is properly called three-fourths of a penny;—a penny being the integer, or unit, of which the fraction is a part.

Vulgar fractions are either proper or improper, single or compound.

A *proper* fraction has its numerator less than its denominator: as, $\frac{2}{3}$ two thirds, $\frac{3}{4}$ three fourths, $\frac{3}{5}$ three fifths; and always stands for less than the integer it is taken from.

An *improper* fraction has the numerator equal to or greater than the denominator, as $\frac{4}{3}$, $\frac{5}{4}$, $\frac{6}{5}$, and always represents as much or more than the integer.

A *single* fraction is only a single expression of any assigned parts of an integer.

A *compound* fraction consists of more than one fraction, and is a fraction of a fraction, and they have the particle *of* placed between them, as $\frac{1}{2}$ of $\frac{3}{4}$, $\frac{2}{3}$ of $\frac{4}{5}$, &c.

There are also mixed numbers, which are whole numbers united with fractions, as $8\frac{1}{2}$, $12\frac{1}{3}$, &c.

The *common measure* of two or more numbers, is that number which will divide each of them without a remainder; thus 4 is the common measure of 12 and 16.

A number that can be exactly measured by two or more numbers is called their *common multiple*; and if it be the least number that can be measured, it is called their *least common multiple*: thus, 45 and 60 are the common multiples of both 3 and 5, but their least common multiple is 15.

Before the learner can proceed to reduction of fractions, it is necessary that he be able to solve the two following Problems.

PROBLEM I.

To find the greatest common Measure of two or more Numbers.

Rule. If there be only two numbers, divide the greater by the less: and if there be no remainder, the divisor is the greatest common measure; but if there be a remainder, the divisor is to be divided by such remainder, and if there still be

be a remainder, the last divisor is still to be divided by the last remainder till there be no remainder; then the last divisor is the greatest common measure.

But if there be more than two numbers after having found the greatest common measure of two of them, the common measure of that common measure and one of the other numbers is to be found in the same manner; and proceed in this manner through all the numbers: then the last common measure will be the common measure of each of them.

Example 1. What is the greatest common measure of 624, 3126, and 336?

$$\begin{array}{r}
 624 \overline{)3126} \begin{array}{l} 5 \\ 3120 \\ \hline 6 \end{array} \quad \begin{array}{l} 6 \overline{)624} \begin{array}{l} 104 \\ 6 \\ \hline 24 \\ 24 \\ \hline 0 \end{array} \end{array}
 \end{array}
 \qquad
 \begin{array}{r}
 6 \overline{)336} \begin{array}{l} 56 \\ 30 \\ \hline 36 \\ 36 \\ \hline 0 \end{array}
 \end{array}$$

Thus 6 is the greater common measure of 624, 3126, and 336.

2. What is the greatest common measure of 81 and 63?

—*Answer* 9.

3. What is the greatest common measure of 720, 356, and 1736?—*Answer* 8.

PROBLEM II.

To find the least common Multiple of two or more Numbers.

Rule. Divide the numbers by any number that will divide two or more of them without a remainder, and set the quotient of each number under the dividend to which it belongs; bring down the undivided numbers, or those which cannot be divided, into the same line with the quotient. Then divide

divide the second line, in the same manner, by any number that will divide two or more of them, and bring down the quotients and undivided numbers, as before. Proceed in this manner as long as there can be found any two numbers that can be exactly divided by one number.

When the numbers are so far divided that there are no two numbers that can be divided by one, then multiply all the divisors and the quotients continually together, and the product is the least common multiple.

Example 1. . . What is the least common multiple of 7, 8, 4, and 2 ?

$$\begin{array}{r}
 2) 7 \text{ „ } 8 \text{ „ } 4 \text{ „ } 2 \\
 \hline
 2) 7 \text{ „ } 4 \text{ „ } 2 \text{ „ } 1 \\
 \hline
 7 \text{ „ } 2 \text{ „ } 1 \text{ „ } 1
 \end{array}
 \qquad
 \begin{array}{r}
 2 \\
 \times 2 \\
 \hline
 4 \\
 \times 7 \\
 \hline
 28 \\
 \times 2 \\
 \hline
 56
 \end{array}$$

Here I divide the numbers first by 2, as that number exactly measures the 8, 4, and 2 ; the quotients I place under their dividends ; I then divide the next line by 2, as that measures 4 and 2 ; the quotient I place below, and the 7 I bring down.

Then, as there are no two others which admit of a common measure, I multiply the divisors and quotients continually together, and the product 56 is the least common multiple of them all.

Before fractions can be wrought by any of the rules in arithmetic, it is necessary to reduce them from compound to simple ones, and to bring them into several other forms ; for which purpose there are eight ways of altering the form of fractions, without changing their value, as follow :

Case 1. To reduce a mixed number to an improper fraction.

Rule. Multiply the integer by the denominator of the fraction, and add thereto the numerator, and the product will

will form a numerator to a fraction, whose denominator is the denominator of the former fraction.

Example 1. Reduce $10\frac{3}{4}$ gallons to an improper fraction.

$$\begin{array}{r} 10 \\ \frac{4}{4} \\ 40 \\ \frac{3}{4} \\ \hline 43 \end{array}$$

$\therefore 43$ the fractions required.

2. Reduce $56\frac{1}{2}$ to an improper fraction.—*Answer* $112\frac{1}{2}$.

Case 2. To reduce an improper fraction to a whole or mixed number.

This is the reverse of the former case.

Rule. Divide the numerator by the denominator, and the quotient will be the whole number, and the remainder (if any) will be the numerator to a fraction, whose denominator is the divisor.

Thus, to reverse the first example in the former case,

Example 1. Reduce $10\frac{3}{4}$ to its equivalent, whole, or mixed number.

$$\begin{array}{r} 4)43(10 \\ \underline{4} \\ 3 \end{array}$$

$10\frac{3}{4}$ the mixed number.

2. Reduce the second example in the former case $112\frac{1}{2}$ to its equivalent, whole, or mixed number.—*Answer* $56\frac{1}{2}$.

Case 3. To reduce a fraction to its lowest terms.

Rule. Divide the two terms of a fraction by any number that will exactly divide them without a remainder, and then divide the quotients by a number that will exactly divide them, and so on till there can be found no number that will exactly divide the last quotients; then these quotients will be the fraction reduced to its lowest terms.

Or, 2dly, find the greatest common measure of the numerator and denominator, as taught in the first problem in this section, and divide them thereby; then the quotients will be the fraction reduced to its least terms.

Examp

Example 1. Reduce $\frac{488}{164}$ to its least terms.

4) $\frac{488}{164}$ | $\frac{122}{41}$ | $\frac{61}{108}$ the *Answer*.

Or thus,

$$\begin{array}{r}
 488)864(1 \\
 \underline{488} \\
 376)488(1 \\
 \underline{376} \\
 112)376(3 \\
 \underline{336} \\
 40)112(2 \\
 \underline{80}
 \end{array}$$

Therefore

$$\begin{array}{r|l}
 8)488(61 & 8)864(108 \\
 \underline{48} & \underline{8} \\
 8 & 64 \\
 \underline{8} & \underline{64} \\
 0 & 0 \\
 \hline & \hline
 \end{array}$$

$$\begin{array}{r}
 32)40(1 \\
 \underline{32} \\
 8)32(4 \\
 \underline{32} \\
 0 \\
 \hline
 \end{array}$$

Thus, 8 is the greatest common measure of 488 and 864, and these, divided by 8, quote 61 and 108 as before *.

2. Reduce $\frac{122}{41}$ to its least terms.—*Answer* $\frac{122}{41}$.

3. Reduce $\frac{61}{108}$ to its least terms.—*Answer* $\frac{61}{108}$.

Case 4. To reduce a whole number to an improper fraction.

Rule. Multiply the whole number by the intended denominator, and the product placed over the denominator will form the fraction required.

* There are several brief methods of reducing a fraction to its least terms; a few of which I shall mention.—1. Any number that ends with 5 or 0 is divisible by 5: as 25, 55, 105, 40, 100, &c.—2. A number that ends with an even number, or a cypher, is divisible by 2: as 28, 50, 64, 100, &c.—3. If the two right-hand figures of a number be divisible by 4, the whole is divisible by 4: as 312, 4728, &c.—4. If the three right-hand figures be divisible by 8, the whole is divisible by 8: as 81104, &c.—5. If the terms of a fraction have a cypher or cyphers on the right-hand, as many cyphers may be cut off from the one as from the other.

Example

Example 1. Reduce 12 to a fraction, whose denominator shall be 7.

$$\frac{12}{7} = \frac{84}{49} \quad \frac{84}{49} \text{ the fraction required.}$$

2. Reduce 135 to an improper fraction, whose denominator shall be 16. — *Answer* $2\frac{15}{16}$.

3. Reduce 1206 to a fraction, whose denominator shall be 20. — *Ans.* $24\frac{3}{10}$.

Case 5. To find the value of a fraction in the known parts of money, weights, or measures.

Rule. Multiply the numerator by the parts of the next inferior denomination, and divide the product by the denominator. If then any thing remain, multiply it as before by the parts of the next inferior denomination, and divide by the denominator; and so on till it be brought to the lowest denomination, and if any thing then remain, it must be set over the denominator for a fraction of the lowest denomination.

Example 1. What is the value of $\frac{7}{11}$ of a pound sterling?

$$\begin{array}{r} \text{multiply by } 20 \text{ shillings} \\ 11 \overline{) 140} (12 \text{ shillings} \\ \underline{11} \\ 30 \\ \underline{22} \\ 8 \text{ remains} \\ 12 \text{ pence} \\ 11 \overline{) 96} (8 \\ \underline{88} \\ 8 \text{ remains} \\ \text{multiplied by } 4 \text{ farthings} \\ 11 \overline{) 32} (2 \text{ farthings} \\ \underline{22} \\ 10 \text{ of a farthing} \\ \underline{11} \\ 1 \text{ remains.} \\ \hline \end{array}$$

2. What is the value of $\frac{4}{5}$ of a guinea?—*Ans.* 9s. 4d.
 3. What is the value of $\frac{3}{10}$ of half a crown?—*Ans.* 9d.
 4. What is the value of $\frac{16}{131}$ of a lb. troy weight?—*Ans.* 3oz. 5pwt. $22\frac{1}{3}\frac{8}{11}$ gr.
 5. What is the value of $\frac{2}{3}$ of an acre?—*Ans.* 2 roods 20 poles.

Case 6. To reduce a compound fraction to a simple one of the same value.

Rule. Multiply all the numerators together for a new numerator, and all the denominators together for a new denominator; and that will be the fraction required.

Example 1. Reduce $\frac{2}{7}$ of $\frac{3}{4}$ of $\frac{1}{2}$ of a pound sterling to a simple fraction.

Thus $\frac{10}{224}$ is the simple fraction, equal in value to the compound one in the example; as the learner may prove by finding the value of each part of the compound fraction, by the foregoing case, and the value of this simple fraction; and the two values will be found to be equal.

$\frac{7}{8}$	$\frac{2}{5}$
$\frac{56}{10}$	
$\frac{4}{3}$	
<u>denom. 224</u>	<u>30 numerator</u>

Answer $\frac{10}{224}$ or $\frac{5}{112}$.

2. Reduce $\frac{2}{3}$ of $\frac{3}{4}$ of $\frac{1}{2}$ to a simple fraction.—*Answer* $\frac{10}{120}$, or $\frac{1}{12}$.

3. Reduce $\frac{2}{3}$ of $\frac{2}{3}$ of $\frac{7}{10}$ of $\frac{6}{4}$ to a simple fraction.—*Ans.* $\frac{168}{2100}$ or $\frac{2}{25}$.

If any part of the compound fraction be a whole or mixed number, it must be reduced to a fraction by one of the foregoing cases, before the compound fraction can be reduced to a simple one.

Case 7. To reduce fractions having different denominators to fractions of the same value having a common denominator.

Rule. Multiply each numerator into all the denominators, except its own; and the product is the numerator of that fraction;

fraction; and multiply all the denominators continually together, and the product is the common denominator.

Example 1. Reduce $\frac{1}{2}$, $\frac{2}{3}$, and $\frac{4}{7}$ to fractions of the same value, and having the same denominators.

			4	
		2	3	
		<u>2</u>	<u>12</u>	
2	1	2	4	
<u>3</u>	<u>3</u>	<u>7</u>	<u>2</u>	
6	7	28	24	the numerator for $\frac{4}{7}$.
				the numerator for $\frac{2}{3}$.
				the numerator for $\frac{1}{2}$.
				42 common denominator.

Therefore the new fractions equal to the former ones are, $\frac{21}{42}$ equal to $\frac{1}{2}$, $\frac{28}{42}$ equal to $\frac{2}{3}$, $\frac{24}{42}$ equal to $\frac{4}{7}$.

2. Reduce $\frac{1}{3}$, $\frac{1}{4}$ of $\frac{1}{5}$, $5\frac{1}{2}$, and $\frac{2}{7}$ to a common denominator.—*Ans.* $\frac{160}{1120}$, $\frac{142}{1120}$, $\frac{12540}{1120}$, $\frac{240}{1120}$.

In this case, as well as the former, if any of the parts be a whole or mixed number, or a compound fraction, they must be reduced to simple fractions, as in the last example.

Case 8. To reduce a fraction of one denomination to that of another, without changing its value.

This is either ascending or descending; ascending is when a fraction of a less denomination is brought into a fraction of a greater; and descending, when a fraction is brought from a greater to a less denomination.

Rule. When a fraction is to be brought from a less to a greater denomination, bring the fraction from a simple to a compound one, by comparing it with the intermediate denominations, which are between the given fraction and the denomination required; and then, by the rule in Case 6, bring the compound fraction to a simple one.

Example 1. Bring $\frac{2}{3}$ of a penny to the fraction of a pound.

A penny is $\frac{1}{12}$ of a shilling, and a shilling is $\frac{1}{20}$ of a pound, therefore $\frac{2}{3}$ of a penny is $\frac{2}{3}$ of $\frac{1}{12}$ of $\frac{1}{20}$ of a pound, which, reduced to a simple fraction, is $\frac{1}{600}$ of a pound, as follows:

D d 2

Numerators.

Numerators.

Denominators.

2

20

11222401521200

1200 $\frac{2}{1200}$ or $\frac{1}{600}$ of a pound.

Rule. When a fraction is to be brought from a greater to a less denomination, multiply the numerator of the fraction by the number of parts of the next inferior denomination, and that product by the number of parts of the next denomination; and so on till it be brought to the denomination required; and the product is the numerator of the fraction required.

Example 2. Reduce $\frac{2}{17}$ of a pound sterling to the fraction of a penny.

Here I multiply the numerator by 20, as that is the number of shillings contained in a pound, and that product by 12, as there are so many pence in a shilling; and the product gives a numerator to the fraction required.

$\frac{2}{17}$
 $\frac{20}{17}$
 $\frac{40}{17}$
 $\frac{12}{17}$
 $\frac{480}{17}$

Thus, if in this case the reduction be ascending, the denominator only of the fraction need be multiplied; and, if descending, the numerator only.

3. Reduce $\frac{1}{17}$ of a pound troy to the fraction of a pwt.
 —*Ans.* $\frac{1200}{17}$.

4. Reduce $\frac{1}{3}$ of a day to the fraction of a minute.—
Ans. $\frac{160}{3}$ or $\frac{53}{1}$.

5. Reduce $\frac{1}{4}$ of a guinea to the fraction of a penny.—
Ans. $\frac{140}{1}$.

Addition of Vulgar Fractions.

Before vulgar fractions can be added together, it is necessary that they be brought into a proper form, for which,

Rule. Reduce the compound fractions to simple ones, and whole and mixed numbers to improper fractions, and
 fractions

fractions of different denominations to fractions of the same denomination. Then reduce all the fractions to fractions of the same common denominator. Add all the numerators together, and place the sum over the common denominator; and this fraction will be equal to the sum of all the given fractions.

The total thus found, if it be an improper fraction, may be reduced to a whole or mixed number.

Example 1. Add $\frac{1}{3}$, $\frac{2}{7}$, and $\frac{3}{13}$ together.

$\frac{13}{7}$	$\frac{9}{2}$	$\frac{7}{2}$	$\frac{9}{7}$	The given fractions reduced to a common denominator are, $\frac{9}{13}$ equal to $\frac{1}{3}$, $\frac{2}{7}$ equal to $\frac{2}{7}$, $\frac{3}{13}$ equal to $\frac{3}{13}$. $\frac{91}{234}$ $\frac{234}{126}$ $\frac{451}{451}$
$\frac{13}{91}$	$\frac{18}{18}$	$\frac{14}{14}$	$\frac{63}{63}$	
	$\frac{13}{54}$	$\frac{9}{120}$	$\frac{13}{819}$	
	$\frac{18}{18}$			
	$\frac{234}{234}$			

$\frac{451}{451}$ the answer, or sum of the given fractions.

2. Add $3\frac{2}{3}$, $\frac{1}{7}$, and $\frac{2}{5}$ of $\frac{1}{10}$ together.

Then $3\frac{2}{3}$ is equal to $\frac{17}{3}$, and $\frac{2}{5}$ of $\frac{1}{10}$ is equal to $\frac{2}{50}$ or $\frac{1}{25}$. Then the given fractions will be $\frac{17}{3}$, $\frac{1}{7}$, $\frac{1}{25}$.

$\frac{17}{7}$	$\frac{3}{5}$	$\frac{3}{7}$	$\frac{5}{7}$	The fractions reduced to a common denominator are, $\frac{292}{105}$ equal to $\frac{17}{3}$, $\frac{1}{7}$ equal to $\frac{1}{7}$, $\frac{1}{25}$ equal to $\frac{1}{25}$. $\frac{105}{375}$ $\frac{2975}{3455}$
$\frac{119}{119}$	$\frac{15}{15}$	$\frac{21}{21}$	$\frac{35}{35}$	
$\frac{25}{25}$	$\frac{25}{75}$	$\frac{5}{105}$	$\frac{25}{175}$	
$\frac{595}{595}$	$\frac{30}{30}$		$\frac{70}{70}$	
$\frac{238}{238}$	$\frac{375}{375}$		$\frac{875}{875}$	

$\frac{3455}{3455}$ or $\frac{691}{691}$ or $\frac{1382}{1382}$ for the answer.

In this example, as in the former, I first (after having reduced the mixed number $3\frac{2}{3}$ and the compound fraction $\frac{2}{5}$ of $\frac{1}{10}$ to simple fractions) reduce all the fractions to a common denominator, by multiplying the numerator of each into all the

the denominators except its own, for a numerator to each fraction, and all the denominators into each other for a common denominator; as taught in reduction of fractions; the numerators of these newly-discovered fractions are then added together, and their sum 3455 placed over the common denominator 875 is the answer or fraction equal to the sum of all the given fractions, which, divided by 5, gives $\frac{691}{175}$, and reduced to a mixed number is $3\frac{66}{175}$.

The proof of addition of fractions is wrought by finding the value of each of the given fractions, and adding them together, and then finding the value of the whole fraction or sum of the fractions, and if these two values be equal the work is right.

Qu. 3. What is the sum of $\frac{1}{2}$ of a pound and $\frac{1}{4}$ of a shilling?—*Ans.* $\frac{716}{1600}$ of a pound.

In this example the fraction $\frac{1}{4}$ of a shilling must be reduced to the fraction of a pound before the work can be wrought, as before directed.

Qu. 4. What is the sum of $\frac{1}{2}$, $\frac{1}{3}$, and $\frac{1}{4}$ of $\frac{1}{2}$?—*Answer* $2\frac{1}{8}$.

Qu. 5. What is the sum of $\frac{1}{2}$ of a week, $\frac{1}{4}$ of a day, and $\frac{1}{2}$ an hour?—*Ans.* 2 days $14\frac{1}{2}$ hours.

Subtraction of Vulgar Fractions.

Rule. Reduce the fractions in the same manner as for addition; then the less numerator is to be subtracted from the greater, and the difference placed over the common denominator will form the fraction required.

Example 1. Subtract $\frac{3}{5}$ from $\frac{7}{3}$.

$$\frac{7}{3}$$

$$\frac{9}{5}$$

$$\frac{9}{7}$$

$$\frac{21}{83} \text{ equal to } \frac{3}{11}$$

$$\frac{43}{83} \text{ equal to } \frac{5}{11}$$

From

$$\begin{array}{r} \text{From } 45 \\ \text{Take } 21 \\ \hline 24 \\ = \end{array}$$

$\frac{24}{63}$ or $\frac{8}{21}$ the Answer.

Proof. Add the less number and the remainder together, as in whole numbers.

Example 2. Subtract $\frac{2}{3}$ from $\frac{7}{8}$ of $\frac{7}{9}$.

$\frac{7}{8}$ of $\frac{7}{9}$ is $\frac{49}{72}$ or $2\frac{5}{8}$, therefore I can take $\frac{2}{3}$ from $2\frac{5}{8}$.

$$\begin{array}{r} 24 \\ 2 \\ \hline 48 \end{array}$$

$$\begin{array}{r} 9 \\ 7 \\ \hline 63 \end{array}$$

$$\begin{array}{r} 24 \\ 9 \\ \hline 216 \end{array}$$

$2\frac{48}{16}$ equal to $2\frac{3}{2}$.

$2\frac{63}{16}$ equal to $2\frac{7}{2}$.

$$\begin{array}{r} \text{From } 63 \\ \text{Take } 48 \\ \hline \text{Rests } 15 \\ = \end{array}$$

$1\frac{15}{16}$ or $\frac{5}{8}$ the Answer.

Qu. 3. What is the difference between $28\frac{3}{4}$ and $27\frac{7}{10}$?—Ans.

Qu. 4. What is the difference between $26\frac{1}{2}$ and $54\frac{1}{8}$?—Ans. $28\frac{1}{4}$.

Qu. 5. What is the difference between $14\frac{1}{2}$ and $\frac{2}{3}$ of 19?—Ans. $17\frac{1}{6}$.

Multiplication of Vulgar Fractions.

Rule. Reduce whole and mixed numbers, compound fractions, and fractions of different denominations, to simple fractions of the same denomination; then multiply all the numerators together for a new numerator, and all the denominators together for a new denominator, and such fraction will be the true product required.

Example 1. Multiply $\frac{3}{5}$ by $\frac{4}{7}$.

$$\begin{array}{r} 3 \\ 5 \\ \hline 15 \end{array}$$

numerator.

$$\begin{array}{r} 4 \\ 7 \\ \hline 28 \end{array}$$

denominator.

$\frac{12}{140}$ Answer.

Example

Example 2. What is the product of $2\frac{1}{2}$ multiplied by $\frac{5}{8}$, by $\frac{3}{4}$, and $\frac{4}{7}$ of $\frac{2}{3}$?

$$\begin{array}{r} 19 \\ \underline{5} \\ 95 \\ \underline{3} \\ 285 \\ \underline{4} \\ 1140 \end{array}$$

$$\begin{array}{r} 7 \\ \underline{6} \\ 42 \\ \underline{9} \\ 378 \\ \underline{21} \\ 378 \\ \underline{756} \\ 7938 \end{array}$$

Ans. ~~$\frac{1140}{7938}$~~ or $\frac{100}{1378}$

In this example $2\frac{1}{2}$ is to be reduced to its equivalent fraction $\frac{5}{2}$, and $\frac{4}{7}$ of $\frac{2}{3}$ must be reduced to its simple fraction, $\frac{8}{21}$ or $\frac{4}{21}$, then the question will be, what is the product of $\frac{5}{2}$, $\frac{3}{4}$, $\frac{5}{8}$ and $\frac{4}{21}$ multiplied together?

Qu. 3. What is the product of $\frac{1}{2}$ of $\frac{2}{3}$, by $\frac{3}{4}$ of $\frac{1}{2}$?—

Ans. $\frac{1}{12}$ or $\frac{1}{12}$.

Qu. 4. What is the product of 24 by $\frac{2}{3}$?—*Ans.* $\frac{16}{3}$ or 16.

Qu. 5. What is the product of $\frac{1}{2}$ of 7 by $\frac{1}{2}$?—*Ans.* $1\frac{1}{2}$.

Qu. 6. What is the product of 36 by $\frac{9}{11}$?—*Ans.* $\frac{324}{11}$ or $29\frac{4}{11}$.

Thus it may be seen, that multiplication of vulgar fractions is performed in the same manner as reducing a compound fraction to a simple one*.

Division of Vulgar Fractions.

Rule. Prepare all the fractions for division in the same manner as for addition, subtraction, and multiplication; and invert the two terms of the divisor, placing the denominator at the top and the numerator at the bottom; then proceed exactly the same as in multiplication, viz. multiply the

* If any number be multiplied by a fraction, the product will be less than the multiplicand, and in proportion as the multiplier is less than an unit.

numerator of the dividend and that figure that is uppermost in the divisor together for a numerator to the quotient: and multiply the denominator of the dividend and the lower figure of the divisor for the denominator of the quotient.

Proof. Multiply the quotient by the divisor, as in whole numbers.

Example 1. Divide $\frac{1}{2}$ by $\frac{3}{7}$.— $\frac{1}{2})\frac{3}{7}(\frac{3}{8}$ or $2\frac{1}{2}$ *Answer.*

2. Divide $\frac{2}{3}$ of 7 by $\frac{3}{4}$ of $\frac{4}{5}$.

In this example, $\frac{2}{3}$ of 7 is equal to $\frac{14}{3}$, and $\frac{3}{4}$ of $\frac{4}{5}$ is equal to $\frac{3}{5}$ or $\frac{6}{10}$; therefore I say, divide $\frac{14}{3}$ by $\frac{6}{10}$.—*Ans.* $\frac{70}{9}$ or $28\frac{2}{9}$.

3. Divide $\frac{7}{8}$ by 3.—*Ans.* $\frac{7}{24}$.

4. Divide 10 by $\frac{1}{4}$.—*Ans.* 40 or $5\frac{1}{4}$.

5. Divide $7\frac{1}{2}$ by $\frac{3}{4}$ of $\frac{1}{2}$.—*Ans.* $40\frac{2}{3}$ or $61\frac{1}{3}$.

The Rule of Three direct in Vulgar Fractions.

Rule. Prepare the fractions as for the four foregoing rules; and then having placed the three numbers in their proper order according to their proportion, invert the first term or fraction, transposing the numerator and denominator, as in the foregoing rule; then multiply all the three numerators together for a numerator, and all the denominators together for a denominator, and this fraction will be the answer required.

Proof. The same as in whole numbers.

Example 1. If $\frac{1}{2}$ of an ell English cost $\frac{1}{4}$ of a pound, what will $\frac{1}{2}$ of an ell Flemish cost?

In this example I say, if $\frac{1}{2}$ of $\frac{1}{4}$ of a yard (which is an ell English) cost $\frac{1}{4}$ of a pound, what will $\frac{1}{2}$ of $\frac{1}{4}$ of a yard (which is an ell Flemish) cost?

I then reduce the compound fraction $\frac{1}{2}$ of $\frac{1}{4}$ to a simple one, and it is $\frac{1}{8}$ or $\frac{2}{16}$, and also the fraction $\frac{1}{2}$ of $\frac{1}{4}$ is $\frac{1}{8}$ of a yard, which is of the same denomination with the first

* If a whole number be divided by a proper fraction, the quotient will be greater than the dividend; but if any fraction be divided by a whole number, the quotient will be less than the dividend.

number, viz. a fraction of a yard; and inverting the first term the question will stand thus,

$\frac{4}{7}$ cost $\frac{3}{8}$, $\frac{2}{5}$.—The answer $\frac{108}{1120}$ or 1s. 11 $\frac{1}{2}$ d.

Then multiplying the three uppermost figures of the fractions together for a numerator, and the three undermost figures for a denominator, the answer is $\frac{108}{1120}$ of a pound, which, reduced to its real value, is 1s. 11d. $\frac{1}{2}$ of a penny.

Here it must be observed, that the first and third fractions must be reduced to the same denomination as in whole numbers, as seen in the foregoing example, where they are both fractions of a yard; and the fourth fraction is of the same denomination with the second.

Qu. 2. If $\frac{3}{4}$ of a gallon of brandy cost $\frac{1}{2}$ of a pound, what will 12 $\frac{3}{4}$ cost at that rate?—Ans. $\frac{190}{27}$, or 7l. 0s. 8 $\frac{1}{2}$ d. $\frac{1}{2}$.

Qu. 3. If $\frac{2}{5}$ of a bale of linen cost 14l. 14s. what will 7 $\frac{1}{2}$ bales cost at that rate?—Answer 122l. 10s.

Qu. 4. If 12 $\frac{1}{2}$ lb. of sugar cost 15s. 9d. what is the price of 48 $\frac{1}{2}$ lb.?—Answer 3l. 0s. 9 $\frac{1}{2}$ d. $\frac{1}{10}$.

Rule of Three inverse in Vulgar Fractions.

Rule. Prepare all the fractions as for the foregoing rule, then consider (as taught in the rule of three in whole numbers) whether the question belongs to the inverse or direct rule, and if it belong to the rule of three inverse, the third fraction is to be inverted, by transposing the numerator and denominator; and the work is wrought exactly in the same manner as in the direct rule, by multiplying the three uppermost terms of the fractions together for a numerator, and the undermost terms of the three fractions for a denominator; and the fraction thus formed will be the answer.

Proof. As before in whole numbers.

Example 1. If A lent B $\frac{1}{2}$ of 1000l. for $\frac{3}{4}$ of a year, how much must B lend A for $\frac{2}{5}$ of a year in return?

After

After disposing of the fractions as before directed, I consider that $\frac{9}{10}$ of a year being a longer time than $\frac{3}{4}$, it will not require so much principal lent, therefore the greater of the first and third numbers must be the divisor (as in whole numbers); the third fraction therefore must be inverted, and the question will stand thus :

If $\frac{3}{4}$ of a year. $\frac{1}{3}$ of 1000*l*. $\frac{9}{10}$ of a year?

Answer $3\frac{0000}{1199}$, or $158\frac{138}{1199}$, equal to 158*l*. 14*s*. 7 $\frac{1}{2}$ *d*.

Questions both in this rule and the former are proved by back-stating the question, as in whole numbers; thus the foregoing example may be proved by saying, if $\frac{9}{10}$ of 1000*l*. principal require $\frac{3}{4}$ of a year, what will $\frac{1}{3}$ of 1000*l*. require? and the *answer* is $3\frac{138}{1199}$, or $\frac{3}{4}$ of a year.

Qn. 2. How much shalloon will it require at $\frac{3}{4}$ of a yard wide to line the garments made with $10\frac{1}{2}$ yards of cloth at $1\frac{1}{2}$ yard wide?—*Ans.* $1\frac{38}{24}$, or $1\frac{9}{2}$, equal to $24\frac{1}{2}$ yards.

Qn. 3. If 12 men can mow $24\frac{1}{2}$ acres in $10\frac{2}{3}$ days, how many days will 6 men require to do the same?—*Answer* $21\frac{1}{3}$ days.

Qn. 4. If a board be $\frac{1}{4}$ of a foot in breadth, how many inches in length will make a square foot?—*Ans.* 16 inches.

From what has been delivered in this section concerning vulgar fractions, it is plain that every other rule in arithmetic may be wrought by vulgar fractions as well as by whole numbers, as the operation in both cases depends upon the same principle; thus, in the rule of three direct in vulgar fractions, inverting the first fraction, and multiplying it by the second and third, is the same as multiplying the second and third fractions together, and dividing by the first: and in the inverse rule, inverting the third fraction and multiplying it by the first and second, is equal to dividing the product of the first and second fractions by the third, as the learner may prove at his leisure.

SECT. XIV.

OF PRACTICE.

PRACTICE is the most expeditious rule in arithmetic, and is of general use among men of business, as it readily discovers the value of any number of integers from having the value of one.

By this rule are answered all questions in the rule of three direct that have an unit for their first number.

Rule. Divide the given number of integers by one or more aliquot parts of a penny, shilling, or pound, or any two or three of them; and the quotient will be the answer, and of the same denomination of which the divisor is a part.

An aliquot part of a number is such a part, that being taken any number of times, will exactly measure that number without a remainder: thus 2 is an aliquot part of 6, for it is contained exactly 3 times in 6; and 5s. is an aliquot part of a pound, for it is contained exactly four times in a pound; but 5s. 2d. is not an aliquot part, for it is not exactly contained any number of times in a pound without leaving a remainder.

Before the learner can perform this rule, he must perfectly understand the following tables of aliquot parts, and retain them in his memory.

The aliquot Parts.

d.			s.	d.	
6	}	$\frac{1}{2}$ half	10	0	}
4		$\frac{1}{3}$ third	6	8	
3		$\frac{1}{4}$ fourth	5	0	
2		$\frac{1}{6}$ sixth	4	0	
$1\frac{1}{2}$		$\frac{1}{8}$ eighth	3	4	
1		$\frac{1}{12}$ twelfth	2	6	
$\frac{1}{2}$	}	$\frac{1}{16}$ sixteenth	2	0	}
$\frac{1}{4}$			1	8	
$\frac{1}{8}$ half	}		1	0	
$\frac{1}{16}$ fourth					
		of a penny.			
		of a shilling.			
					of a pound.

or

Cafe

Case 2. When the price is an aliquot part of a shilling, divide the given number by such aliquot part, and the quotient is the answer in shillings, which must be reduced into pounds.

Example 5. What is the value of 879lb. of cheese, at 4d. per lb.?

Here the given number of pounds is 879 divided by 3, as 4d. is $\frac{1}{3}$ of a shilling, and it quotes 293 shillings, which are brought into pounds; and the Answer is 14l. 13s.

$$\begin{array}{r} 3 \overline{)879} \\ 2,0 \overline{)29,3} \text{ shillings} \\ \underline{14} \\ 13 \end{array}$$

Q. 6. What is the value of 297lb. of tallow at 3d. per lb.?—Here the given number must be divided by 4, as 3d. is $\frac{3}{4}$ of a shilling, and the Answer is 3l. 14s. 3d.

Example 7. What is the value of 3cwt. of sugar, at 6d. per lb.?—Ans. 8l. 8s.

$$\begin{array}{r} 3 \overline{)336} \\ 2,0 \overline{)16,8} \\ \underline{8} \\ = \end{array}$$

Example 8. What is the value of 2178lb. of alum, at $1\frac{1}{2}$ d. per lb.?—Ans. 13l. 12s. 3d.

$$\begin{array}{r} 8 \overline{)2178} \\ 2,0 \overline{)27,24} \\ \underline{13} \\ 12 \\ = \end{array}$$

Example 9. What is the value of 486lb. at 2d. per lb.?—Ans. 4l. 1s.

$$\begin{array}{r} 6 \overline{)486} \\ 2,0 \overline{)8,1} \\ \underline{4} \\ 1 \end{array}$$

Case 3. When the price of the integer is pence and farthings, and not an aliquot part of a shilling, find what aliquot part of a shilling is the nearest to the given price, and less than it, and divide the given number by that aliquot part; and for the remainder of the price consider what part it is of the

the given price, and divide the quotient by it; and if there be still a remainder of the given price, consider what aliquot part this is of the last, and divide the last quotient thereby; then add all the quotients together for the answer.

Example 10. What is the value of $5\frac{1}{2}$ cwt. of butter, at $8\frac{1}{2}d.$ per lb.?

$$\begin{array}{rcl}
 6d. \text{ is } \frac{1}{4} \text{ of a shilling} & 2)616 & \text{equal to } 5\frac{1}{2} \text{ cwt.} \\
 2d. \text{ is } \frac{1}{3} \text{ of } 6d. \text{ therefore divide by } 3 & 3)308 & \text{equal to } 4d. \\
 \frac{1}{2}d. \text{ is } \frac{1}{4} \text{ of } 2d. \text{ therefore divide by } 4 & 4)102 & \text{remains } 2, \text{ or } \frac{2}{3} \text{ of } 6d. \\
 & 25 & \text{remains } 2, \text{ or } \frac{2}{3} \text{ of } 2d. \\
 & 2,0)43,5 & \text{equal to } 1d. \\
 & 21 & \\
 & \underline{15} & \\
 \text{Answer } 21l. \ 15s. \ 6d. & &
 \end{array}$$

In this example, I divide the given number first by 2, as 6d. is the nearest aliquot part to the price, and the quotient is 308s. which is the price at 6d. per lb.: I then divide that quotient by 3, for the other 2d. in the price, and it quotes 102s. which is the price of the article at 2d. per lb. and 2 remains; and for the halfpenny I divide the last quotient by 4, as one halfpenny is the fourth part of 2d. and the quotient is 25s. which is the price of the butter at a halfpenny per lb.: the three quotients added together give the answer.

Example 11. What is the value of 137 yards of cloth, at $10\frac{1}{2}d.$ per yard?

$$\begin{array}{rcl}
 \text{For the } 6d. \text{ I divide by } 2 & 2)137 & \\
 \text{For the } 3d. \text{ I divide by } 2 & 2)68 & \text{remains } 1, \text{ or } 6d. \\
 \text{For the } 1\frac{1}{2}d. \text{ I divide by } 2 & 2)34 & \\
 & 17 & \\
 & 2,0)11,0 & \\
 \text{Answer } & \underline{\underline{5}} & 19 \quad 10\frac{1}{2}
 \end{array}$$

Qu. 12. What is the value of 520lb. of soap, at $7\frac{1}{2}d.$ per lb.?—*Ans.* 16l. 5s.

Qu. 13. What is the value of 860 yards of linen, at $11\frac{1}{2}d.$ per yard?—*Ans.* 41l. 4s. 2d.

Cuse

Case 4. When the price is any number of shillings under 20.

First. If it be an even number of shillings, multiply the given number by half the price, and double the first figure on the right hand, which will be shillings, and all the other figures are pounds.

Secondly. If the price be an odd number of shillings, multiply the quantity by half the next less number, which will be an even number, then for the odd shilling add $\frac{1}{20}$ of the given number to the last product, and the sum of these two quantities will be the answer.

Q^a. 14. What is the value of 476 yards of cloth, at 4s. per yard?

Here I multiply the given number 476 by 2, half the price, saying, 2 times 6 is 12, which doubled is 24; I set down the 4

$$\begin{array}{r} 476 \\ 2 \\ \hline 954 \end{array}$$

Answer 95l. 4s.

shillings, and carry the 1 pound to be added to the next product, saying, 2 times 7 is 14, and 1 I carried is 15, 5 and carry 1; then 2 times 4 is 8, and 1 is 9; then I cut off the first figure 4 for shillings, and the rest are pounds.

Example 15. What is the value of 462 yards of cloth, at 7s. per yard?

In this example, 7 being an odd number, I take 3 for the multiplier (being the half of 6 the next even number), and multiply as in the former ex-

$$\begin{array}{r} 2,0)46,2 \\ 3 \\ \hline 138,12 \\ 23 \ 2 \\ \hline \end{array}$$

Answer £161 14

ample, doubling the first number for shillings, and the product is 138l. 12s. which is the price of the cloth at 6s. per yard; then for the other-shilling I take a twentieth part of the given number, and the quotient is 23l. 2s. which is the price of the cloth at 1s. per yard, and which, added to the former price at 6s. gives 161l. 14s. the answer.

Examples for Practice.

2,0)151 yards at 13s. per yd.

258	12
31	11

£.280 3 *Answer.*

2,0)324 ells at 17s. per ell.

259	4
16	4

£.275 8 *Answer.*

2,0)279 yards at 3s. per yd.

27	18
13	19

£.41 17 *Answer.*

2,0)2880 yards at 19s. per yd.

2592	0
144	0

£.2736 0 *Answer.*

Qu. 16. What is the value of 5514 yards, at 1s. per yard?

Ans. 275l. 14s.

Qu. 17. What is the value of 2468 yards, at 5s. per yard?

Ans. 617l.

Note. When the price is 5s. as in the last question, the given number may be divided by 4, as 5 is $\frac{1}{4}$ of a pound, and the quotient is the answer in pounds.

And when the price is 2s. it is done at sight, by doubling the first figure on the right hand for shillings, and the other figures are pounds.

Example 18. What is the value of 896 gallons of cyder at 2s. per gallon?

89l. 12s. *Answer.*

By this method of working by 2s. a variety of examples may be wrought very expeditiously, by dividing the given price into parts of 2s. each, and finding the value of the overplus, if any.

Example 19. What is the value of 444 gallons of Hollands, at 5s. 9d. per gallon?

For 2s. per gallon I take

For the other 2s. per gallon

For the 1s. half the 2s. line

For the 6d. half the 1s. line

For the 3d. half the 6d. line

444 gallons

44l. 8s.

44 8

22 4

11 2

5 11

And then add them together for the *Ans.*

127 13

In this manner examples in this rule may be wrought, though the price consist of any number of shillings and pence, and which would supersede the necessity of some of the following rules. I shall nevertheless insert them, that the work appear not deficient.

Case 5. When the price is shillings and pence, which make an aliquot part of a pound, divide the given quantity by such aliquot part.

Example 20. What is the value of 2796 dollars, at 3s. 4d. per dollar?

$$\begin{array}{r} 6 \overline{) 2796} \\ \text{Answer } \underline{\underline{466\text{ l.}}} \end{array}$$

Here I divide the given number by 6, as 3s. 4d. is the sixth part of a pound.

Qu. 21. What is the value of 3575 yards, at 1s. 8d. per yard?—*Ans.* 297l. 18s. 4d.

Qu. 22. What is the value of 2478 gallons of spirits, at 6s. 8d. per gallon?—*Ans.* 826l.

Qu. 23. What is the value of 2793 yards of linen, at 2s. 6d. per yard?—*Ans.* 349l. 2s. 6d.

Case 6. When the price consists of shillings and pence, or shillings, pence, and farthings, which do not make an aliquot part of a pound, divide the given number by the greatest aliquot part of a pound that the price contains; and for the remainder of the price, if any, consider what aliquot part it is of the former aliquot part, and divide the quotient thereby; and if there still be a remainder of the price, consider what aliquot part it is of the last aliquot part, and divide the last quotient thereby: and proceed in this manner as long as there is any remainder of the price: then the sum of all the quotients will be the answer.

Example 24. What is the value of 3726 yards of cloth, at 7s. 4½d. per yard?

$$\begin{array}{r} 3 \overline{) 3726} \\ 10 \overline{) 1242} \\ 16 \overline{) 124 \quad 4 \quad 0} \\ \quad \quad \quad 7 \quad 15 \quad 3 \\ \text{Answer } \underline{\underline{\text{£} 1373 \quad 19 \quad 3}} \end{array}$$

In this example, I divide the given number by 3, for 6s. 8d. as that is the third part of a pound; the quotient I again divide by 10, for the 8d. remaining, as 8d. is the tenth part of 6s. 8d.; and this quotient I again divide by 16, for the halfpenny, as that is the sixteenth part of 8d.: then the three quotients added together give the answer.

Or the price may be divided into the aliquot parts of a pound, and then the given number must be divided by each of them; thus, in the foregoing example, the given number of yards may be divided by 5 for 4s. the fifth part of a pound, and 6 for the other 3s. 4d. the sixth part of a pound, and for the halfpenny the given number may be divided by 24, the aliquot part of a shilling, and this last quotient is shillings; and the three quotients added together is the answer.

Qu. 25. What is the value of 784 gallons, at 6s. 9d. per gallon?—*Ans.* 264*l.* 12*s.*

Qu. 26. What is the value of 1464 gallons, at 12s. 7d. per gallon?—*Ans.* 981*l.* 2*s.*

Case 7. When the price is pounds, shillings, pence, and farthings, multiply the given number by the number of pounds, and the product is pounds; then for the remainder of the price, work according to some of the former rules, as the case may require; and these sums added together will give the answer.

Example 27. What is the value of 416 cwt. of sugar, at 2*l.* 9*s.* 3½*d.* per cwt.?

	416cwt.
	2
	<hr/>
For the 2 <i>l.</i> take	832 0 0
For 8 <i>s.</i> —	166 8 0
For 1 <i>s.</i> —	20 16 0
For 3 <i>d.</i> —	5 4 0
For ½ <i>d.</i> —	1 6 0
	<hr/>
<i>Answer</i>	<i>£.</i> 1025 14 0

F f 2

For

For the 2*l*. I multiply the given number by 2, and the product 832 is pounds; then for the 8*s*. I multiply the given number by 4, setting the double of the unit figure apart for shillings, as directed in the fourth case; for the 1*s*. I take the eighth part of the 8*s*. line; for the 3*d*. I take the fourth part of the 1*s*. line; and for the $\frac{1}{4}$ I take the fourth part of the 3*d*. line: and the sum of these is the answer.

Qⁿ 28. What is the value of 1894 pieces of linen, at 4*l*. 15*s*. 10 $\frac{1}{2}$ *d*. per piece?—*Ans.* 9077*l*. 7*s*. 9 $\frac{1}{2}$ *d*.

Case 8. When the number of integers whose value is required is a whole number with fractions annexed, find the value of the whole number by some of the foregoing rules, to which add the value of the parts represented by the fractions.

Example 29. What is the value of 278*cwt*. 84*lb*. of soap, at 1*l*. 19*s*. 4*d*. per *cwt*.?

	<u>278$\frac{1}{2}$ <i>cwt</i>.</u>		
For 1 <i>l</i> . take	278	0	0
For 16 <i>s</i> .	222	8	0
For 3 <i>s</i> . 4 <i>d</i> .	46	6	8
For $\frac{1}{2}$ <i>cwt</i> .	0	19	8
For $\frac{1}{4}$ <i>cwt</i> .	0	9	10
<i>Answer</i>	<u>548</u>	<u>4</u>	<u>2</u>

In this example, after finding the value of the 278 *cwt*. at the given price, I find that 84*lb*. is $\frac{1}{2}$ of an *cwt*. I therefore take $\frac{1}{2}$ of the given price, viz. 19*s*. 8*d*. for $\frac{1}{2}$ the *cwt*. and for the other $\frac{1}{4}$ *cwt*. I take $\frac{1}{4}$ of the price, 9*s*. 10*d*.

Qⁿ 30. What is the value of 568 $\frac{1}{2}$ pieces of cloth, at 2*l*. 1*cs*. 6*d*. per piece?—*Ans.* 1435*l*. 9*s*. 3*d*.

Case 9. When the quantity whose value is sought is of several denominations, first, find the value of the greatest denomination; and for the other denominations, take an equivalent part or parts of the given price; and add the several sums together for the answer.

Example 31. What is the value of 10*cwt*. 2*qrs*. 21*lb*. of tobacco, at 4*l*. 5*s*. 6*d*. per *cwt*.?

	4	5	6 per cwt.
			10 cwt.
For 10cwt. take	42	15	0
For 2qrs.	2	2	9
For 14lb.	0	10	8½
For 7lb.	0	5	4½
<i>Answer</i>	<u>£.45</u>	<u>13</u>	<u>9½</u>

The cwt. being the greatest denomination, is found first, by multiplying the price of an cwt. by 10, and for the value of the 2qrs. I take half the price of an cwt. then for the 14lb. I first find the value of 14lb. by taking half the price of a qr. or a fourth part of 2qrs. and half this price for the other 7lb.; these sums added together give the answer.

Qu. 32. What is the value of 28yds. 3¼qrs. of superfine cloth, at 30s. per yard? *Ans.* 43l. 6s. 3d.

SECT. XV.

OF BARTER.

BARTER is that rule which instructs traders to exchange one commodity for another, so that neither party may sustain any loss.

Rule. Find the value of that commodity whose quantity is given, by the rule of three or practice; then by the rule of three, practice, or division, find what quantity of the other commodity should be given in exchange.

Example 1. How many gallons of brandy, at 6s. per gallon, must be given in barter for 7cwt. 2qrs. 14lb. of sugar, at 2l. 10s. per cwt.?

cwt.

$$\begin{array}{r}
 \text{cwt. grs. lb.} \\
 7 \quad 2 \quad 14 \\
 2 \\
 \hline
 £.14 \quad 0 \quad 0 \\
 3 \quad 10 \quad 0 \\
 1 \quad 5 \quad 0 \\
 0 \quad 6 \quad 3 \\
 \hline
 19 \quad 1 \quad 3 \\
 20 \\
 \hline
 381 \\
 12 \\
 \hline
 72)4575(63 \text{ gallons.} \\
 432 \\
 \hline
 255 \\
 216 \\
 \hline
 39 \\
 8 \\
 \hline
 72)312(4\frac{2}{3} \text{ pints.} \\
 288 \\
 \hline
 24 \\
 \hline
 \hline
 \end{array}$$

Answer 63 gallons 4 $\frac{2}{3}$ pints.

In this example, I first find the value of the given quantity of sugar by the rule of practice, which I reduce into pence, and it produces 4575 pence; these pence are then divided by 72, the pence in 1 gallon of brandy, and it quotes 63 gallons and $4\frac{2}{3}$ of a pint for the answer.

Thus it appears that questions that concern only one price of each sort, of two different kinds of goods, may be wrought by practice and division only, as the foregoing; but those of a more complex nature must be resolved by the rule of three.

Qu. 2. A grocer has 120lb. of tea, which cost him 6s. per lb. but he intends to barter it at the rate of 8s. per lb. with a distiller, for Hollands that cost him 4s. per gallon. At what price must the distiller rate his Hollands, that he may have as much profit as the grocer; and how many gallons must he give for the 120lb. of tea?—*Answer*, he must rate his Hollands at 5s. 4d. per gallon, and give 180 gallons for the tea.

In

In resolving this question, first find what the *Hollandis* must be rated at, by the rule of three, saying, if 6 cwt . require 2 s . what will 4 s . require?—*Answer* 5 s . 4 d . Then by practice (as in the first example) find the value of the tea at 2 s . per lb. which, divided by the price of 1 gallon of *Hollandis*, as before, quotes the answer.

Qu. 3. A vintner barter 196 gallons of wine for 14 cwt . of sugar worth 6 d . per lb. how much was the wine worth at that rate?—*Ans.* 4 s . per gallon.

Qu. 4. A barter 320 gallons of gin, at 4 s . 6 d . per gallon, with B for 6lb. of tea at 5 s . per lb. and for sugar at 2 d . per lb.; how much sugar will A receive?—*Answer* : 10 cwt . 1 qr .

Qu. 5. A vintner barter 608 gallons of brandy at 1 s . per gallon, for sugar at 3 l . 10 s . per cwt. and 12 s . 12 d . in cash: how much sugar should the vintner receive?—*Answer* 85 cwt . 2 qrs . 24 lb .

SECT. XVI.

OF LOSS AND GAIN.

Loss and gain is that rule which discovers the loss or gain from buying and selling goods; and instructs traders how to fix their price, in order to gain or lose any certain sum.

Rule. By the rule of three direct. Though questions in this rule may often be answered by practice, or other rules.

Example 1. As how much per lb. must a grocer sell tea which cost him 4 s . 10 d . per lb. so as to gain 2 s . per cwt. profit?

As 100*l.* is to 127*l.* so is 4*s.* 10*d.* to

$$\begin{array}{r} 58 \quad 12 \\ \hline 1016 \quad 58 \\ 635 \\ \hline 1,00 \overline{) 73,66} (73 \text{ pence} \\ 73 \\ \hline 66 \end{array}$$

$$\begin{array}{r} 4 \\ \hline 1,00 \overline{) 2,64} (2 \text{ farthings} \end{array}$$

$$\begin{array}{r} 2 \\ \hline 64 \\ \hline \hline \end{array}$$

Ans. 6*s.* 1½*d.* $\frac{64}{100}$ or $\frac{16}{25}$ per lb.

When the gain or loss is required at any rate per cent. where the interest has a 5 or a cypher on the right hand, as is most commonly the case, the answer may be readily found, by adding to or subtracting from the given price such a part as the interest is of the principal; thus, if it be required to gain or lose 5 per cent. (as 5 is the twentieth part of an hundred) the answer is found by adding to or subtracting from the given price one twentieth part; and if the gain or loss be 10 per cent. then it is one tenth part; and if 15 per cent. it is $\frac{3}{20}$; and 20 per cent. is $\frac{1}{5}$; and 25 per cent. is $\frac{1}{4}$, &c.

Qu. 2. A grocer bought 8½ cwt. of sugar, which cost 3*l.* 14*s.* 8*d.*; but, it being damaged, he is willing to lose 12*l.* 10*s.* per cent. in the sale of it; at how much per lb. must he sell it?—*Answer* 7*d.* per lb.

In this example I subtract the loss per cent. from the principal, and the remainder is the second number in the rule of three, the principal the first number, the whole price of the sugar the third number; and the fourth number will be the whole price at the reduced rate, which divided by the number of pounds, gives 7*d.* the price of 1 lb.

Qu. 3. A wholesale factor in Ireland made linen, which cost him 12½*d.* per yard, the expense of sending it to London 1½*d.* per yard, it was sold in London at 1*s.* 9*d.* per yard, and the retail trader was allowed 26 per cent. profit, what profit had the wholesale factor?—*Ans.* 24 per cent.

In resolving this question, I say, as 1s. 2d. the expense of making and exporting the linen, is to 1s. 9d. the retail price, so i. 100l. to 150l.; thus there is 50 per cent. profit, which, after deducting 26 per cent. the retail trader's profit, leaves 24 per cent. for the factor.

Qu. 4. A merchant bought 100 gallons of brandy, at 6s. per gallon, of which quantity 40 gallons were lost; at what price per gallon must he sell the remainder, that he may gain 10 per cent. profit upon the money it cost him?—*Ans.* 11s. per gallon.

SECT. XVII.

OF EQUATION OF PAYMENTS.

EQUATION of payments is that rule whereby is discovered the time to pay at one payment several sums due at different times, so that neither party may sustain any loss.

Rule. Multiply each debt by the time at which it is due, and add all the products together; divide the sum of the products by the sum of all the debts, and the quotient will be the answer, or the equated time to pay the whole.

Example 1. A is indebted to B in the sum of 200l. to be paid as follows: 60l. in 4 months, 40l. in 6 months, and 100l. in 10 months; what is the equated time to pay the whole?

Debts.		Months.	Products.
60l.	in	4	240
40		6	240
100		10	1000
<u>200</u>			<u>1480</u>
		2,00)	1480
			7
			<u>80</u>
			=

Answer $7\frac{2}{5}$ months.

Qu. 2. A owes B 1000*l.* of which $\frac{1}{3}$ is to be paid in 6 months, $\frac{2}{3}$ in 8 months, and the remainder in 10 months; what is the equated time to pay the whole?—*Answer* $7\frac{1}{3}$ months.

In this example, $\frac{1}{3}$ of 1000*l.* multiplied by 6 months produces 2, and $\frac{2}{3}$ multiplied by 8, produces $3\frac{1}{3}$, and $\frac{1}{3}$ (the remainder of the 1000*l.*) multiplied by 10, produces $2\frac{1}{3}$; and these three products added together gives $7\frac{1}{3}$ months for the answer.

And, *note*, when the sums or times of payment are given in fractions, the sum of the products is not to be divided by the sum of the debts, as it is in whole numbers.

Qu. 3. A tradesman owes his creditor 144*l.*; 44*l.* he pays in ready money; 60*l.* is to be paid at the expiration of 6 months, and the remaining 40*l.* at the expiration of 8 months; but the tradesman desiring to have more time for the payment of the last 40*l.* pays his creditor the 60*l.* due 6 months after, in ready money; how long may he defer paying this last 40*l.* to make him amends for this prompt payment?—*Ans.* 17 months.

In this example, the 44*l.* to be paid in ready money is neglected; but for the 60*l.* paid 6 months before due, I find by the rule of three what interest it would gain at any rate per cent. in that time, and then how long the 40*l.* may be lent for that interest at the same rate, which I find is 9 months, and which added to 8 months, its time of payment, gives 17 months for the answer*.

* This rule, though greatly used by men of business, is not mathematically exact. The reason of the rule given by many writers, is, that for the debtor paying a sum before the time it is due, an equal sum should be forborn, for as long a time after it is due; but this is a mistake, for by the debtor paying money before it is due, he has the discount only; but keeping the money after it is due, he gains the interest, which is greater than the discount.

SECT. XVIII.

OF THE RULE OF FALSE, SINGLE AND DOUBLE
(GENERALLY CALLED POSITION,.)

THIS rule teacheth to answer such questions as cannot be resolved by any direct rule in vulgar arithmetic, and must be performed by false or feigned numbers.

Position is either single or double.

Single position is when the question can be resolved by one false position or set of feigned numbers, and one operation in the rule of three direct.

Rule. Take any number, and proceed exactly the same as if it were the true number through all the proportions mentioned in the question.

Then say (by the rule of three) as the result of this false operation is to any of its parts, so is the true result in the question to the corresponding part required.

Example 1. A son asking his father his age, the father answered, I am double the age of your eldest brother John, and he is three times the age of your youngest brother Henry, and the sum of all our ages is 80 years; what is each person's age?

False Supposition.

Suppose Henry's age is	6
Then John's age will be	18
And the father's age	36
The sum of the ages	<u>60</u>

The Truth.

Henry's age	3
John's	24
The father's	53
The sum	<u>80</u>

Here I suppose Henry's age to be 6, at which supposition John's age must be 18, the father's 36, and the sum of these three is 60, whereas it should be 80; then I say, as 60 the sum of the false supposition, is to 80 the sum of the true one,

so is 6 the supposed age of Henry to 8 his true age; therefore John's age is 24, and the father's 48, as in the example.

Qn. 2. A asked B how much money he had in his pocket; B answered, If you give me 4 guineas of the money in your pocket, I shall have five times as much as you will then have; but if, instead of that, I should give you 2 guineas of the money in my pocket, you will then have twice as much as I shall then have: how much money had each?—*Ans.* 6 guineas.

Qn. 3. A person hired a horse for 9 days, on the following terms: for the first 3 days he was to pay $\frac{1}{3}$ of the hire for the next 3 days, and for each of the last 3 days as much as the hire for the first 6 days; the whole was 2*l.* 8*s.*; what was it per day?—*Ans.* 1*s.* per day the first 3 days, 3*s.* per day the next 3 days, and 12*s.* per day the 3 last days.

Double position, or the *double rule of false*, is when two false positions are requisite to give an answer to the question.

Rule 1. Take any two convenient numbers, and work with each of them according to the question, as in single position. 2. Find the difference between the result of each of these false positions and the result of the question; these differences are called the errors. 3. Multiply each error by the contrary position, that is, the first error by the second position, and the second error by the first position; then find the sum and difference of the products. 4. If the errors are both alike, that is, if the result of the two positions be both greater or both less than the result of the question, divide the difference of the products by the difference of the errors, and the quotient will be the answer. 5. But if the errors be unlike, that is, if the result of one position be greater and the other less than the truth, then the sum of the products must be divided by the sum of the errors, and the quotient will be the answer.

Example 1. Three persons, A, B, C, built a house, which cost 50*l.* of which B paid half as much again as A, and C paid as much as A and B together; what did each pay?

First

First Supposition.

Suppose A paid £90
 Then B must have paid 135
 And C must have paid 225
 Result 450

First error 50
 Second position 96
300

450

First product 4800
 Second product 1800

Difference of errors 3,0)300,0 difference of the products.

£100 for A's share, wherefore

B must have paid 150 being half as much again as A,
 and D must have paid 250 being as much as both A and B,
 which added together gives £500 the original sum.

Second Supposition.

Suppose A paid £96
 Then B must have paid 144
 And C have paid 240
 Result 480

Second error 20

First position 90

Second product 1800

From this example may be seen the method of working this rule, which is always the same, except when the errors are unlike; then the sum of the products is to be the dividend, and the sum of the errors the divisor as above directed.

Qu. 2. A salesman bought a number of oxen, sheep, and lambs, for which he paid 115*l.*; for the oxen he paid 10*l.* each, for the sheep 20*s.* each, and for the lambs 10*s.* each; how many of each sort did he buy?—*Ans.* 10 of each.

Qu. 3. Three persons, A, B, C, have equal incomes; A saves $\frac{1}{8}$ of his income every year; B spends 10*l.* per annum more than A, and C spends 10*l.* per annum more than B. At the expiration of five years, C finds himself in debt 50*l.* what is each person's income, and what has each saved or spent?—*Ans.* the income of each is 100*l.* per annum; A has saved 50*l.*; B has saved nothing; and C has spent 50*l.* more than his income.

Qu. 4. A labourer was hired for 40 days: for every day he wrought he was to receive 2*s.* and for every day he was idle he was to forfeit 1*s.*; at the end of the time he had to receive 44*s.*; how many days did he work, and how many was he idle?—*Ans.* he wrought 28 days and was idle 12 days.

SECT.

SECT. XIX.

OF EXCHANGE.

EXCHANGE is that rule which teacheth to find what sum of the money of one country is equal to a given sum of the money of another country, the course of exchange being known.

The *course of exchange* is that sum of the money of one country which is proposed to be given for a certain constant sum of that of another country: thus, when we say the course of exchange between England and Holland is 34s. Flemish per pound sterling, it signifies that 1 pound sterling is equal to the value of 34s. in Flemish money. This course of exchange varies on the part of the foreign coins, according to the state of public affairs.

The *par of exchange* is that quantity of the coin of one country which is intrinsically equal to a certain quantity of the coin of another country, according to the value of the metal.

Most foreign countries have two sorts of coins, called *current money* and *banco money*; the first is that in general use throughout the country; the latter is that kept in the banks of those places, and is finer than the other; the difference between any sum as it is valued in current money and banco money is called the *agio*.

The money used in exchange is generally imaginary, and different from that in which the accompts are kept in most places: the money used in exchange also differs from current money in its value.

Before

Before the learner can resolve any question in this rule, it is necessary that he know how the country with which the exchange is to be made keep their accounts.

Holland, Flanders, and Germany.

In these countries accounts are kept in *guilder*, *shilling*, and *pennings*, similar to the English *pound*, *shilling*, and *pence*.

But the different denominations of their money are contained in the following table:

8 pennings	} make one	grote or penny
2 grotes		shilling
6 stivers		florin or guilder
20 stivers		rix-dollar
2½ florins		pound Flemish
6 florins		

Note. The money of Holland and Flanders is called Flemish money, and they exchange by the pound sterling.

The course of exchange with these countries has not long been (except during the troubles on the Continent, from 33s. 4d. to 36s. 6d. Flemish per pound sterling; and the *agio* from 3 to 6 per cent.

As the exchange with all countries is supposed to be made in *banco* money, the current money must be reduced to *banco* before the exchange can be made.

Rule. By the rule of three, or practice.

Proof. By reversing the question.

To change current money into *banco*, and *banco* money into current.

Say, by the rule of three, as 100*l.* with the *agio* added to it, is to 100*l.* so is the given sum current money to *banco* in *banco*.

And, as 100*l.* is to 100*l.* with the agio added to it, so is the given sum banco to its value in current money.

Example 1. In 110*l.* 10*s.* 6*d.* sterling, how many florins, stivers, and pence current, the course of exchange being at 35*s.* 7½*d.* Flemish per pound sterling, agio 4*l.* per cent.?

For 1 <i>l.</i> take	£ 110	10	6
For 10 <i>s.</i> take	55	5	3
For 5 <i>s.</i>	27	12	7½
For 6 <i>d.</i>	2	15	3 ⅞
For 1¼ <i>d.</i>	0	13	9 ⅞
	£ 196	17	5 ⅞

2 deniers gros	} make one	sol lub
6 sols lub		sol gros
16 sols lub		mark
2 marks		drick, or Hamburgh dollar
3 marks		six-dollar
7½ marks		five gros, or pound Flemish

The exchange with this place has lately been from 32 to 35s. Flemish per pound sterling, and *agio bank* 18 to 20 per cent.

Example 4. In 886 marks 12 sols lub banco, how many pounds sterling, exchange 36 sols gros 2 deniers per pound sterling?

$$\begin{array}{r}
 36 \text{ sols gros 2 den.} \\
 12 \\
 \hline
 434
 \end{array}
 \qquad
 \begin{array}{r}
 886 \text{ marks : 2 sols lub} \\
 15 \\
 \hline
 53:8 \\
 887 \\
 \hline
 14188 \text{ sols lub} \\
 2
 \end{array}$$

Answer 65*l.* 7*s.* 7½*d.* 434)14188(65*l.*

$$\begin{array}{r}
 9604 \\
 3336 \\
 2170 \\
 \hline
 166 \\
 20 \\
 \hline
 434)3320(7*l.* \\
 3038 \\
 \hline
 282 \\
 13 \\
 \hline
 434)3384(7*l.* \\
 3038 \\
 \hline
 346 \\
 4 \\
 \hline
 434)1384(3*l.* \\
 1302 \\
 \hline
 82
 \end{array}$$

Here the marks and sols are reduced into deniers, as are also those in the course of exchange; then the course of exchange I make the divisor, and the foreign coin the dividend, and the quotient is 65*l.* 7*s.* 7½*d.* for the answer.

Qn. 5. In 1072*l.* sterling, how many marks, the exchange at 36*s.* 4*d.* Flemish per pound sterling?—*Ans.* 14605 marks.

Qn. 6. In 1686*l.* 2*s.* 5½*d.* sterling, how many six-dollars and sols lub current, exchange at 33 sols gros 9½ deniers *agio* 18½ per cent.?—*Answer* 8434 six-dollars 23 sols lub.

In France

Accompts are kept in livres, sols, and deniers; and the exchange is by the ecu, or crown tournois.

The exchange has mostly been from 30*d.* to 32*d.* sterling per ecu *.

12 deniers	} make one {	sol
20 sols		livre
3 livres		crown tournois, or ecu
10 livres		pistole
24 livres		Louis d'or, or guinea

Example 7. In 2465 livres 12 sols 9 deniers, how many pounds sterling, exchange 31½*d.* per ecu?

Here the livres, sols, and deniers are brought into crowns tournois, by dividing by 3, and that quotient divided by the aliquot parts of the course of exchange gives the answer.

	liv.	sol.	den.
3)2465	12	9	
30 <i>d.</i> 8)	821	17	7
1½ <i>d.</i> 20)	102	14	8½
	5	2	8½
<i>Answer</i> £	107	17	5

Qu. 8. In 1543*l.* 15*s.* sterling, how many French pistoles, exchange 30½*d.* per ecu?—*Answer* 3600.

In Spain

Accompts are kept in piastras, rials, and marvadies; and the course of exchange is by the piastra, and is generally from 38 to 42*d.* sterling per piastra.

4 marvadies vellon, or	} make one {	quarta
2½ marvadies of plate		rial vellon
2½ quartas, or		rial of plate, or dollar
34 marvadies vellon		piso, piastra, or piece of
16 quartas, or		Spanish pistole [eight
34 marvadies of plate	}	doubloon
8 rials of plate		
5 piastras		
2 pistoles		

* The course of exchange mentioned in every part of this section means the exchange as it generally stood before the late unhappy affairs on the Continent.

Example

Example 9. In 8746 rials of plate, how many pounds sterling, exchange at $4\frac{1}{2}d.$ per piaſtre or piſo?

Here the rials are brought into piaſtres or piſos, by dividing by 8, as the exchange is by that piece, and the remainder of the work is wrought as the 7th example.

$$\begin{array}{r}
 \text{rials.} \\
 8)8746 \quad 0 \quad 0 \\
 \hline
 \text{For } 40d. \quad 6)1093 \quad 5 \quad 0 \\
 \hline
 \text{For } 1d. \quad 40)182 \quad 4 \quad 2 \\
 \hline
 \text{For } \frac{1}{2}d. \quad 2) \quad 4 \quad 11 \quad 1\frac{1}{2} \\
 \hline
 \quad \quad \quad 2 \quad 5 \quad 6\frac{1}{2} \\
 \hline
 \text{Answer } \underline{\underline{\pounds 189 \quad 0 \quad 9\frac{1}{2}}}
 \end{array}$$

Qn. 10. In 178*l.* 5*s.* 11*d.* ſterling how many rials of plate, &c. exchange at $40\frac{1}{2}d.$ per piaſtre?—*Answer* 8452 rials plate 8 quartas.

In Portugal

Accompts are kept in reas and milreas; and the exchange is by the milrea, and generally from 60 to 67*d.* per milrea.

$\left. \begin{array}{l} 400 \text{ reas} \\ 1000 \text{ reas, or} \\ 2\frac{1}{2} \text{ cruſadoes} \end{array} \right\} \text{make one} \left\{ \begin{array}{l} \text{cruſado} \\ \text{milrea} \end{array} \right.$

Example 11. In 1864 milreas 108 reas, how many pounds ſterling, exchange at 5*s.* 9*d.* per milrea?

$$\begin{array}{r}
 \text{For } 5s. \quad 4)1864 \quad 0 \quad 0 \\
 \hline
 \text{For } 6d. \quad 10)466 \quad 0 \quad 0 \\
 \hline
 \text{For } 3d. \quad 2) \quad 46 \quad 12 \quad 0 \\
 \hline
 \quad \quad \quad 23 \quad 6 \quad 0
 \end{array}$$

Here the milreas are brought into pounds and ſhillings by practice.

$$\begin{array}{r}
 \text{For } 108 \text{ reas take} \quad 0 \quad 0 \quad 7\frac{1}{2} \\
 \hline
 \text{Answer } \underline{\underline{\pounds 535 \quad 18 \quad 7\frac{1}{2}}}
 \end{array}$$

Qn. 12. In 1509*l.* 17*s.* how many cruſadoes, the exchange at $64\frac{1}{2}d.$ per milrea?—*Answer* 14045 cruſadoes.

In Venice and Leghorn

The accompts are kept in dollars, foldi, and denari. The exchange is by the ducat and piaſtre, and from 52*d.* to 54*d.* per ducat, and from 45*d.* to 54*d.* per piaſtre, and the agio 20 per cent.

12 denari	} make one	{	foldi
20 foldi			lira, or piaſtre of Leghorn
$5\frac{1}{8}$ foldi			grosſe
24 grosſe			ducat

Example 13. In 846 piaſtres 10 foldi 8 denari; how many pounds ſterling, exchange $48\frac{1}{2}d.$ per piaſtre?

		pie.	ſol.	den.
For 40 <i>d.</i>	6)	846	10	8
For 8 <i>d.</i>	5)	141	2	$9\frac{1}{2}$
For $\frac{1}{2}d.$	16)	28	4	4
		1	15	$3\frac{1}{2}$
<i>Answer</i>		<u>£171</u>	<u>1</u>	<u>$4\frac{1}{2}$</u>

Q^{u.} 14. In 831*l.* 14*s.* 8*d.* ſterling, how many ducats, &c. current, exchange at 53*d.* per ducat, agio 20 per cent.?—
Anſ. 4519 ducats 14 grosſe.

The foregoing examples will be found ſufficient to inſtruct the learner in the method of performing this rule; I ſhall therefore inſert the remainder of the tables of foreign coin, and a few queſtions with their anſwers, omitting the operation for the practice of the learner.

In Ruſſia

Accounts are kept in rubles and copecs; and the exchange is by the ruble; and when made immediately with London, is from 4*s.* to 5*s.* per ruble; but when made by the way of Hamburgh or Amſterdam is from 48 to 50 ſtivers per ruble.

3 copecs	} make one	{	altine
10 copecs			grivena
25 copecs			polpolitin
2 polpolitins			politin
2 politins			ruble
2 rubles			ducat

Q^{u.} 15. In 2337 rubles 73 copecs, how many pounds ſterling; exchange, by way of Amſterdam, at 122 copecs per rix-dollar of 50 ſtivers; and exchange between Holland and England at 34*s.* 7*d.* Flemiſh, per pound ſterling?—*Anſ.* 230*l.* 17*s.* 3*d.*

Qu. 16. For 943*l.* 14*s.* 8*d.* paid in London, how many rubles must be received at Peterburgh, exchange by way of Holland at 34*s.* 9*d.* Flemish per pound sterling, and exchange from Holland to Peterburgh at 50 stivers per ruble?—*Ans.* 7870 rubles 36 copecs.

In Poland and Prussia

Accompts are kept in florins, gros, and penins. The exchange is by the way of Holland, and from 240 to 295 gros per pound Flemish.

18 penins	} make one {	gros
18 gros		oort
30 gros		florin, or Polish gilder
3 florins		rix-dollar
2 rix-dollars		gold ducat

Qu. 17. In 437*l.* 17*s.* 4*d.* sterling, how many rix-dollars Polish, exchange 290 gros per pound Flemish, and 34*s.* 4*d.* Flemish per pound sterling?—*Ans.* 2422 rix-dollars 4 gros 13 penins.

In Sweden

Accompts are kept in copper dollars, and oorts or silver dollars. The exchange is by the copper dollar, and mostly from 46 to 56 copper dollars per pound sterling.

8 penins	} make one {	runstychen
3 runstychens		stiver
8 stivers		marc
10 stivers and 2 runstychens, or		copper dollar
32 runstychens		
3 copper dollars and 32 stivers, or		silver dollar
96 runstychens, or		
4 marcs	}	
24 marcs		copper rix-dol.

Qu. 18. In 5838 silver dollars 9 runstychens 3½ penins, how many pounds sterling, exchange at 49 copper dollars per pound sterling?—*Ans.* 357*l.* 8*s.* 8½*d.*

In Ireland, America, and the West Indies,

Accompts are kept in pounds, shillings, and pence, as in England, but the course of exchange between England and Ireland is from 5 to 12 per cent. in favour of England, and the course of exchange of the paper of America and the West Indies is never at any certain standard.

Qu. 19. If 2172*l.* be remitted to Ireland, how much money sterling may be drawn for it, exchange at 8 per cent. ?
—*Ans.* 2345*l.* 15*s.* 2½*d.*

Qu. 20. A merchant sells goods in London, and remits to his correspondent in Boston, the value amounting to 1151*l.* 19*s.* how much must the merchant at Boston receive in paper currency, exchange at 33½ per cent. in favour of England ?—*Ans.* 1535*l.* 18*s.* 8*d.*

Questions of the nature of these two are resolved by the rule of three: thus, in the 19th question, I say, if 100*l.* require 108*l.* what will 2172*l.* require? and the answer is 2345*l.* 15*s.* 2½*d.*

The *arbitration of exchange* is the method of finding the course of exchange between any two places, by having the course of exchange between each of these two places and a third place.

By comparing the par of exchange thus found with the course of exchange, a person can tell which way to draw or remit his money to the most advantage.

All questions in arbitration of exchange are resolved by the rule of three direct, comparing the course of exchange between any two places, with that of a third place to a fourth.

Example 21. If the exchange between London and Paris be 33*d.* per ecu, and the exchange between London and Amsterdam be 34*s.* 6*d.* per pound sterling, what is the par of arbitration between Paris and Amsterdam?

As

As 240*d.* London is to 34*s.* 6*d.* Amsterdam, so is 33*d.* Paris to

$$\begin{array}{r}
 12 \\
 \hline
 414 \\
 33 \\
 \hline
 1242 \\
 1242 \\
 \hline
 24,0 \overline{) 1366,2} (56*d.* \\
 120 \\
 \hline
 166 \\
 144 \\
 \hline
 222 \\
 4 \\
 \hline
 24,0 \overline{) 88,8} (3*f.* \\
 72 \\
 \hline
 168 \\
 \hline
 \end{array}$$

Answer 56½*d.* ¾ Flemish per ecu for Paris.

Here I say, as 240*d.* the pence in a pound sterling of London, is to 34*s.* 6*d.* the Flemish exchange, so is 33*d.* the exchange for an ecu, or crown tournois of Paris, to 56*d.* ¾ farthings the Flemish exchange for an ecu, and is the course of exchange between Paris and Amsterdam.

This example will be sufficient, as the rule never varies, though the course of exchange between several countries be given to find a proportional course between any two; in which case they may be resolved into as many questions as is necessary in the rule of three, and all the first numbers multiplied together for a divisor, and the second and third numbers in each question multiplied together for a dividend, then the quotient will be the answer.

Qu. 22. A merchant in London is drawn upon by his correspondent in Russia for money to the amount of 12500 rubles; the exchange between England and Russia being at 50*d.* per ruble; between Russia and Holland 90*d.* per ruble; and between England and Holland 36*s.* 4*d.* per pound sterling; which is the most advantageous method for the London merchant to pay by; directly to Russia or by way of Holland, and what is the advantage?—*Answer,* the London merchant will gain 23*l.* 17*s.* 10½*d.* by making payment by way of Holland.

SECT,

SECT. XX.

OF THE RELATION OF NUMBERS.

ANY set of numbers that constantly increase or decrease by a common difference are said to bear a relation to each other, which relation is called progression, and is divided into two kinds: arithmetical progression and geometrical progression.

Arithmetical progression is, when any set or series of numbers constantly increase or decrease by a given number, called from thence their common difference; such are the natural orders of numbers, 1, 2, 3, 4, 5, 6, 7, 8, &c. each of which increases by 1; and the same in inverted order 8, 7, 6, 5, 4, &c. decrease by 1; 1 is therefore their common difference.

Again, the numbers 2, 5, 8, 11, 14, 17, &c. increase by 3; and 27, 23, 19, 15, 11, 7, &c. decrease by 4: therefore 3 is the common difference of the former, and 4 that of the latter.

The numbers themselves that form the series are called the *terms* of the progression.

Of every series of arithmetical progression having any three of the five following terms the other two may be found readily;

- | | |
|------------------------------|--------------------------------|
| 1, the first term or number | } called the <i>extremes</i> . |
| 2, the last term or number | |
| 3, the number of terms. | |
| 4, the common difference. | |
| 5, the sum of all the terms. | |

PROBLEM I.

The first and last terms, and number of terms, being given, to find the sum of all the terms, and the common difference.

Rule. To find the sum of all the terms, multiply the sum of the extremes by the number of terms, and half the product will be the answer; and to find the common difference of the terms, divide the difference of the extremes by the number of terms made less by 1, and the quotient will be the answer.

Example 1. What is the sum of the series, and the common difference of that progression, whose first term is 3, last term 67, and number of terms 33?

last term 67
first term 3
70
number of terms 33
210

210
2)210
105

last term 67
first term 3
difference 64
32

32)64

2 common difference

1155 Answer, sum of the series.

In the first operation, 70, the sum of the extremes, is multiplied by 33 the number of terms, and half the product 1155 is the sum of the series, or sum of all the numbers, 3, 5, 7, 9, 11, 13, 15, 17, 19, 21, 23, 25, 27, 29, 31, 33, 35, 37, 39, 41, 43, 45, 47, 49, 51, 53, 55, 57, 59, 61, 63, 65, 67, added together.

In the second operation the difference of the two extremes 67 and 3, or 64, is divided by 32, which is one less than the number of terms, and the quotient 2 is the common difference.

PROBLEM II.

Having the two extremes and the common difference, to find the number of terms.

Rule. Divide the difference of the extremes by the com-

mon difference, and 1 added to the quotient will be the number of terms.

Example 2. What is the number of terms of that progression whose extremes are 3 and 67, and common difference 2?

Thus 64, the difference of the extremes, divided by 2, the common difference, quotes 32, to which adding 1 there is 33, the number of terms; as may be seen annexed.

$$\begin{array}{r} 67 \} \text{ extremes} \\ \underline{3} \\ 2)64 \text{ difference} \\ \underline{32} \\ 1 \\ \underline{33} \text{ number of terms.} \end{array}$$

In every arithmetical progression the sum of any two terms is equal to the sum of any two other terms, taken at an equal distance from the former, and on opposite sides: thus, in the foregoing progression, 15 and 19 is equal to 23 and 11; viz. 34; and 31 and 33 is equal to 23 and 41, viz. 64. And the double of any one term is equal to the sum of any two terms taken on each side of it, and at an equal distance from it; thus the double of 25 is equal to 19 and 31, viz. 50.

Qⁿ. 3. How many strokes does an English clock strike in 8 days?—*Ans.* 1248 strokes.

Qⁿ. 4. If a traveller go a journey of 10 days, travelling 3 miles the first day, and increasing three miles every day, how many miles will he travel in the 10 days? and how many miles the last day?—*Ans.* 165 miles in the whole, and 30 miles the last day.

Geometrical Progression.

When any series of numbers increase or decrease by a constant multiplication or division, they are said to be in *geometrical progression*.

Thus the numbers 4, 8, 16, 32, 64, &c. and 243, 81, 27, 9, 3, 1, are in geometrical progression, the former *increasing* by multiplying each preceding number by 2, and the latter *decreasing* by dividing each preceding number by 3.

Thus, in geometrical progression, the numbers are increased by multiplication, and decreased by division; whereas in arithmetical progression they are increased by addition and decreased by subtraction.

The number in geometrical progression by which the series are multiplied or divided, is called the *ratio*; whereas in arithmetical progression the number by which the series are increased or diminished is called the *common difference*.

In geometrical progression, as in arithmetical progression, having any three of the following terms, the other two may be readily found.

- 1, the first term } the extremes.
- 2, the last term }
- 3, the number of terms.
- 4, the ratio.
- 5, the sum of all the terms.

PROBLEM I.

Having the first and last term and the ratio, to find the sum of the terms.

Rule. Multiply the last term by the ratio, and from the product subtract the first term, and the remainder divided by one less than the ratio will quote the sum of the series.

Example 1. What is the sum of the series of a geometrical progression whose extremes are 1 and 65536, and the ratio 4?

$$\begin{array}{r}
 \text{last term} \quad 65536 \\
 \text{ratio} \quad 4 \\
 \hline
 262144 \\
 \text{first term} \quad 1 \\
 \hline
 \text{one less than the ratio} \quad 3 \overline{)262143} \\
 \text{the sum of the series} \quad \underline{\underline{87381}}
 \end{array}$$

Here the last term is multiplied by the ratio 4, the first term 1 subtracted from the product, and the remainder divided by 3, which is 1 less than the ratio, and the quotient 87381 is the sum of the series.

Q. 2. What is the sum of a series of numbers, in geometrical progression, whose extremes are 1924 and 59049 and the ratio 3? — *Answer* 88061.

PROBLEM II.

Having the first term and the ratio, to find any other term required.

Rule. Write down a few of the leading terms of the series, and place their indices over them, beginning with a cypher*.

Then add any of those indices together, which will make an index an unit less than the number which expresses the place of the term required.

Multiply the terms of the series together belonging to those indices, and the product will be a dividend, to be divided by the product of the first term, multiplied by a number an unit less than the number of terms multiplied; and the quotient will be the answer.

Example 9. What is the last term of that geometrical series whose first term is 3, ratio 2, and number of terms 10?

Indices 0, 1, 2, 3, 4, 5, 6,
Leading terms 3, 6, 12, 24, 48, 96, 192.

$$\begin{array}{r}
 48 \\
 96 \\
 \hline
 288 \\
 432 \\
 3)4608 \\
 \hline
 1536
 \end{array}
 \qquad
 \begin{array}{r}
 4 \\
 5 \\
 \hline
 9 \text{ index to the 10th term;} \\
 \hline
 \hline
 \end{array}$$

1536 the Answer, or the 10th term.

* The indices of the terms in this rule are numbers expressing the place of each term according to the natural order of numbers; thus the index of the first term is 0, that of the second term 1, the third term 2, the fourth 3, &c.; but when the first term of the series is equal to the ratio, the indices must begin with an unit, and the product of the terms will be the answer, without dividing by the product of the first term, as directed above.

In

In this example, the indices 4 and 5 added together are 9, which is 1 less than the number of the term sought; then the term belonging to those indices, 48 and 96, are multiplied together for a dividend, the first term 3 multiplied by 1 less than the number of terms, which are multiplied together, is the divisor; and the quotient 1536 is the answer.

Note. The number of terms multiplied together in this example are 3; therefore 3 the first term multiplied by 1 less than 2, or 1 only, produces 3.

Example 4. What is the 15th term of a series whose first term is 3, and ratio 3?

1,	2,	3,	4,	5,	6,	Indices
3,	9,	27,	81,	243,	729,	Leading terms
		729			6	
		243			4	
		2187			5	
		2916			15	index to the 15th term
		1458				
		179649				
		81				
		177147				
		1417176				
		14348907				the 15th term, or Answer.

Here, as the first term is equal to the ratio, the indices must begin with 1, instead of 0; and as many indices must be taken as will make the entire index to the term sought, viz. 15, and the product of the terms must not be divided, as in the former case.

Qu. 5. What is the last term of that geometrical series where the first term is 1, ratio 2, and number of terms 23?
—*Ans.* 4194304.

Qu. 6. A person having an elegant house to dispose of, offered it to sale on the following terms: for the first window the purchaser was to pay 1 farthing, for the second window 4 farthings, for the third window 4 farthings, and so on, doubling
bling

bling the price for every succeeding window; there were 32 windows in the house: what would be the price of the house at that rate?—*Ans.* 4,473,924*l.* 5*s.* 4*d.*

Q^a. 7. An Indian of the name of Sessa having invented the game of chess, shewed it to his prince Shehram, who was so pleased with the invention, that he bade Sessa say what he would have as a reward for his ingenuity. Sessa requested 1 grain of wheat for the first square on the chess-board, 2 grains of wheat for the second square, 4 for the third, 8 for the fourth, and so on, doubling the quantity of grains of wheat for every succeeding square: now the whole number of squares on the chess-board is 64. Supposing a bushel to contain 640,000 of these grains, how many ships would it require to export the whole quantity of wheat, each ship being 100 tons burden?—*Ans.* 7,205,759,403 ships, and about $\frac{1}{4}$ of a ship.

There are many other questions in progression which favour more of curiosity than real utility. This rule was much admired formerly, before the nature of numbers was so well understood as at present, on account of its surprising increasing power: it is, however, of little use, except in calculating tables, &c.

SECT. XXI.

OF EVOLUTION, OR THE EXTRACTION OF ROOTS, SQUARE AND CUBIC.

BEFORE the learner proceeds to extraction of roots, he should understand the nature of involution, or the raising of a number to any power required.

A power

A *power* is the product of any number multiplied into itself any certain number of times.

If a number be multiplied into itself only once, the product is called the *second power*, or the *square*; thus 4 is the second power, or square of 2, being the product of 2, multiplied by 2; and 9 is the second power of 3. But if a number be multiplied twice into itself continually, the product is called the *third power*, or *cube*; thus 27 is the cube, or third power of 3; and 64 is the third power of 4.

When a number is multiplied 3, 4, 5, or 6 times into itself, &c. the product is called the *fourth*, *fifth*, *sixth*, or *seventh power* respectively.

Example 1. What is the fifth power of 6?

$$\begin{array}{r}
 6 \\
 \times 6 \\
 \hline
 36 \text{ second power} \\
 \times 6 \\
 \hline
 216 \text{ third power} \\
 \times 6 \\
 \hline
 1296 \text{ fourth power} \\
 \times 6 \\
 \hline
 7776 \text{ fifth power}
 \end{array}$$

Fractions are raised to powers in the same manner as whole numbers; multiplying them as taught in multiplication of fractions.

A TABLE

OF THE

First nine Powers of each of the nine single Numbers.

1 st	2 ^d	3 ^d	4 th	5 th	6 th	7 th	8 th	9 th
1	1	1	1	1	1	1	1	1
2	4	8	16	32	64	128	256	512
3	9	27	81	243	729	2187	6561	19683
4	16	64	256	1024	4096	16384	65536	262144
5	25	125	625	3125	15625	78125	390625	1953125
6	36	216	1296	7776	46656	279936	1679616	10077096
7	49	343	2401	16807	117649	823543	5764801	40353607
8	64	512	4096	32768	262144	2097152	16777216	134217728
9	81	729	6561	59049	531441	4782969	43040721	387420481

The

The use of this table is, to find any power less than the 10th power of any number under 10; thus, to find the 7th power of 6, I look in that column which has 7 at the top, and in that line which has 6 on the left hand, and in the junction of these two lines is 279936.

The *root* of any number is that number which, being multiplied into itself a certain number of times, will produce the given number; thus 2 is the square root of 4, because 2 multiplied by 2 produces 4; and 3 is the cube root of 27, for 3 multiplied 3 times into itself continually produces 27.

There are many numbers the given roots of which can never be exactly found, though by decimal fractions we may approximate towards it to any degree of exactness; but the power of a given number can always be found exactly.

Those roots, which only approximate towards the truth, are called *furd* roots, and those which perfectly express the root of the given power are called *rational* roots: thus the square root of 5 is a furd root, as it cannot be exactly expressed; but the cube root of 8 is a rational root, being exactly 2.

To extract the Square Root.

Rule 1. Divide the given number into periods of two figures each, by placing a point over every other figure, beginning with that in the place of units.

1. Find the greatest square root of the first period, and place it on the right hand of the given number, like a quotient in division.

2. Subtract the square of this root from the aforesaid period, and to the remainder bring down the next period, and call it the resolvend.

3. Double the figures in the quotient for a new divisor to the resolvend, and find how often this divisor is contained in the resolvend, exclusive of the unit figure, and place the answer both in the quotient, and on the right hand of the divisor.

4. Then

5. Multiply the divisor thus increased by the last quotient figure, and subtract the product from the resolvend, and to the remainder bring down the next period for a new resolvend.

6. Double the figures in the quotient for a new divisor to this last resolvend, and seek how often it is contained therein, exclusive of the unit figure, placing the answer both in the quotient and divisor as before : and proceed in the same manner through each new resolvend, and the figures placed in the quotient will be the answer, or square root of the given number.

Example 1. What is the square root of 5934096?

$$\begin{array}{r}
 \begin{array}{c} \cdot \cdot \cdot \cdot \\ 5934096 \end{array} \begin{array}{c} \text{2436 root} \\ 4 \end{array} \\
 \begin{array}{r}
 44 \overline{) 193} \\
 4 \overline{) 176} \\
 483 \overline{) 1740} \\
 3 \overline{) 1449} \\
 4866 \overline{) 29196} \\
 6 \overline{) 29196}
 \end{array} \\
 \text{The Answer} \quad \underline{\underline{2436}}
 \end{array}$$

After dividing the given number into periods of two figures each by the points, I find the nearest square root of 5 (as that is pointed for the first period), which is 2 ; I therefore place 2 in the quotient, and the square of 2, which is 4, I subtract from this first period 5, and to the remainder 1 I bring down the next period 93 ; thus I have 19 for a new resolvend (for the unit figure 3 is to be neglected), then I take the double of the figure 2 in the quotient for a divisor, and seeking how often it is contained in 19, the resolvend, the answer is 4, which I place in the quotient, and also on the right hand of the divisor 4 ; then I multiply the divisor thus increased by 4, and the product 176 I place under and subtract from the last resolvend, and to the remainder 17 I bring down the next period 40 ; thus I have 174 for a new resolvend, and 48 the double of the figures in the quotient for a

new divisor; then I seek how often the divisor 48 is contained in 174, and the answer 3 I place both in the quotient and divisor as before, and multiply the divisor also thereby, and the product 1449 placed under and subtracted from the resolvend, there remains 291, to which bringing down the last period 96, I have 2919 for the last resolvend, and the double of 243 or 486 for a divisor: this last quotient is 6, which I place in the quotient and divisor, and multiply the divisor thereby, and the product is equal to the last resolvend: thus 2436 is the answer.

Qu. 2. What is the square root of 152399025?—*Answer* 12345.

Qu. 3. What is the square root of 22071204?—*Answer* 4698.

To extract the Cube Root.

Rule 1. Divide the given number into periods of three figures each, by placing a point over every third figure, beginning with the unit figure.

2. Find the greatest cube in the first period, and place its cube root on the right hand of the given number.

3. Subtract the cube from the said period, and to the remainder bring down the next period, which is called the resolvend.

4. Place three times the root under the resolvend, and also three times the square of the root, the latter being removed one place to the left; these two added together form a divisor.

5. Seek how often this divisor is contained in the resolvend, exclusive of the place of units, and place one less than the answer in the quotient.

6. Under the divisor place the cube of the last quotient figure, the square of it multiplied by three times the rest of the quotient, and three times the root multiplied by the square of the quotient; each of the two latter numbers being removed one place farther towards the left hand than the foregoing.

foregoing, and the sum of these three numbers is called the *subtrahend*.

7. Then subtract this subtrahend from the resolvend, and to the remainder bring down the next period for a new resolvend, with which proceed as before, and so on till the work be finished.

Example 4. What is the cube root of 164566592?

$$\begin{array}{r}
 \begin{array}{r}
 \overset{\cdot}{1}\overset{\cdot}{6}\overset{\cdot}{4}\overset{\cdot}{5}\overset{\cdot}{6}\overset{\cdot}{6}\overset{\cdot}{5}\overset{\cdot}{9}\overset{\cdot}{2} \\
 125 \\
 \hline
 765 \overline{) 39566} \\
 \underline{15} \\
 75 \\
 \underline{765} \\
 64 \\
 \underline{240} \\
 300 \\
 \underline{32464} \\
 86742 \overline{) 7102592} \\
 \underline{162} \\
 8748 \\
 \underline{87642} \\
 512 \\
 \underline{10368} \\
 69984 \\
 \underline{7102592}
 \end{array}
 \end{array}$$

cube of the first period
 first resolvend
 three times the root 5
 three times the square of 5
 first divisor
 cube of 4
 square of 4 multiplied by 3 times 5
 three times 4 multiplied by the square of 5
 subtrahend
 second resolvend
 three times 54
 three times the square 54
 second divisor
 cube of 8
 square of 8 multiplied by 3 times 54
 three times 8 multiplied by the square of 54
 second subtrahend

Answer 548 the cube root required

Qn. 5. What is the cube root of 389017?—*Ans.* 73.

Qn. 6. What is the cube root of 40512?—*Ans.* 34.

There are many other methods of extracting the cube root given by different mathematicians; but none of which I have seen are so simple and easy to be remembered as the method here laid down.

After what has been said, it need hardly be mentioned, that the extraction of the square root is proved by multiplying the root into itself; and the extraction of the cube root, by multiplying the root three times into itself.

CHAP. IV.

OF DECIMAL ARITHMETIC.

SECT. I.

REDUCTION OF DECIMAL FRACTIONS.

A DECIMAL fraction is that fraction whose denominator has an unit in the first place on the left hand, with as many cyphers annexed as necessary : thus $\frac{1}{10}$, $\frac{25}{100}$, $\frac{48}{1000}$, $\frac{572}{10000}$, &c. are decimal fractions. But these fractions are usually expressed in writing without a denominator, by writing the numerator, and prefixing as many points or cyphers before it on the left hand, as there are more places of figures in the denominator than in the numerator. Thus the foregoing fractions are written .5, .25, .048, .0572, and expressed five tenths, twenty-five hundredths, forty-eight thousandths, &c.

Cyphers placed on the right hand of decimal fractions make no alteration in their value : thus, if a cypher be annexed to the foregoing fraction .5, it will be then .50 fifty hundredths or half an integer, as before ; if two cyphers .500 it will be five hundred thousandths, or $\frac{1}{2}$, and the same of the others.

But cyphers placed on the left hand of a decimal fraction decrease their value, every cypher decreasing it in a tenfold proportion : thus .5, .05, .005, are five tenths, five hundredths, five thousandths parts respectively.

The

The value of every figure in a decimal fraction increases in a tenfold proportion from the first place on the right hand, as in whole numbers.

The figures in the first place of a decimal fraction on the left hand are called *primes*, those in the second place *seconds*, those in the third place *thirds*, &c.

DECIMAL TABLES

or

Coin, Weight, and Measure.

TABLE I. <i>English Coin.</i> 1l. and 1s. the Integer.				TABLE II. <i>English Coin 1s.</i> Long Meas. 1 foot Troy Weight 10z. <i>Pence &c.</i>				4 3 2 1		,016666 ,0155 ,008333 ,0041666		6 5 4 3 2 1		,0125 ,010416 ,008333 ,00625 ,004166 ,002083			
Sh.	Dec.	Sh.	Dec.	lacker.	Decimals.	Grains.	Decimals.										
19	,95	9	,45	6	,5	12	,002083										
18	,9	8	,4	5	,416666	11	,001910										
17	,85	7	,35	4	,333333	10	,001736										
16	,8	6	,3	3	,25	9	,001562										
15	,75	5	,25	2	,166666	8	,001389										
14	,7	4	,2	1	,083333	7	,001215										
13	,65	3	,15			6	,001042										
12	,6	2	,1			5	,000868										
11	,55	1	,05			4	,000694										
10	,5					3	,000521										
						2	,000347										
						1	,000173										
TABLE III. <i>Troy Weight.</i> 1lb. the Integer. <i>Penny-weight.</i>				10z. the Integer.		Pennyweights the <i>same as Shillings</i> <i>in the First Table.</i>		Grains.		Decimals		TABLE IV. <i>Avoirdupois Wt.</i> 12lb. the Integer <i>Gr.</i>				Decimals.	
Farths.	Decimals																
3	,003125			10	,041666			12	,025			14	,125				
2	,0020833			9	,0375			11	,022916			13	,116071				
1	,0010416			8	,033333			10	,020833			12	,107143				
				7	,029166			9	,01875			11	,098214				
				6	,025			8	,016666			10	,089286				
				5	,020833			7	,014583			9	,080357				
												8	,071428				
												7	,0625				
												6	,053571				
												5	,044643				
												4	,035714				
												3	,026786				
												2	,017857				
												1	,008928				

Ounces.	Decimals.	Drams.	Decimals.	7	,027777	Pints.	Decimals.
8	,004166	8	,03125	6	,023809	3	,005952
7	,003906	7	,027343	5	,019847	2	,003968
6	,003646	6	,023437	4	,015873	1	,001984
5	,002790	5	,019531	3	,011904	TABLE VII. Measure. Liquid. Dry. 1 Gall. 1 Quarter the Integer. Pints. Dec. Bu.	
4	,002532	4	,015625	2	,007936		
3	,001874	3	,011718	1	,003968		
2	,001116	2	,007812	Pints.	Decimals.		
1	,000558	1	,003906	4	,001984	4	,5
$\frac{1}{2}$ oz.	Decimals.	TABLE VI. Liquid Measure. 1 Tun the Integer Gallons Decimals		3	,001488	3	,375
3	,000418			2	,000992	2	,25
2	,000279			1	,000496	1	,125
1	,000139			Hoghead the Integer. Gallons Decimals.		3	,09375
TABLE V. Avoirdupois Wt 1 lb. the Integer. Ounces. Decimals.		100	,396825	30	,476190	2	,0625
		90	,357142	20	,317460	1	,03125
		80	,317460	10	,158730	Decimals. $\frac{1}{2}$ pcks.	
		70	,277772	9	,142857		
8	,5	60	,238095	8	,126984		
7	,4375	50	,198412	7	,111111	Decimals.	Pints.
6	,375	40	,158730	6	,095238	,0234375	3
5	,3125	30	,119047	5	,079365	,015625	2
4	,25	20	,079365	4	,063492	,0078125	1
3	,1875	10	,039682	3	,047619	Decimals.	Pints.
2	,125	9	,035714	2	,031746	,005859	3
1	,0625	8	,031746	1	,015873	,003906	2
						,001953	1

Case 1: To reduce a vulgar fraction to a decimal one of the same value.

Rule. Divide the numerator by the denominator, and the quotient will be the decimal fraction required.

If the numerator be too small to be divided by the denominator, add one or more cyphers thereto, in which case as many places must be separated from the quotient for decimals as the difference of the number of places of decimals in the dividend and the divisor, and the remaining figures in the quotients (if any) are integers.

But if the number of places in the quotient be less than the required number of decimal places, as many cyphers must be prefixed on the left hand thereof as are necessary.

Example 1. What is the decimal fraction of a pound equal to $\frac{1}{4}$?

Here

Here two cyphers must be added to the numerator, which divided by 4 quotes 75, which I mark for decimals, as there are two decimals added to the dividend, and the quotient is the seventy-five hundredth part of a pound, or $\frac{3}{4}$, equal to 15s.

Example 2. What is the decimal fraction of a pound equal to 9d.?—*Ans.* .0375.

As there are but three places of figures in the quotient, I place a cypher on the left thereof, for there should be four figures pointed off for decimals, being so many cyphers annexed to 9; thus the quotient is $\frac{3}{80}$ of a pound.

Example 3. What is the decimal fraction of a pound for 3 farthings?—*Ans.* .003125.

Here the vulgar fraction for 3 farthings is $\frac{3}{80}$; I therefore place as many cyphers as are necessary to the numerator 3, which in this case is 6 cyphers, and the quotient is 3125, but there must be 6 decimals in the quotient, as there are 6 in the dividend and none in the divisor; I therefore prefix two cyphers to the left hand thereof as before directed.

Example 4. What is the decimal fraction of a year for 73 days?—*Answer* .2, equal to $\frac{2}{5}$.

Case 2. To find the value of a decimal fraction in money, weight, or measure.

Rule. Multiply the decimal fraction by the number of parts of the next inferior denomination, and from the product cut off as many figures from the right hand side as there are decimal places in the fraction.

Then multiply these figures so separated on the right hand by the number of parts of the next inferior denomination, and from the product cut off as many figures as this last multiplicand has decimal places, as before.

Proceed in the same manner through all the denominations, then the separated figures on the left hand side will be the answer.

Example

Example 1. What is the value of .564 of a pound sterling?

Here I multiply the given decimal by 20, the number of parts of the next less denomination, viz. shillings, and from the product I separate the three figures on the right hand, 280, as there are three places of decimals in the given number.

$$\begin{array}{r}
 .564 \\
 \times 20 \text{ shillings in a pound} \\
 \hline
 11,280 \\
 \times 12 \text{ pence in a shilling} \\
 \hline
 3,360 \\
 \times 4 \text{ farthings in a penny} \\
 \hline
 1,440 \\
 \hline
 \text{Answer } 11s. 3\frac{1}{4}d. 440 \text{ of a farthing.}
 \end{array}$$

I then multiply these separated figures 280 by 12, and from the product 3360 I separate the three first figures 360, as before, to be multiplied by the parts of the next denomination, viz. 4; then the figures standing on the left hand of these separated figures will be the answer.—11s. 3 $\frac{1}{4}$ d.

The same rule serves to discover the value of any other integers, having regard to the number of parts contained in each inferior denomination.

Thus to find the value of a decimal fraction of a pound troy, I first multiply the fraction by 12 to find the ounces, then the separated figures by 20 to find the pennyweights, and lastly by 24 to discover the grains.

Qu. 2. What is the value of .6725 of a cwt.?—*Answer* 2qrs. 19lb. 5oz.

Qu. 3. What is the value of .61 of a ton of wine?—*Ans.* 2hhds. 27galls. 2qts. 1pt.

Case 3. To find the value of a decimal of a pound sterling, by inspection.

Rule. Double the first figure of the decimal on the left hand for shillings; and if the next figure be 5, or above 5, add 1 shilling more to it: then call the figures in the second and third places farthings (after deducting 5 from the second figure if necessary), and if the number of farthings be above 12 abate one, and if above 37 abate 2. The other figures in the fraction, if more, are neglected.

Example

Example 1. Find the value of .564 of a pound by inspection.

Here I take the double of 5, the first figure, for shillings, which is 10, also 1 shilling more for the 5 contained in the 6, and the remaining 14 (viz. 1 in the 6, and 4) I consider as farthings, abating 1 because they are above 12, which make $3\frac{1}{4}d.$: thus the decimal .564 of a pound is 11s. $3\frac{1}{4}d.$ as may be seen in Case 4.

Qu. 2. Find the value of .7880 of a pound by inspection,
—*Answer* 15s. 9d.

Qu. 3. Find the value of .14729 of a pound by inspection,
—*Ans.* 2s. $11\frac{1}{4}d.$

Case 4. To find the decimal fraction of a pound equal to any given number of shillings and pence.

Rule. Write half the greatest even number of shillings for the first figure in the decimal, and the number of farthings in the pence and farthings for the second and third figures, observing to add 5 more to the second figure, if the number of shillings be odd; also add one more to the third figure, if the farthings exceed 12, and add 2 if they exceed 37.

Example 1. Find the decimal fraction of a pound equal to 11s. $3\frac{1}{4}d.$

Here for the 11s. I take $\frac{1}{2}$ of the next greatest even number, which is 5, for the first figure of the decimal, and for the odd shilling, place 5 to be added to the second figure; then for the $3\frac{1}{4}d.$ I take 14 to be added to the second and third numbers, as there are 13 farthings in $3\frac{1}{4}d.$ and 1 is added, as the farthings are above 12; thus, the decimal is .564 of a pound.

Qu. 2. Find the decimal equal to 18s. $4\frac{1}{4}d.$ —*Ans.* .919.

Qu. 3. Find the decimal equal to 17s. $6\frac{1}{4}d.$ —*Ans.* .878.

SECT. II.

OF ADDITION OF DECIMALS.

ADDITION of decimal fractions is performed in the same manner as addition of whole numbers, except in the disposition of the figures.

Rule. Place the figures exactly under each other, according to the value of their places, that is, the whole numbers (if any) under each other, as in addition of whole numbers, and the fractions are also to be placed according to their values, viz. primes under primes, seconds under seconds, &c.

Then find their sum as in whole numbers, and point off as many places for decimals as are equal to the greatest number of decimal places in any of the given numbers.

Example 1. What is the sum of 22.5709, 1.03, 1492.001, 2.971, .00726?

Here the numbers are added together like whole numbers, and the number of places pointed off for decimals are 5, equal to the greatest number of decimal places in any of the given numbers. The figures on the left hand of the decimal point are whole numbers or integers.

$$\begin{array}{r}
 22.5709 \\
 1.03 \\
 1492.001 \\
 2.971 \\
 .00726 \\
 \hline
 1518.58016
 \end{array}$$

More Examples.

$$\begin{array}{r}
 2.219 \\
 24.002 \\
 .125 \\
 75.002 \\
 100.1002 \\
 \hline
 201.4482
 \end{array}$$

$$\begin{array}{r}
 22.179 \\
 .025 \\
 .3006 \\
 41.2278 \\
 112.002 \\
 \hline
 175.7344
 \end{array}$$

$$\begin{array}{r}
 224.010 \\
 .120 \\
 2.24 \\
 112.01 \\
 7.8602 \\
 \hline
 346.2402
 \end{array}$$

SECT.

SECT. III.

OF SUBTRACTION OF DECIMALS.

DECIMAL fractions are subtracted in the same manner as whole numbers, but the numbers are placed and the decimals pointed off according to the rule given in the foregoing section.

Example 1. What is the difference of 247.0729 and 3746.805732?

Here the uppermost number consists of 6 places of decimals, wherefore the remainder sought must have as many decimal places; in every other respect it is wrought like subtraction of whole numbers.

Examples.

$$\begin{array}{r} 20.0192 \\ 14.129732 \\ \hline 5.889468 \end{array}$$

$$\begin{array}{r} 928.12004 \\ 400.002 \\ \hline 528.11804 \end{array}$$

$$\begin{array}{r} 3.7031 \\ .002397 \\ \hline 3.700703 \end{array}$$

SECT. IV.

OF MULTIPLICATION OF DECIMALS.

RULE. Multiply the given numbers one by the other, as in whole numbers.

Then point off as many figures for decimals as there are decimal places in both the numbers; but if there be not so many places in the product, as many cyphers must be prefixed on the left hand thereof as will make up the number of decimal places required.

Example 1. Multiply 2.03271 by .0056.

In this example the number of decimal places in both the multiplier and multiplicand is 9; but the product consists of only 8 figures, therefore I must prefix one cypher on the left hand thereof, which makes the true product.

$$\begin{array}{r}
 2.03271 \\
 .0056 \\
 \hline
 1219626 \\
 1016355 \\
 \hline
 .011383176
 \end{array}$$

Examples.

$$\begin{array}{r}
 27.02943 \\
 4.30020 \\
 \hline
 54058860 \\
 810882900 \\
 10811772 \\
 \hline
 116.2319548860
 \end{array}$$

$$\begin{array}{r}
 2.20434 \\
 .43970 \\
 \hline
 15430380 \\
 1983906 \\
 661302 \\
 881736 \\
 \hline
 .9692482980
 \end{array}$$

Thus it appears that in multiplication of decimal fractions there is no necessity for placing those figures of the same value under each other, as in the two foregoing rules *.

SECT. V.

OF DIVISION OF DECIMALS.

THIS rule is also performed as division of whole numbers, but from the quotient point off as many figures for decimals as the decimal places in the dividend exceed those in the divisor. But if the places in the quotient be not so many as should be pointed off for decimals, as many cyphers must be placed on the left hand as necessary.

* There is a contracted method of working both this and the following rule given by many writers on arithmetic. They are, however, not very simple, and may very well be superseded, by omitting some of the decimals in both numbers, and working according to the general rule.

If at any time there be a remainder, and if it be required to have an answer to a greater degree of exactness, as many cyphers may be affixed to the right hand of the dividend as is thought proper.

Example 1. Divide 1836.88305 by 23.15.

$$\begin{array}{r}
 23.15 \overline{)1836.88305} (79.347 \\
 \underline{1620\ 5} \\
 216\ 38 \\
 \underline{208\ 35} \\
 8\ 033 \\
 \underline{6\ 945} \\
 1\ 0880 \\
 \underline{9260} \\
 16205 \\
 \underline{16205} \\
 \hline
 \hline
 \end{array}$$

In this example there are three more decimal places in the dividend than in the divisor; there must therefore be three decimals in the quotient.

Examples.

$$\begin{array}{r}
 2 \\
 146 \overline{)27.0024} (.1849 \\
 \underline{146} \\
 1240 \\
 \underline{1168} \\
 722 \\
 \underline{584} \\
 1384 \\
 \underline{1314} \\
 70 \\
 \hline
 \hline
 \end{array}$$

$$\begin{array}{r}
 3 \\
 11027 \overline{)472.0000} (4595 \\
 \underline{4108} \\
 6120 \\
 \underline{5135} \\
 9850 \\
 \underline{9243} \\
 6070 \\
 \underline{5135} \\
 935 \\
 \hline
 \hline
 \end{array}$$

In the second example there are 4 decimal places in the dividend, and none in the divisor, therefore there must be 4 places of decimals in the quotient.

And in the third example as the number of decimal places in the divisor and dividend are equal, being 4 in each, there must be no decimals in the quotient.

Every other rule in arithmetic may be performed by decimal fractions as well as by whole numbers, as the same rule serves for both; thus in the rule of three direct in decimals, the second and third numbers are multiplied together for a dividend, and the first number is the divisor as in whole numbers; and the like may be observed of every other rule.

CHAP. V.

OF STOCK-HOLDING.

SECT. I.

THE NATURE OF THE PUBLIC STOCKS.

THE public stocks of Great Britain is an invention in imitation of the state of Florence, which, in the year 1344, owing 60,000*l.* and being unable to pay the principal, converted the debt into a public stock, transferable from one individual to another, with interest at 5 per cent.: these public stocks were called a *bank* or *mount*, and the prices varied according to the state of public affairs.

In imitation of this, soon after the revolution in Great Britain, the expenses of settling the new establishment, carrying on long wars on the Continent for reducing the French monarchy, securing the Dutch barrier, settling the Spanish succession, &c. increased to such a degree, that it was thought advisable to draw upon posterity, rather than reduce the state by an immediate payment.

It was therefore the policy of ministers of that day to raise a public stock with transferable shares, thus converting the debt into a new species of property, for the security of which Parliament pledged their faith; and this example has been closely followed ever since.

This system laid the foundation of what is called the national debt, to which every stock-holder is a creditor.

Thus

Thus the property in the kingdom in idea has been greatly increased, compared to that of former times; but if we inquire into the nature of this property, we shall find it exists only in name, in paper, and public faith; but that is sufficient for the public creditor to rely upon, for the pledge is nothing less than the land, the trade, and personal industry of the subject, from which sources the money must arise to pay the interest; in these, therefore, the property of the public creditors does really exist, consequently the value of the land, the trade, and industry of the subject, are diminished in-proportion as the debt increases.

To demonstrate this, if a debtor owes 100*l.* the creditor has a demand upon him to that amount, and though he may not call upon him for the principal, yet so much of the debtor's estate is always transferable to the creditor; and the same principle holds good with regard to the public creditor.

Therefore the property of every stockholder consists in a certain portion of the national debt.

To pay the interest, and, if required, the principal of this debt, certain taxes are appropriated: these taxes were originally separate and distinct funds, and each of them was a security for a certain sum advanced; they were afterwards united together in three capital funds, which are the principal funds of any account, called the *general fund*, the *aggregate fund*, and *South Sea fund*; and the funds thus united together are become mutual securities for each other, and the whole produce of them liable to pay the interest charged upon each.

The surplusses of these three funds, over and above the interest charged upon them, are directed to be at the disposal of the Parliament, and are usually called the *sinking fund*, because originally designed to reduce the national debt: to this have lately been added several other duties granted in later years.

This sinking fund, with the other duties added thereto, is the last resort of the nation, on which must depend all our

hope.

hopes of discharging our incumbrances. But by a statute passed in the 26th year of Geo. III. Parliament vested one million annually in commissioners for the reduction of the debt: in which act several provisions are made to prevent this fund from being appropriated to any other purpose at any future time. In consequence of this act 8,200,000*l.* had been redeemed in February 1792.

Since which period imposts have been applied to the same purpose, particularly the redemption of the land-tax; and several new loans have been opened on different conditions.

SECT. II.

SIMPLE AND COMPOUND INTEREST.

SIMPLE interest is the allowance made by the borrower of any sum of money to the lender, according to a certain rate per cent. per annum, which is limited by law to 5 per cent. per annum, that is, 5*l.* for the use of 100*l.* for one year, and 10*l.* for the use of it for two years, or 10*l.* for the use of 200*l.* for one year.

Principal is the sum of money lent.

Interest or *rate* is the sum agreed upon for the use of the principal.

Amount is the principal and interest added together *.

* In the courts of law interest for money lent is computed for years, quarters of years, and single days. But for the interest on public bonds belonging to the South Sea and India Companies, and the securities of the Bank of England, the time is generally computed in calendar months and days, and on Exchequer bills in quarters of a year and days.

Simple Interest.

Rule. Questions that concern interest are resolved by the rule of three; thus—

Multiply the principal by the rate of interest, and divide the product by 100, and the quotient will be the interest for one year.

Then multiply the interest for one year by the number of years given, and the product is the interest for that time.

But when parts of a year are given, then find what aliquot part or parts of a year such part is, and take an equivalent part of the interest for one year; or find the interest for such part by the rule of three direct.

Example 1. What is the interest of 245*l.* 10*s.* for 2 years 4 months and 50 days, at $3\frac{1}{2}$ per cent. per annum?

<i>£.</i>	<i>s.</i>	<i>d.</i>	<i>Days.</i>	<i>£.</i>	<i>s.</i>	<i>d.</i>	<i>Days.</i>
245	10			8	11	10	
			3½ per cent.				5
	736	10			42	19	2
		122	15				10
1,00)	8.59	5	(8 <i>l.</i>	365)	429	11	8(1 <i>l.</i>
	59				365		
	20				64		
1,00)	11.85	(11 <i>s.</i>			20		
	85			365)	1291	(3 <i>s.</i>	
	12				1095		
1,00)	10.20	(10 <i>d.</i>			196		
	20				12		
	4			365)	2360	(6 <i>d.</i>	
	80				2160		
					170		
					4		
<i>£.</i>	<i>s.</i>	<i>d.</i>		365)	650	(1 <i>s.</i>	
8	11	10	1 year's interest		365		
		2					
17	3	8	2 years interest		315		
2	17	3½	4 months interest				
1	3	6½	50 days interest				
21	4	5½					

In this example I first multiply the 245*l.* 10*s.* by the rate per cent. $3\frac{1}{2}$, and the product divided by 100 quotes 8*l.* 11*s.* 10*d.* for the interest of one year. Then by the rule of three direct I find the interest for 50 days, which is 1*l.* 3*s.* 6*d.*

And by the rule of practice I find the interest for 2 years 17*l.* 3*s.* 8*d.* and also the interest for 4 months, or one third of the year, 2*l.* 17*s.* $3\frac{1}{4}$ *d.* which 3 added together, the sum is 21*l.* 4*s.* $5\frac{1}{2}$ *d.* for the answer.

Questions concerning interest also are resolved by the double rule of three; thus the foregoing example may be wrought, saying, if 100*l.* gain 3*l.* 10*s.* per cent. in 1 year, what will 245*l.* 10*s.* gain in 2 years 4 months and 50 days?

Qu. 2. What is the interest of 461*l.* for 1 year, at 4 per cent. per annum?—*Ans.* 18*l.* 8*s.* $9\frac{1}{2}$ *d.*

Qu. 3. What is the amount of 345*l.* for 3 years, at $4\frac{1}{2}$ per cent. per annum?—*Ans.* 391*l.* 11*s.* 6*d.*

Qu. 4. What is the interest of 411*l.* 10*s.* for 3 months, at 4 per cent. per annum?—*Ans.* 4*l.* 2*s.* $3\frac{1}{2}$ *d.*

Qu. 5. What is the amount of 241*l.* for $2\frac{1}{2}$ years, at $4\frac{1}{2}$ per cent. per annum?—*Answer* 269*l.* 12*s.* 4*d.*

Qu. 6. What is the amount of 400 guineas for 4 years 7 months and 25 days, at $4\frac{1}{2}$ per cent. per annum?—*Answer* 507*l.* 18*s.* 9*d.*

Qu. 7. What is the interest due upon an Exchequer bill of 900*l.* at $3\frac{1}{2}$ per cent. per annum, for $2\frac{1}{2}$ years and 67 days?—*Answer* 99*l.* 9*s.* $1\frac{1}{2}$ *d.*

A TABLE

SHOWING

The Number of Days from any Day of one Month to the same Day of another Month.

From any day of												
To	Jan.	Feb.	Mar.	Apr.	May	June	July	Aug.	Sept.	Oct.	Nov.	Dec.
Jan.	365	334	306	275	245	214	184	153	122	92	61	31
Feb.	31	365	337	306	276	245	215	184	153	123	92	62
Mar.	59	28	365	334	304	273	243	212	181	151	120	90
Apr.	90	59	31	365	335	304	274	243	212	182	151	121
May	120	89	61	30	365	334	304	273	242	212	181	151
June	151	120	92	61	31	365	335	304	273	243	212	182
July	181	150	122	91	61	30	365	334	303	273	242	212
Aug.	212	181	153	122	92	61	31	365	334	304	273	243
Sept.	243	212	184	153	123	92	62	31	365	335	304	274
Oct.	273	242	214	183	153	122	92	62	31	365	335	304
Nov.	304	273	245	214	184	153	123	92	61	31	365	335
Dec.	334	303	275	244	214	183	153	122	91	61	30	365

The use of this table is, to find how many days are contained from any day of one month to the same day of any other month: thus, to find how many days there are from the 10th of March to the 10th of October following, I look in that column which has March at the top, and in that line which has October on the left hand, and in the junction of these two lines I find 214 for the answer.

Compound Interest.

Compound interest is that interest which arises from the principal and a former interest taken together as a principal: thus, if 100*l.* be put out to interest for any number of years

M in a

at

at simple and compound interest, the interest as it arises is added to the principal, and their sum becomes a new principal; and the interest due upon this is called compound interest.

Rule. Find the amount of the given principal and the first payment of interest, by the rule of simple interest. Then this amount will be the principal for the second payment of interest, which second payment of interest being added to the last amount, forms a third principal, and so on through all the payments to the last, always accounting the last amount the principal for the next interest.

Example 1. What is the amount of 540*l.* 10*s.* for 5 years at 5 per cent. per annum, compound interest?

<i>l.</i>	<i>s.</i>	<i>d.</i>	
540	10	0	first year's principal
27	0	6	first year's interest
567	10	6	second year's principal
28	7	6½	second year's interest
595	18	0½	third year's principal
29	15	10½	third year's interest
625	13	11	fourth year's principal
31	5	8½	fourth year's interest
656	19	7½	fifth year's principal
32	16	11½	fifth year's interest
689	16	7	the Answer.

After finding the first year's interest by the foregoing rule of simple interest, I add it to the principal; thus I have 567*l.* 10*s.* 6*d.* for the second year's principal, the interest of which is 28*l.* 7*s.* 6½*d.* and which added to its principal gives 595*l.* 18*s.* 0½*d.* for the third year's principal, to which is also added its interest 29*l.* 15*s.* 10½*d.* and the sum 625*l.* 13*s.* 11*d.* forms a principal for the fourth year, which is also added to its interest 31*l.* 5*s.* 8½*d.* and the sum 656*l.* 19*s.* 7½*d.* is the principal for the fifth year, which added to the last interest gives 689*l.* 16*s.* 7*d.* for the answer required.

Q. 2. What is the amount of $\text{£}100$ at 5 per cent. per annum, compounded interest, for 10 years?

Q. 3. What is the compound amount of $\text{£}100$ at 5 per cent. per annum, for 10 years?

Interest by Decimal Fractions.

Simple interest is also sought by decimal fractions as well as by whole numbers.

Rule. Multiply the principal, the rate, and the time together, and the product will be the interest required.

Ratio is the simple interest of $\text{£}1$ for 1 year at the rate agreed upon; thus the ratio

1 per cent. is $\text{£}0$		
$\frac{1}{2}$	_____	$\text{£}0$
1	_____	$\text{£}0$
$\frac{3}{4}$	_____	$\text{£}0$
3	_____	$\text{£}0$
$3\frac{1}{4}$	_____	$\text{£}0$
4	_____	$\text{£}0$
$4\frac{1}{2}$	_____	$\text{£}0$
5	_____	$\text{£}0$
$5\frac{1}{2}$	_____	$\text{£}0$
6	_____	$\text{£}0$
$6\frac{1}{2}$	_____	$\text{£}0$
7	_____	$\text{£}0$
$7\frac{1}{2}$	_____	$\text{£}0$
8	_____	$\text{£}0$
$8\frac{1}{2}$	_____	$\text{£}0$
9	_____	$\text{£}0$
$9\frac{1}{2}$	_____	$\text{£}0$
10	_____	$\text{£}1$

The fraction opposite each rate per cent. is the decimal of a pound, equal to the interest of $\text{£}1$ for a year; it also shows the decimal parts the interest is of the principal.

Thus, $.05$ of a pound is the interest of a pound for one year at 5 per cent. being equal to $1s.$; it also shows that the interest

interest of any principal for one year, at 5 per cent. is .05 parts of the principal.

Example 1. What is the interest of 846*l.* 15*s.* for 3 years, at 4 per cent. per annum, simple interest?

	Principal	846.75
	Ratio	.04
		<u>33.8700</u>
Here for the 15 <i>s.</i> in the principal, I take the equivalent fraction .75, as taught in Case 4 of reduction of decimal fractions; then multiplying the principal by the ratio, and the product by the number of years, I have 101.6100 <i>l.</i> for the interest.	Number of years	<u>3</u>
	Pounds	101.6100
		<u>20</u>
	Shillings	12.2000
		<u>12</u>
	Pence	2.4000
		<u>4</u>
	Farthings	<u>1.6000</u>

Then I find the value of the decimal .6100 in shillings and pence, by the rule in Case 2, of reduction of decimals, and it produces 12*s.* 2½*d.*; thus the answer is 101*l.* 12*s.* 2½*d.*

Qu. 2. What is the simple interest of 254*l.* 17*s.* 6*d.* for 5 years, at 4 per cent. per annum?—*Answer* 50*l.* 19*s.* 6*d.*

Qu. 3. What is the simple interest of 1760*l.* for 2½ years, at 3½ per cent. per annum?—*Ans.* 154*l.*

Compound interest is also wrought by decimal fractions.

Rule for which is, to find the amount of 1*l.* for 1 year, at the given rate per cent. Then raise the amount to such a power as is expressed by the number of years. Multiply this power by the principal given, and the product will be the amount, from which if the principal is extracted the remainder will be the interest.

Example 1. What is the compound interest of 520*l.* for 4 years, at 4 per cent. per annum?

1.04	
1.04	
<u>4 16</u>	
1 04	
<u>1.08 16</u>	the second power of 1.04
1.04	
<u>432 64</u>	
1 0816	
<u>1.1248 64</u>	the third power of 1.04
1.04	
<u>4 4994 56</u>	
1 12 4864	
<u>1.16 9858 56</u>	the fourth power of 1.04
520	the principal
<u>2 33 9717 120</u>	
58 49 2928 0	
<u>608.3 2645 120</u>	the whole amount
520	
<u>88.3 2645 120</u>	<i>Answer</i> , or interest required.

In this example I raise the amount of 1*l.* for 1 year, 1.04, to the fourth power, and multiplying this power by the 520*l.* principal, the product is 608.32645120 for the whole amount of principal and interest, and subtracting the principal therefrom, the remainder is 88.32645120 or 88*l.* 6*s.* 6½*d.*

Qn. 2. What is the amount of 450*l.* for 5 years, at 4 per cent. per annum?—*Ans.* 547*l.* 9*s.* 10½*d.*

Most writers on compound interest give tables to discover at once the compound interest of any sum for any certain time, at different rates per cent.; I shall, therefore, give one to shew the manner in which they are constructed.

A TABLE

OF

*Compound Interest, at 3½ per Cent. per Annum, calculated
without the Loss of a single Fraction,
for Twenty Years.*

Year.	L.
1st	.035
2d	.036225
3d	.037492875
4th	.038805125625
5th	.04016330501875
6th	.041569020697640625
7th	.043021936422058046875
8th	.044529774196830078515625
9th	.046088316293719131263671875
10th	.047701407363999300857900390625
11th	.049370956621739276387926904296875
12th	.051098940103500151061505145947265625
13th	.052887403007122656348656998055419921875
14th	.054738462112371949320859992987359619140625
15th	.056644308286104967547096092741917205810546875
16th	.0586037209076125641411238245987884308013916015625
17th	.0606289511393997038860631584597460258794403076171875
18th	.0627213644292786935220753690058371367852207183819890625
19th	.06488195515270798670859120297553306486698150634767625
20th	.067287546107540684681851871632778868527480640506744384765625

The foregoing table shews the amount of 1*l.* principal, at $3\frac{1}{2}$ per cent. per annum, compound interest, for any number of years not exceeding 20: thus, to find the amount of 1*l.* for 15 years, I seek in the table in the first line of the fifteenth year, and the answer is £1.675348, &c. equal to 1*l.* 13*s.* 6*d.*

Though the table be formed only for 1*l.* principal, yet questions that concern 10*l.*, 100*l.*, 1000*l.*, or any multiple of 10, may be readily answered by it, by only removing the decimal point as many places further towards the right hand as the pounds principal contain cyphers: thus, to find the amount of 10*l.* for 15 years at $3\frac{1}{2}$ per cent. compound interest, I place the decimal point one place farther towards the right hand, and the answer is £16.753, &c. or 16*l.* 15*s.* 0*d.* and for 100*l.* the answer is £167.5348, &c. or 167*l.* 10*s.* 8*d.* by inspection, &c.

The Construction of the Table.

The foregoing table, and all others of the same nature, are formed by multiplying the principal by the ratio, and the product gives the simple interest for one year, which added to the principal, the sum is the amount or principal for the next year; which is to be again multiplied by the ratio, and the product again added to the last principal for a new principal, &c.: thus, in the foregoing table, I multiply the principal 1*l.* by .035, the ratio at $3\frac{1}{2}$ per cent. and the product is .035 for the interest for one year, which added to its principal 1*l.* is 1.035 for the second year's principal, which is again multiplied by the ratio, and the product is .036225, which is again added to its principal 1.035, and the sum is 1.071225 for the principal for the third year, &c.

In forming tables of compound interest, it is sufficient, in general, if there be only three places of decimals, which will give the answer to a farthing.

SECT. III.

OF DISCOUNT OR REVERSION.

DISCOUNT is that allowance which is made for the payment of any sum before it is due, according to a certain rate per cent.

Discount may be wrought either by whole numbers or decimal fractions.

The present worth of any sum which is due some time hence is that sum which if put to interest for that time, at the same rate per cent. will amount to the sum or debt then due, whether the interest be simple or compound. Discount is wrought by the rule of three.

Rule. As the amount of 100*l.* at the given rate and time is to 100*l.* so is the sum given to the present worth. Then subtract the present worth from the given sum, and the remainder is the discount required.

Or, as the amount of 100*l.* at the given rate and time, is to the interest of 100*l.* for that time, so is the given sum to the discount required:

Example 1. What is the discount of 573*l.* 15*s.* due 3 years hence, at $4\frac{1}{2}$ per cent. per annum, simple interest?

$$\begin{array}{r}
 \text{£. s.} \\
 113 \ 10 \\
 \underline{20} \\
 2270
 \end{array}
 \qquad
 \begin{array}{r}
 \text{£. s.} \\
 4 \ 10 \\
 \underline{3} \\
 13 \ 10 \\
 \underline{20} \\
 270
 \end{array}
 \qquad
 \begin{array}{r}
 \text{£. s.} \\
 573 \ 15 \\
 \underline{20} \\
 11479 \\
 \underline{270} \\
 803250 \\
 \underline{22950} \ 20) \\
 227,0)3098250(1364 \\
 \underline{227} \qquad \underline{68 \ 4} \\
 828 \\
 \underline{681} \\
 1472 \\
 \underline{1362} \\
 1105 \\
 \underline{908} \\
 197 \\
 \underline{12} \\
 227)2364(10 \\
 \underline{227} \\
 94 \\
 \underline{4} \\
 227)376(1 \\
 \underline{227} \\
 149
 \end{array}$$

Answer 68l. 4s. 10½d.

In this example, the amount of 100l. for 3 years, at 4½ per cent. is made the first number in the rule of three direct, the interest for the 100l. for that time, at that rate per cent. is the second number; and the given sum, whose discount is required, is the third number; and the fourth number is the answer.—68l. 4s. 10½d.

Discount by Decimals.

Rule. As the amount of 1l. for the given time, at the given rate, is to 1l. so is the interest of the debt for the same time to the discount required.

Example 1. What is the discount of 573l. 15s. due 3 years hence, at 4½ per cent. per annum?

N B 2

As

Questions of this nature are resolved by the rule of practice: thus, in the foregoing example, I take the principal sum of 2220*l.* 10*s.* for the price of the Bank stock, at 100*l.* per cent. and for the other 5 per cent. in the price of the stocks, I take a twentieth part of the above sum 111*l.* 0*s.* 6*d.* and for the remaining $\frac{1}{2}$ per cent. I take a tenth part of the 5*l.* line, which is 1*l.* 2*s.* 0*d.*; and these three lines added together is the answer.

Qu. 2. What is the purchase of 1975*l.* 10*s.* India stock, at 113 $\frac{1}{2}$ per cent.?—*Ans.* 2249*l.* 12*s.*

Qu. 3. What is the purchase of 1633*l.* 4*s.* Bank annuities, at 89 $\frac{1}{2}$ per cent.?—*Ans.* 1459*l.* 13*s.* 5 $\frac{1}{2}$ *d.*

Qu. 4. What is the purchase of 4109*l.* 12*s.* in the 3 per cent. consolidated Bank annuities, at 55 $\frac{1}{2}$ per cent.?—*Ans.* 2265*l.* 8*s.* 3 $\frac{1}{2}$ *d.*

CHAP. VI.

OF MERCHANTS ACCOUNTS.

SECT. I.

OF BOOK-KEEPING THE ITALIAN WAY; OR, BY DOUBLE ENTRY.

THERE are several books used by mercantile men, of which the principal are the *waste-book*, the *journal*, and the *ledger*. These books are generally used by all men of business;

business; I shall therefore give an example of the use of each of them.

Of the Waste-Book.

This book, by some called the *day-book*, is used to note down whatever occurs in the way of trade, as buying, selling, paying, receiving, delivering, &c. &c. without omitting any one thing.

This book has one marginal line on the left hand, and three lines for pounds, shillings, and pence, on the right hand; the day of the month and the date of the year are inserted in the middle of the page.

Of the Journal.

Every thing which is written in the waste-book must be transcribed into the journal in the proper mercantile terms, and in a fairer hand; and every article is to be set down according to the order of time, without any intermission, to make the book of more validity in case of any controversy or dispute.

In this book every article bought or sold is distinguished by the name of debtor or creditor, and to this book recourse must be had for the particulars of any account.

In the journal the day of the month is also placed in the middle of the page; it is ruled with three lines, for pounds, shillings, and pence, as the waste-book is, and one, and sometimes two marginal lines on the left hand, to refer to the ledger.

Of the Ledger.

Every article is posted into this book from the journal, but in short, in one line; every left-hand page of this book is called the debtor page, and each right-hand page the

creditor.

creditor. Every debtor page must have a creditor page of the same number; thus, the articles or persons which are called the debtors (in the mercantile phrase) must be entered on a page of the same number with those called the creditors.

The day of the month, in this book, is set in a narrow column on the left hand; at the top of each page is the name, the place of residence, and the year of our Lord.

I shall give a few examples of the use of each of these books; but it must first be observed, that in the waste-book every article is written down in ordinary plain language, and entering the same articles into the journal is nothing more than the setting them down in the mercantile phrase, distinguishing each person or article by the name of debtor or creditor, for which the following rule must be strictly observed:

All things received, or the receiver, are debtors to the delivered, or the deliverer.

Examples of the Use of the Waste-Book.

April 2d, 1808.		£.	s.	d.
Bought of Henry Smith, of Cheapside, 100 yards of Irish linen, at 2s. 6d. per yard, to be paid in 3 months	—	12	10	0
Sold to John Jones, 20 yards of broad cloth, at 18s. per yard, to be paid in 6 months	—	36	0	0
Bought of William Williams, 1 cwt. of sugar, at 12. 18s. per cwt.	—	12	18	0
Bought of John Edwards, 60lb. of tea, at 5s. per lb.	—	3	0	0
		16	18	0

Journal Entry.

April 2d, 1808.		£.	s.	d.
1/2 Irish linen debtor to Henry Smith, 12l. 10s. for 100 yards, at 2s. 6d. per yard, to be paid in 3 months	—	12	10	0
3/4 John Jones debtor to broad cloth for 20 yards, at 18s. per yard, to pay in 6 months.	—	36	0	0

Lisbon

3	Lilbon sugar debtor to William Williams for	£.	s.	d.
	10 cwt. at 12. 18s. per cwt.	—	18	0
4	Green tea debtor to John Edwards for 60 lb.	—	—	—
	at 5s. per lb.	—	35	0

In the first of these examples I make Irish linen the debtor, as that is the article received, and Henry Smith creditor, as he is the deliverer.

In the second example John Jones is the debtor, as he is the receiver, and broad cloth the creditor. The same may be observed in all the other examples which follow.

Thus an experienced person may keep a journal without a waste-book, as they both import the same thing.

I shall, however, give a few more examples of each: and first the form of an inventory, as that is always taken down

Waste-Book.

London, January 1st, 1808.

An inventory of all the money, goods, and debts, belonging to me, A. B. of London, merchant: viz.

In cash	—	—	—	4269	15	0
In Canary wine	—	—	—	124	9	10
In French brandy	—	—	—	386	13	3
In Florence oil	—	—	—	224	0	8
In 1000 ells of Irish linen at 2s. 4d.	—	—	—	—	—	—
per ell	—	—	—	116	13	4
Due from John Smith, by note of hand	—	—	—	50	0	0
				6292	15	5

January 20, 1808.

Received from Benjamin Hughes, in Jamaica, 50 puncheons of rum, at 18s. per puncheon, to pay in 6 months

Vol. I. 00 January

January 30, 1808.		£.	s.	d.
Bought of Henry Jones, 20 yards of broad cloth, at 17s. 6d. per yard	£17 10			
Bought 20 yards of serge, at 3s. 6d. per yard, to pay in 6 months	3 10	21	0	0
February 3, 1808.				
Sold to Henry Brown 20 gallons of rum, at 2s. per gallon, in 3 months	—	8	0	0
February 4, 1808.				
Received of George Jones 100l. 10s. due upon bonds	—	100	10	0
March 4, 1808.				
Paid to Thomas Williamson 50l. due by note of my hand	—	50	0	0
March 5, 1808.				
Bought of John Edwards 2 pipes of Madeira, at 30s. per pipe, to pay in 3 months	—	60	0	0
March 6, 1808.				
Sold to Henry Williams 40 gallons of brandy, at 12s. per gallon, to pay in 6 months	—	24	0	0
March 6, 1808.				
Bought of John Thomas 12 puncheons of Highlands, at 15l. per puncheon, to be paid in 9 months	—	180	0	0
March 6, 1808.				
Bought 2 pieces of muslin, at 4l. per piece, for ready money	—	8	0	0

Journal.

Journal.

Inventory, &c. London, January 1, 1808.

$\frac{1}{2}$	Sundry accounts Dr. to stock	£6298 1 5	£.	1.	d.
	Cash	4250 5 0			
	Canary wine	1124 9 10			
	French brandy	526 13 3			
	Florence oil	224 0 0			
	1000 ells of Irish linen, at 2s. 4d. per ell	116 13 4			
	Note of hand of John Smith	50 0 0	6298	1	5
<hr/>					
	January 20, 1808.				
$\frac{2}{2}$	Rum Dr. to Benjamin Hughes, in Jamaica, for 50 puncheons, at 18l. per puncheon, to pay in 6 months		900	0	0
<hr/>					
	January 30, 1808.				
$\frac{1}{2}$	Sundries Dr. to Henry Jones, viz. Broad cloth, for 20 yards, at 17s. 6d. per yard	£17 10			
	Serge, for 20 yards, at 3s. 6d. per yard, to pay in 6 months	3 10	21	0	0
<hr/>					
	February 3, 1808.				
$\frac{1}{2}$	Henry Brown Dr. to rum, for 20 gallons, at 8s. per gallon, to pay in 3 months		8	0	0
<hr/>					
	February 4, 1808.				
$\frac{1}{2}$	Cash Dr. to George Jones, for 100l. 10s. due upon bonds		100	10	0
<hr/>					
	March 4, 1808.				
$\frac{1}{2}$	Thomas Williamson Dr. to cash, for 50l. due by note of hand		50	0	0
<hr/>					
	March 5, 1808.				
$\frac{1}{2}$	Madeira Dr. to John Edwards, for 2 pipes, at 30l. per pipe, to pay in 3 months		60	0	0
<hr/>					
	March 6, 1808.				
$\frac{1}{2}$	Henry Williams Dr. to brandy, for 40 gallons, at 12s. per gallon, to pay in 6 months		24	0	0
<hr/>					
	March 6, 1808.				
$\frac{1}{2}$	Hollands Dr. to John Thomas, for 12 puncheons, to pay in 6 months		18	0	0
<hr/>					
	March 6, 1808.				
$\frac{1}{2}$	Muslin Dr. to cash, for 2 pieces, at 4l. per piece		8	0	0

002

Ledger.

Ledger.

		(1)		(1)			
		Dr.		Cr.		Dr.	Cr.
1808	Cash						
Jan. 1	To sundry accounts					46292	1 5
Feb. 28	R To balance the net produce of my stock	36920	5 1			714	16 11
						7006	18 4
Jan. 1	Cash						
Feb. 2	To stock					506	7 0
	To sundry accounts received this month	201	10 0			101	2 6
						73844	5 6
						4451	15 0
Jan. 1	Canary						
Feb. 2	To stock, at 4s. 3s. 2d. per pipe						
	To cash, at 50s. per pipe					4500	0 0
	To profit and loss gained by this account					250	0 0
						900	0 0
						294	10 0
						1944	10 0

(a)

1808	Per contra	Cr. Ml	1808	Dr. Ml	1808	Dr. Ml		
Jan. 2	By John Green, at 44 ^d . per pound	6 10 0	Jan. 2	To stock, at 40 ^d . 10 ^d . 3 ^d . per pound	526 11 3	Jan. 2	To stock, at 21 ^d . 4 ^d . per ell	116 13 4
—	By John Thomas, at 43 ^d . 10 ^d .	3 04 10 0	—	—	39 16 9	—	—	20 16 8
—	—	564 10 0	—	—	—	—	—	—
Jan. 3	By John Jones, at 56 ^d . per jar	104 0 0	Jan. 3	To stock, at 21 ^d . 4 ^d . per jar	224 0 0	Jan. 3	To stock, at 21 ^d . 4 ^d . per jar	224 0 0
—	By John Hawkins, at 30 ^d . per jar	280 0 0	—	—	—	—	—	—
—	—	224 0 0	—	—	—	—	—	—
Jan. 7	By John Edwards, at 31 ^d . per ell	71 0 0	Jan. 7	To stock, at 21 ^d . 4 ^d . per ell	116 13 4	Jan. 7	To stock, at 21 ^d . 4 ^d . per ell	116 13 4
—	By John Thomas, at 21 ^d . 6 ^d . per ell	62 10 0	—	—	20 16 8	—	—	20 16 8
—	—	137 10 0	—	—	—	—	—	—
Jan. 5	By cash received in full	20 0 0	Jan. 5	By cash received in full	20 0 0	Jan. 5	By cash received in full	20 0 0

(a)

French brandy	Dr.	Per. Ml.
To stock, at 40 ^d . 10 ^d . 3 ^d . per pound		— 11 3 1 A
To profit and loss		—
		—
		—
		—
		—
		—
		—
		—
		—
		—
		—
		—
		—
		—
		—
		—
		—
		—
		—
		—
		—
		—
		—
		—
		—
		—
		—
		—
		—
		—
		—
		—
		—
		—
		—
		—
		—
		—
		—
		—
		—
		—
		—
		—
		—
		—
		—
		—
		—
		—
		—
		—
		—
		—
		—
		—
		—
		—
		—
		—
		—
		—
		—
		—
		—
		—
		—
		—
		—
		—
		—
		—
		—
		—
		—
		—
		—
		—
		—
		—
		—
		—
		—
		—
		—
		—
		—
		—
		—
		—
		—
		—
		—
		—
		—
		—
		—
		—
		—
		—
		—
		—
		—
		—
		—
		—
		—
		—
		—
		—
		—
		—
		—
		—
		—
		—
		—
		—
		—
		—
		—
		—
		—
		—
		—
		—
		—
		—
		—
		—
		—
		—
		—
		—
		—
		—
		—
		—
		—
		—
		—
		—
		—
		—
		—
		—
		—
		—
		—
		—
		—
		—
		—
		—
		—
		—
		—
		—
		—
		—
		—
		—
		—
		—
		—
		—
		—
		—
		—
		—
		—
		—
		—
		—
		—
		—
		—
		—
		—
		—
		—
		—
		—
		—
		—
		—
		—
		—
		—
		—
		—
		—
		—
		—
		—
		—
		—
		—
		—
		—
		—
		—
		—
		—
		—
		—
		—
		—
		—
		—
		—
		—
		—
		—
		—
		—
		—
		—
		—
		—
		—
		—
		—
		—
		—
		—
		—
		—
		—
		—
		—
		—
		—
		—
		—
		—
		—
		—
		—
		—
		—
		—
		—
		—
		—
		—
		—
		—
		—
		—
		—
		—
		—
		—
		—
		—
		—
		—
		—
		—
		—
		—
		—
		—
		—
		—
		—
		—
		—
		—
		—
		—
		—
		—
		—
		—
		—
		—
		—
		—
		—
		—
		—
		—
		—
		—
		—
		—
		—
		—
		—
		—
		—
		—
		—
		—
		—
		—
		—
		—
		—
		—
		—
		—
		—
		—
		—
		—
		—
		—
		—
		—
		—
		—
		—
		—
		—
		—
		—
		—
		—
		—
		—
		—
		—
		—
		—
		—
		—
		—
		—
		—
		—
		—
		—
		—
		—
		—
		—
		—
		—
		—
		—
		—
		—
		—
		—
		—
		—
		—
		—
		—
		—
	</	

Dr.		Cr.	
	Paid		Paid
Jan. 20	To Benjamin Hughes, at 22d. per punchoon, to pay in 6 months	5	900 0 0
— 26	To profit and loss gained by this account		70 0 0
			970 0 0
Jan. 30	Sundries		
	To Henry Jones, viz.		
	To broad cloth, 20 yds. at 17s. 6d. per yd.	8	1710 0
	To serge, 20 yards, at 3s. 6d. per yard,		210 0
	to pay in 6 months		210 0
Feb. 8	To profit and loss gained by this account	7	2810 0
			2810 0
Feb. 3	Henry Brown		
	To rum, at 8s. per gallon, to pay in 3 months	3	24 0 0
Feb. 4	Cash		
	To George Jones, due on bond	4	100 10 0
			100 10 0

(4)

1808	(4)		1808	(4)	
	Thomas Williamson	Dr.		Per contra	Cr.
Mar. 4	7 To cash, on note of hand	—	Mar. 7	2 By Thomas Williamson, on note of hand	50
Mar. 5	7 To John Edwards, 2 pipes	—	Mar. 3	3 By cash, at 18l. per bhd.	58
—	7 To profit and loss gained by this account	—	—	4 By John Evans, at 18l. 10s. per bhd.	37
			—	3 By balance, remaining unpaid	15
					80
Mar. 6	8 To brandy, at 12s. per gallon	140	Mar. 6	2 By Henry Williams, at 12s. per gallon	24
Mar. 6	7 To John Thomas, at 12l. per purchase	—	Mar. 6	2 By John Abraham, at 12l. per purchase	18
Mar. 6	7 To cash	—	Mar. 6	2 By John Thomas, in part of Holland	20
Mar. 6	7 To cash, at 4l. per piece	—	Mar. 6	2 By muslin unpaid, at 4l. per piece	20

Every article in the journal is posted into the ledger, and those persons or things to which the other party is said to be debtor are entered on the debtor side, or left-hand page, with the word *To*; and those persons or things belonging to the same account, to which the other party is said to be creditor, are entered on the right-hand side, or creditor page, with the word *By*, as may be seen in the foregoing examples.

The figures in the narrow column in the ledger after the day of the month, both on the debtor and creditor side, shew what pages in the journal the entries are taken from; and the letter R in these columns signifies the remainder on the balance of that particular account. Those figures in the narrow column on the debtor side, before the money, shew in what page of the ledger that entry stands on the creditor side; and the figures in the like column on the creditor side shew what page of the ledger the entry stands in on the debtor side.

Each debtor page of the ledger must be numbered the same as that part of the ledger where the creditors to the same accounts are entered.

The small figures at the beginning of each journal entry shew in what pages of the ledger such entry is posted; the upper figure shews the debtor side, and the lower figure the creditor.

These three books are or may be used by every man of business, in any line whatever, as well as by the merchant.

There are several other books used by merchants, as follows:

First, the *debt-book*, in which is entered the day whereon all sums become due, whether to be paid or to be received, by bills of exchange, notes, bonds, &c. that by comparing receipts and payments together, provision may be made in time.

Second, the *cash-book*, which is ruled and pages like the ledger, wherein all receipts of money are entered on the left-hand folio, and all payments on the right, with the day of the month (the year being set on the top of the page), for whose account the money was received or paid; and the total debit or credit on each side is usually once a month posted into the ledger to the account of cash therein, with references in the cash-book to the several folios in the ledger.

Third, the *invoice-book*, or book of factories, wherein are copied all invoices of goods shipped, and also of goods received from abroad, which must always be entered on the left-hand side, leaving the right-hand side blank; and on the advice of the sale of goods which are sent abroad, and also on the sale of goods received from abroad; which are to be entered on the blank or right side. The use of this book is, to save the journal from any errors, which would be unavoidable in taking invoices of the several goods received, sent, or sold.

Fourth, the *receipt-book*, wherein are given receipts for money paid: the receipt to be given by the receiver; for whose account or use, or for what it is received; and the day of the month, and date of the year.

Fifth, the *book of charges of merchandise*, wherein are entered all the charges and expenses of any goods bought, sold, received, or shipped; such as portage, wharfage, warehouse-room. &c. &c.; the whole account is once a month transferred into the cash-book, on the creditor side, with references to the book of charges; and the same in the debtor side on the same account in the ledger.

Sixth, the *book of household expenses*, wherein are entered the charges of what is expended in housekeeping, such as apparel, house-rent, servants' wages, pocket expenses, vic-tuals, &c. &c.: this is once a month summed up, and carried to the cash-book.

Seventh, the *letter-book*, which contains copies of all letters sent or received from abroad, and also inland letters.

Lastly, the *note* or *memorandum book*, wherein are inserted affairs that occur, for the help of memory, and of great use where there is a multiplicity of business.

General Rules.

The principal rule in book-keeping has already been delivered, in the beginning of this section, viz. To make all things received or the receiver debtor to the delivered, or the deliverers. But I shall add a few more rules, which may serve to exemplify the foregoing rule more fully.

First, *Stock is debtor to profit and loss* for all money received and nothing given for it; as the profit of every commodity in the trade, legacies left, money received with an apprentice, &c. &c.

Per contra creditor, by profit and loss, for all money paid and nothing received for it; as the discount of money received before due, loss of goods untured, loss by accidents, abatement by composition, household expenses, gifts, &c. &c.

Secondly, for money received make cash debtor to the party that paid it, and the party creditor by cash.

Thirdly, for money paid make the receiver debtor to cash, and cash creditor by the party.

Fourthly, for goods bought for ready money make the goods debtor to cash, and cash creditor by the goods.

Fifthly, for goods sold for ready money make cash debtor to the goods, and goods creditor by cash.

Sixthly, goods bought on credit, make the goods debtor to the seller, and the seller creditor by the goods.

Seventhly, goods sold upon credit just the contrary; that is, the person who buys them must be debtor to the goods, and the goods creditor by the party.

Eighthly, for goods bought part for ready money and the rest on credit; first make the goods debtor to the party for the whole; secondly, make the party debtor to cash for the money paid him in part of these goods, and cash creditor by the party.

Ninthly, for goods sold part for ready money and the rest on credit; first make the party debtor to the goods for the whole, and the goods creditor; secondly, make cash debtor to the party for the money received of him in part of these goods, and the party creditor.

Tenthly, when discount is allowed for money paid before it is due, make the party who receives the money debtor to cash for so much as is paid him, and stock debtor to profit and loss for the discount.

To remove an Account full written to another Folio.

First, if the debtor side be more than the creditor, make the old account creditor by the new; but if the creditor side be more than the debtor, make the old account debtor to the new, and the new account creditor by the old for the remaining sum.

Secondly, in an account of company, wherein there is placed more received of another than his stock amounts to, as much must be added to the debtor side as there is placed on the creditor side, in order that in the new account there may be as much debit as was put in, and as much credit as was received.

Thirdly, in all accounts of merchandise, the gain or loss must be entered before the old account is made creditor by the new, and the new debtor to the old.

To balance or clear an Account when full written.

Find the sum of the debtor and creditor side, and the difference place to its opposite: thus, if the creditor side exceed the debtor, then the line is to be written in the old account to balance on the debtor side, to answer the line on the creditor side in the new account.

How to balance the Account at the Year's End, and thereby to know the State of Affairs.

First, even the account of cash, and bear the nett remainder to balance debtor.

Secondly, find the value of all goods bought and sold; and for goods bought which remain unsold, value them at prime cost, and bear the nett remainder to balance debtor.

Thirdly, find what all the goods cost which are bought, and also how much they were sold for, and bear the nett gain or loss to the account of profit and loss.

Fourthly, even all personal accounts with your debtors and creditors in the order in which they are written down, and bear the nett remainder to balance, &c.

Fifthly, even all voyages and factors accounts wherein there is either gain or loss, and bear the nett gain or loss to the account of profit and loss, and the goods unsold to balance.

Sixthly, even the account of profit and loss, and bear the nett remainder to stock.

Seventhly, even the stock, and bear the nett remainder to balance creditor.

Lastly, find the sum of both the debtor and creditor sides of the balance, and if they be both alike the account has been kept truly, otherwise not.

If the debtor and creditor sides of the balance be not both alike there is an error, which must be found by pricking over the books again, to see whether every debtor and creditor is entered in the ledger *.

* Pricking over the books is comparing every article of the journal with the same in the ledger, and marking it thus 7, and on the second examination thus 1, &c.

The Form of an Invoice.

March 6, 1808.

Invoice of 100qrs of wheat in sacks, shipped on board the good ship Neptune, William Browne master, and consigned to Henry Jones of Jamaica, for the account and risk of John Dalton and Thomas Edwards of London, merchants, marked and numbered as per margin :

A D W B O No. 1 to 100	100qrs. of wheat, at 12s. per quarter		60	0	0
	200 sacks, at 1s. each	—	10	0	0
	Meterage	—	2	0	0
	Porterage, at 1s. 6d. per quarter	—	7	10	0
	Cartage	—	5	18	4
	Lighterage	—	3	0	0
	Freight, at 2s. per quarter	—	10	0	0
			98	8	4
	The commission on 98l. 8s. 4d. at 1 per cent.		0	19	0
			<u>108</u>	<u>7</u>	<u>4</u>

The Form of a Bill of Lading.

Shipped by the grace of God, in good order and well conditioned, by Henry Williams of London, merchant, in and upon the good ship called the Mercury of London, whereof is master under God for this present voyage John Dickson of London, mariner, and now riding at anchor in the port of London, and by God's grace bound for Antigua; so say,

One bale of Irish linen, and one bale of men's shoes, and five hundred pair of silk stockings, No. 3, 4, 7. contents, &c. as per invoice, being marked and numbered as per margin, and are to be delivered in the like good order at the aforesaid place (Antigua), the danger of the sea only excepted, unto Mr. Edward Brown, merchant there, or to his assigns, he or they paying freight for the said goods, three pence of eight per cent. with primage and average accustomed. In witness whereof, the master or

purser of the said ship hath affirmed to three bills of lading, all of this tenour and date, one of which three bills being accomplished the other two to stand void; and so God send the good ship to her desired port in safety. Amen.

Dated in London, the 6th of March 1808, inside and contents unknown to
John Dickson.

An Account of a Sale.

London, March 6th, 1808.

Account of a sale of two thousand four hundred yards of muslin, two thousand two hundred yards of dimity, two hogsheds of rum, five cwt. of sugar, and five hundred pair of cotton stockings, received from on board the ship Maria, Captain Thomas Jbass commander, for account of Edward Wills of Jamaica, merchant, is debtor

	£.	s.	d.	£.	s.	d.
To portage of ditto	—	1	2	6		
To commission of sales	—	15	2	2		
To storage, at $\frac{1}{2}$ per cwt.	—	6	10	0		
				22	14	8
To the nett product, bad debts excepted	—	3	10	12	0	
				333	6	8
Per contra						
By two thousand four hundred yards of muslin, at 1s. per yard				120	0	0
By two thousand two hundred yards of dimity, sold Henry Brett, at 1s. 8d. per yard				128	6	8
By two hogsheds of rum, at 20s. per hoghead				40	0	0
By Henry Jones, for 5 cwt. of sugar, at 17. 10s. per cwt.				7	10	0
By Henry Williams, for five hundred pair of cotton stockings, at 1s. 6d. per pair				37	10	0
				333	6	8

Errors excepted, London, 6th March 1808,

Per John Smith.

Most

Most writers on merchants accounts give a variety of examples and rules to inform the learner how to enter the several articles according to debitor and creditor; see the foregoing examples, and a strict attention to the general rules before delivered, will be sufficient for any ordinary capacity.

Concerning the Exporting and Importing of Goods.

Goods to be exported must have a bill of entry, in the following form:—

In the Maria, Captain Thomas Jones, for Jamaica.

For export.

Two thousand four hundred yards of muslin.

Two thousand two hundred yards of dimity.

Two hogheads of rum.

Five cwt. of sugar.

Five hundred pair of cotton stockings.

There must be seven of these bills, one of which must be in words at length, the others may be in figures; these bills are entered into books by the Clerks of the Customs House. When some goods pay custom and others do not, there must be two entries made, one for those which pay custom and the other for those which do not pay.

Importing Goods.

When goods are to be landed from any ship abroad, the entry-book of the Customs House must be searched, to find the names of the ship and the captain, as also the names of those that are to attend the landing of the goods, &c.

The Form of a Bill of Entry Inwards.

In the Maria, Captain Thomas Jones, from Jamaica.

Fifty hogheads of sugar.

Twenty bags of cotton.

For import.



Forty hogheads of rum.

Twenty bales of muslin, &c.

Of these bills of entry inwards there must be eight; one of which must be expressed in words at length; this must be signed by the person in whose name the goods are entered, with the mark in the margin: then the customs and the fee for entry being paid, the land-waiter will grant a warrant for landing the goods.

Of Tare and Trett.

Tare and trett are rules for making a deduction from the value of any goods which are sold by weight.

Tare is that allowance which is made to the buyer for the box, barrel, bag, &c. which contains the goods, and is either so much per box, barrel, bag, &c. at so much per cwt. or at so much in the gross weight.

Trett is an allowance of 4lb. in every 104lb. for waste, dust, &c.

Cliff is an allowance made to the buyer after tare and trett are deducted, and is generally 1lb. in every 3cwt.

Gross weight is the whole weight of the goods at first, together with the box, barrel, bag, &c.

Suttle is when a part only of the allowance is deducted, as the tare or the trett.

Nett weight is the weight that remains after all the foregoing deductions are made.

Case 1. When the tare is at so much per box, barrel, bag, &c.

Rule. Multiply the number of boxes, barrels, bags, &c. by the rate per tare, and subtract the product from the gross weight, and the remainder will be theuttle.

Example 1. What is the nett weight of 20 hogheads of tobacco, each weighing 4cwt. 2qrs. 20lb. at 80lb. per hoghead tare?

20 hhd.	The weight of each hhd.	Cwt. qrs. lb.
80 lb. tare	4	2 20
1600 or 14cwt. 1qr. 4lb. tare		<u>5</u>
		23 1 16
		<u>4</u>
		93 2 8
	Tare	<u>14 1 4</u>
	Answer	<u>79 1 4</u>

Qⁿ. 2. What is the nett weight of 28 frails of raisins, each frail 5cwt. 2qrs. 17lb. gross, tare 50lb. per frail?—*Answer* 145cwt. 3qrs.

Case 2. When the tare is at so much per cwt.

Rule. Divide the gross weight by such aliquot parts of a cwt. as the tare consists of, as in practice, and subtract the sum of the quotient from the gross weight as before, and the remainder will be the answer.

Qⁿ. 3. What is the nett weight of 22cwt. 3qrs. 20lb. at 21lb. per cwt.?—*Ans.* 179cwt. 2qrs. 11lb.

Qⁿ. 4. What is the nett weight of 14 barrels of figs, each barrel weighing 201lb. gross, tare 10lb. per cwt.?—*Answer* 2562lb. 12oz.

Case 3. When there is an allowance for both tare and trett.

Rule. Subtract the tare from the gross weight by the foregoing rule, and the remainder is called futtle weight, which divided by 26 gives the trett, and which being subtracted from the futtle weight gives the nett weight.

Example 5. In 250cwt. 19lb. gross, how much nett, tare 2qrs. 14lb. trett as usual?

	Cwt.	qrs.	lb.	
	250	0	19	gross
		2	14	tare
26)	249	2	5	futtle
	9	2	10	15oz. trett
	230	3	22	10z. nett weight

Case 4. When there is an allowance made for tare, trett, and cloff.

Rule. After the tare and trett are deducted as before, divide the remainder or futtle by 168, and the quotient is the cloff; then subtract the cloff from the futtle, and the remainder is the nett weight.

Example 6. What is the nett weight of 27hhds. of tobacco, weighing 140cwt. 2qrs. 20lb. gross, tare 14lb. per cwt. trett as before?

	<i>Cwt. qrs. lb.</i>
For 14lb. per cwt. tare, divide by 8	140 2 20 gross
	17 2 9 tare
	<hr/> 26)128 0 11
	4 2 25 trett
	<hr/> 168)118 1 14 futtle
	2 22
	<hr/> <hr/> 117 2 20 nett weight.

Qz. 7. What is the value of 58hhds. of sugar, each hhd. weighing 6cwt. 1qr. gross, at a guinea and a half per cwt. tare 8lb. per cwt. trett and cloff as usual?—*Ans.* 505*l.* 14*s.* 7½*d.*

Commission, Brokerage, and Insurance.

Commission is an allowance of a certain sum per cent. to an agent abroad, for buying or selling goods for his principal.

Brokerage is an allowance to a broker for assisting a person in procuring or disposing of goods, as auctioneers, stock-brokers, &c. And the allowance is generally at a certain rate per cent.

Insurance is a premium at so much per cent. given to certain offices or individuals who engage to make good the loss of any ship or merchandise, which may happen from storms, fire, &c. &c.

Rule. Questions concerning these rules are resolved in the same manner as those in simple interest, viz. by multiplying the

Example 1. How much is the concentration of Fe^{2+} in a solution of FeSO_4 at pH 2.0?

[illegible]

Qr. 2. What is the commission if $\frac{1}{2}$ per cent? — Ans. $\frac{1}{2}$ per cent.

22.3. What is the advantage of power tax in the amount of 1017/ 34 8d. at $\frac{1}{2}$ per cent? — Ans: $\frac{1}{2}$ p. 1017/ 34 8d.

23. What is the influence of a tax on the supply of money at 17863/100. 9d at 177 per cent? — Ans. 2155 3/4

SECT. XXI

**PRECEDENTS OF RECEIPTS, PROMISSORY NOTES,
BILLS OF EXCHANGE, BILLS OF DEBT, BILLS OF
LADING, BILLS OF PARCELS, &c. WITH THE
LAWS CONCERNING THEM.**

Bills of Parcels.

It is usual in some cases for shopkeepers to deliver to their customers a bill of the articles sold them, with the total value put up at the bottom; these are called ~~invoices~~.

6:44

A Linen-draper's Bill.

Mr. Francis,

March 10, 1808.

Bought of John Green.

		s.	d.		£.	s.	d.
25 yards of dowlas	at	2	6	per yd.	1	17	6
30 yards of diaper	—	1	6	do.	2	5	0
20 yards of Holland	—	5	6	do.	5	10	0
40 yards of Irish cloth	—	2	6	do.	5	0	0
10 yards of muslin	—	7	0	do.	3	10	0
5 yards of cambric	—	12	0	do.	3	0	0
20 yards of printed cotton		2	6	do.	2	10	0

 £23 12 6

A Woollen-draper's Bill.

Mr. W. Smith,

March 12, 1808.

Bought of Henry Jones.

		s.	d.		£.	s.	d.
17 yards of fine serge	at	4	0	per yd.	3	8	0
18 yards of drugget	—	8	0	do.	7	4	0
15 yards of superfine scarlet		1	6	do.	1	2	6
16 yards of super black cloth		18	0	do.	14	8	0
25 yards of shalloon	—	2	0	do.	2	10	0
17 yards of drab	—	18	0	do.	15	6	0

 £43 18 6

A Hosiery's Bill.

Mr. Holmes,

March 14, 1808.

Bought of James Richards.

		s.	d.		£.	s.	d.
16 pair of worsted stockings	at	6	6	per pair	5	4	0
16 pair of do.	—	3	2	do.	2	10	8
6 pair of black silk do.	—	14	0	do.	4	4	0
12 pair of black worsted do.	—	6	0	do.	3	12	0
8 pair of cotton do.	—	8	0	do.	3	4	0
4 yards of fine flannel	—	1	8	per yd.	0	6	8

 £19 1 4

A War.

A Wine merchant's Bill.

Mr. Harrison,

March 16, 1808.

Bought of James Simpson.

		£.	s.	d.		£.	s.	d.
12 gallons of palm sack	at	0	8	0	per gall.	4	16	0
24 gallons of red port	—	0	12	0	do.	14	8	0
20 gallons of claret	—	1	0	0	do.	20	0	0
30 gallons of white Lisbon		0	6	0	do.	9	0	0
22½ gallons of shenish	—	1	10	0	do.	33	15	0
30 gallons of sherry	—	0	10	0	do.	15	0	0
						<u>96</u>	<u>19</u>	<u>0</u>

RECEIPTS.

Received, January 19th, 1808, of Mr. John Smith, the sum of ten pounds ten shillings, for a quarter's rent, due at Christmas last. Per me,

£ 10 10 0

Thomas Jones.

Received, January 10th, 1808, of Mr. Charles Hamilton, fifty pounds, in part of indigo sold him the 8th instant. Per

£ 50 0 0

Henry Holmes.

A Receipt given in a Receipt Book.

Received, January 11th, 1808, the sum of forty-five pounds, by the order, and for the account of William Johnson, Esq. Per me,

£ 45 0 0

John Leck.

Received, January 12th, 1808, of Messrs. James Williams and Co. one hundred pounds, on account of self and partner.

£ 100 0 0

Per me, William Martin.

Received,

Received, January 13th, 1808, of Mr. George Goodman, fifty-five pounds sixteen shillings and nine pence, in part for tobacco sold him the 24th of December last. Per

£55 16 9

Samuel Thompson.

Received, January 14, 1808, of the Honourable East India Company, three hundred and fifteen pounds ten shillings, per order, and for the account of Philip Fox. Per

£315 10 0

Benjamin Hamilton.

Received, January 15th, 1808, of the Governor and Company of the Bank of England, one thousand six hundred pounds ten shillings, for self and company. Per

£1600 10 0

Stephen Barber.

Received, January 16, 1808, of the Worshipful Company of Grocers, forty-nine pounds fifteen shillings, in full payment, for my father, John Harrison. Per me,

£49 15 0

John Harrison, jun.

Received, January 17th, 1808, of Richard Clarke, Esq. chamberlain of London, the sum of sixty pounds, for the use of the Worshipful Company of Joiners. Per

£60 0 0

Richard Stevens, Clerk.

A Rent-gatherer's Receipt.

Received, February 7th, 1808, of Mr. Stephen Dickson, in money, eighteen pounds, and allowed him for land-tax five pounds, and for repairs two pounds, in all twenty-five pounds, in full for half a year's rent due at Christmas last: I say received for the use of Thomas Cox, Esq., by virtue of his letter of attorney. Per

£25 0 0

Francis Ward.

Received

Received of Mr. William Morgan, this 9th of February 1808, six pounds, for one quarter's rent, due at Christmas last, for my master, Robert Kerr. Per

£6 0 0

Richard Heath.

Received, February 10th, 1808, of Mr. Samuel Wilkinson, twenty-nine pounds six shillings, in part of a bill of sixty pounds, due the 3d of May next, to Mr. John Lewis. Per

£29 6 0

Nicholas Marsel.

A Receipt on the Back of a Bill of Exchange.

March 1st, 1808. Received the full contents of the within mentioned, being 500 pieces of eight. Per

500 pieces 8

John Wilson.

PROMISSORY NOTES.

I promise to pay to Mr. Edward Brown, or order, the sum of sixty pounds, on the 29th of this instant March. Witness my hand this 6th day of March 1808.

£60 0 0

Edward Jones.

March 10th, 1808.

I promise to pay to the Honourable the East India Company, or bearer, upon demand, two hundred pounds, for Edward Williams.

£200 0 0

Henry Smith.

March 12th, 1808.

I promise to pay to Mr. Joseph Brown, or order, five months after date, ten pounds ten shillings, for value received.

£10 10 0

William Holmes.

March 14th, 1808. —

I promise to pay to Messrs. John Edwards and Co. or bearer, the sum of twenty pounds ten shillings, on demand.

£20 10 0

Robert Martin.

A Note

A Note from two to one.

We or either of us promise to pay to Mr. Robert Stokes, or his order, on demand, twelve pounds ten shillings, for value received. Witness our hands,

£ 12 10 0

Thomas Williams.
Edward Morris.

BILLS OF EXCHANGE.

A Bill payable at Sight.

London, March 5th, 1808.

At sight hereof pay Mr. Robert Winds, or his order, the sum of twenty-five pounds, for value received of Edward Mills, and place it to account, as per advice from

To Mr. John Willis, Your humble servant,
Merchant, High Street, Bristol. *John Miller.*

Dover, March 13, 1808.

Ten days after sight hereof pay to Mr. Thomas Trust, or his order, fifty pounds ten shillings, and place it to account, as per advice from

To Mr. John Francis, Your humble servant,
Linen-draper, Cheapside, London. *Henry Hill*

Foreign Bills of Exchange.

London, March 6, 1808, for 540 crowns,
at 56½d. sterling per crown.

At usance * pay this my first bill of exchange (my second and third not being yet paid) unto Mr. Henry Brown, or order, five hundred and forty crowns, at 56½d. per crown, for the value received here of Mr. Edward Willis, and place it to account, as per advice from

To Mr. John Edwards, Your humble servant,
Merchant in Amsterdam. *John James.*

* Usance between England and France, or England and Holland, is one calendar month; between England and Spain, or Portugal, two months; between England and Italy, three months; and between England and Turkey, four months, &c.

Naples, March 12, 1808, for 640*l.* 10*s.*

At three usances pay this my first bill of exchange, unto Mr. John Stocks, or order, six hundred and forty pounds ten shillings sterling, for value received of himself, and place it to account, as per advice from

To Messrs. Bardy and Edwards,
Merchants in London.

Your humble servant,
John Smith.

Liverpool, March 14, 1808, for 6000 pieces
of eight, at 53½*l.* per piece.

At double usance pay this my fourth bill of exchange, unto Mr. John Jervis, or order, six thousand pieces of eight, Mexico, at 53½*l.* sterling per piece, for value received of Mr. Joseph Hunt, and place it to account, as per advice from

To Mr. Matthew Marsh,
Merchant in Leghorn.

Your humble servant,
Henry Edwards.

A Bill of Debt.

Know all men by these presents, that I John Briton, of London, merchant, do owe and am indebted unto Henry Haynes, of Southwark, salter, in the sum of four hundred and forty-nine pounds, of lawful money of Great Britain; which said sum I ~~do~~ hereby promise to pay unto the said Henry Haynes, his heirs, executors, administrators, and assigns, on or before the twenty-fourth day of September next ensuing the date hereof. Witness my hand and seal, this fourteenth day of March 1808.

John Briton, (J B).

Sealed and delivered } *William Smith.*
in the presence of us, } *Thomas Harrison.*

A Bond.

Know all men by these presents, that I John Drew, of the parish of Saint Giles, in the county of Middlesex, carpenter, am held and firmly bound to Samuel Price, of the city of Westminster, in the county aforesaid, vintner, in the sum of one hundred pounds, of good and lawful money of Great Britain, to be paid to the said Samuel Price, or to his certain

attorney or attornies, heirs, executors, administrators, or assigns; for which payment well and truly to be made, I do hereby bind myself, my heirs, executors, administrators, or assigns, firmly by these presents. In witness whereof I have hereunto set my hand and seal this eighth day of March, in the forty-eighth year of the reign of our sovereign Lord George the Third, &c. and in the year of our Lord 1808.

John Drew, (J D).

The *condition* of the above written obligation is such, that if the above bound John Drew, his heirs, executors, administrators, or assigns, do well and truly pay, or cause to be paid, unto the aforesaid Samuel Price, his heirs, executors, administrators, or assigns, the full sum^d of forty pounds, of lawful money of Great Britain, on or before the twenty-sixth day of December next ensuing the date hereof, then this obligation to be void, otherwise to remain in full force and virtue.

A Bond from two to one.

Know all men by these presents, that we Edward Willis and Joseph Jenks, of the city of London, merchants, are held and firmly bound to Thomas Smith, of Liverpool, merchant, in the sum of five thousand pounds, of good and lawful money of Great Britain, to be paid to the said Thomas Smith, his heirs, executors, administrators, or assigns, and for which payment well and truly to be made, we and either of us do jointly and severally bind ourselves, our joint and several heirs, executors, administrators, or assigns, firmly by these presents. In witness whereof we have hereunto set our hands and seals this twelfth day of March, in the forty-eighth year of the reign of our sovereign Lord George the Third, &c. and in the year of our Lord 1808.

Edward Willis, (E W).

Joseph Jenks, (J J).

The *condition* of this obligation is such, that if the above bounden Edward Willis and Joseph Jenks, each or either of them,

them, or each or either of their heirs, executors, administrators, or assigns, shall well and truly pay, or cause to be paid unto the above-named Thomas Smith, his executors, administrators, or assigns, the sum of one thousand pounds, of lawful money of Great Britain, on or before the twenty-fifth day of December next ensuing the date hereof, then this obligation to be void, otherwise to remain in full force and virtue.

A Penal Bill.

Know all men by these presents, that I Andrew Jones, of the borough of Southwark, do owe unto Edward Mills, of the parish of St. Clement Danes, Westminster, the sum of one hundred pounds, of lawful money of Great Britain, to be paid unto the same Edward Mills, his executors, administrators, or assigns, on or before the fifth day of November next ensuing the date hereof; for which payment well and truly to be made, I do hereby bind myself, my heirs, executors, administrators, or assigns, to the said Edward Mills, his executors, administrators, or assigns, in two hundred pounds, of like lawful money of Great Britain, firmly by these presents. In witness whereof, &c.

Andrew Jones, (A J).

Of Receipts.

A receipt is a discharge for money owing, and is a bar to all suits in law and equity. When given *in full of all demands*, it discharges all debts between the two parties prior to the date thereof.

If a receipt be given, it is good, though no money be paid, except it be obtained by fraud, in which case relief may be had in equity.

A servant may give a receipt in his own name for his master, if he be accustomed to receive or pay money for his master's use, or a wife may give a receipt for her husband in her own name, and it is equally valid.

If a receipt be given *in full*, though there be a condition annexed to it specifying a former debt between the parties, yet the receipt stands as a receipt *in full*.

Of Promissory Notes and Bills.

A promissory note is an engagement for a sum of money to be paid at a certain time, or on demand; when the sum is to be paid on demand, it is due presently, and there needs no actual demand; but it is otherwise where the money is to be paid to a third person, or where there is a penalty, for in that case there must a demand.

A bill is a single bond, without any condition. If a person acknowledges himself indebted to another by bill in the sum of fifty pounds, and by the same bill binds himself and his heirs for the payment of the said sum, in a hundred pounds, without inserting the person's name to whom he is bound in the penalty, it shall nevertheless be taken as a bond against him for the said sum and penalty.

If a man insert in a bill these words: *I do owe and promise to pay to A B 50l. &c. for the payment whereof I bind myself to C D, &c.* (another person), yet this is a good bill by the words of the first part, and the words obligatory to another person, C D, are void.

If a man says in a bill, *That I A B had received of C D the sum of twenty pounds, which I promise to pay to E F; or if the bill be, I shall pay to C D 20l.; or if it be, I owe to C D 20l. to be paid at, &c.; or, I had of C D 20l. &c.; to be repaid him again; or, I A B do bind myself to C D, that he shall receive 20l.*—either of these phrases binds the drawer of the bill, and constitutes a good bill.

Of Bills of Exchange.

A bill of exchange is a writing given among merchants and traders for money, and by the credit of the drawer generally

generally passes as money; these bills are drawn sometimes payable at sight, sometimes at a certain time. An inland bill of exchange is of the nature of a letter, and is drawn by one merchant upon another in the same country, as, by A B of Bristol upon C D of London; but a foreign bill of exchange is drawn by a merchant of one country upon a merchant of another country. These foreign bills are of more consequence in the eye of the law; a foreign bill being refused to be accepted, an action may be had against the drawer, and every person who indorses a foreign bill becomes liable to the payment as if he were the drawer, because every indorsement is in the nature of a new bill; but the indorser is not liable to pay if the drawer can be found; but if the drawer's hand cannot be proved, the acceptor may come upon the last indorser. If a person only signs his name at the back of the bill, the acceptor of the bill may fill up the indorsement, or the bare signature will pass in law for an indorsement.

There is a material difference between a bill of exchange payable to a person or *bearer*, and one payable to a person or *order*. A bill payable to a person or bearer is not assignable, so as to enable the acceptor to bring an action if the drawer refuses payment. In a bill payable to A B or bearer, the acceptor may sue in the name of him to whom it is made payable. If a bank bill payable to A B or bearer be lost, and found by a stranger, payment to the stranger will indemnify the Bank, yet A B may have an action of trover against the finder, but not against any person to whom the finder has paid it for valuable consideration.—A bill of exchange payable to a person or order may always be assigned, and the acceptor can bring his action in his own name. A bill payable to order is within the custom of merchants, and may be negotiated and assigned by custom.

If a person accepts a bill, though the date of payment be past, he cannot return it, but he may bring an action for the money, if the bill be tendered in time.

A servant cannot accept a bill for his master, except there

be good evidence that he has authority to do so, as where a master allows a servant to draw bills, &c.

If one partner in company draws a bill of exchange for self and company, all the other partners are bound by it.

If the acceptor of a bill makes no demand when the bill becomes due, or in reasonable time, and the party on whom it is drawn fails, the acceptor must sustain the loss, as it is his negligence.

If a bill of exchange be lost, notice must be given by a notary public before witnesses, that the bill is lost, or mislaid, requiring that payment be made to no person without his knowledge.

All inland bills of exchange of the sum of 5*l.* or upwards, in which the value is said to be received, and which are payable at a certain time after the date thereof, may, three days after they become due, be protested by a notary public, or any other substantial person of the place, if there be no notary public, and before two witnesses, on the refusal or neglect of payment; which protest shall be made under a copy of the said bill, and be notified within fourteen days after to the party from whom the bill was received, who, upon producing the same protest, is to pay the bill with interest, and the charges of protesting; but if the acceptor neglects to make a protest, and give notice, he shall be liable to the loss.

Of Bonds.

A bond is an obligatory deed in writing, whereby one party binds himself to another to pay a sum of money, or to perform any other lawful act, as, to make a release, surrender an estate, to perform covenants, &c. &c. and it contains a penalty of non-performance; it may be made either on parchment or paper, though it is usually on paper; and the condition to the bond may be either in the same deed or in another; the condition must be to do a thing which is lawful, otherwise
it

it is not good; thus a condition not to use a trade, till or sow ground, &c. is unlawful, being against the public good, and therefore void. When a bond depends upon a deed, and the deed becomes void, the bond is void also; a condition to indemnify a person from any legal prosecution is void, being against law. Conditions of bonds are to be not only lawful, but possible: if no time is limited in a bond for payment of money, it is due presently, and payable on demand.

In a bond where several persons are bound *severally*, the obligee has his option, either to sue all the bondmen together, or all of them apart, and have several judgments and executions, but he can have satisfaction but once; but if a bond be made to three to pay money to one of them, they must all join in the action. An heir is not bound by a bond, unless he be expressly named, but the administrators and executors are.

A bond may be made from one to any number of persons, or from any number of persons to one. If the surname only of the drawer be subscribed, it is sufficient, though there be a blank for his Christian name. If a bond has no date, or a false date, or an impossible date (as the thirtieth day of February), yet if it be sealed and delivered it is good; a person cannot be charged by a bond without delivery of the bond to a creditor.

Where a bond is made for the payment of a sum of money by instalments, the holder of the bond in some cases may come upon the drawer for failure of the first payment, but in other cases he cannot sue for the money till the last payment becomes due: as for instance, if A gives a bond to B to pay 20*l.* in the following manner; viz. 10*l.* to be paid at a certain day, and 10*l.* at another certain day; in this case B cannot sue for the debt till the last payment becomes due, as the 20*l.* is mentioned as an entire debt; but if it be expressed to pay 10*l.* at a certain day, and 10*l.* at another certain day, *making in all 20*l.** the creditor may sue upon the bond on failure of the first payment.

A bond

A bond may be good though it contains false Latin or false English, or though it be of a doubtful interpretation, as, if A binds himself to B to pay a sum of money to A (whereas it should be to B), here the obligation is good, and the *solvendum* void. If a bond be interlined in a place not material, it will not hurt the bond; but if it be in a material part, it will make the bond void. In short, all bonds, obligations, bills, promissory notes, &c. when any doubt arises, are always interpreted in favour of the party to whom the money is due.

These are the principal legal precedents used by mercantile men, to which I shall add a power or letter of attorney, as being in most general use by all descriptions of people.

A Letter of Attorney from one, or two, to settle Accounts and receive Money.

Know all men by these presents, that I Edward Stokes, of the parish of St. Andrew, Holborn, London, chiuaman [or Edward Smith and John James, upholsterers, of the parish of St. Luke, Middlesex] have made, ordained, constituted, and appointed, and do by these presents make, ordain, constitute, and appoint my friend Henry Brown, of the parish of St. James, Clerkenwell, in the said county of Middlesex, watchmaker, my [or our] true and lawful attorney for me, in my name, and on my behalf [or for us, in our names, and on our behalf], to adjust and settle all and every account and accounts with all and every person and persons with whom I [or we] have had, or shall or may have any transactions or dealings whatsoever, and to compromise, agree, and determine all disputes and differences that have or shall arise between me [or us] and any other person or persons whomsoever, and to execute all such deeds, instruments, and writings, as he the said Henry Brown shall judge necessary, and to ask, demand, sue for, recover, and receive, to and for my [or our] use, stand from all and every person

person or persons, that now is, are, or hereafter shall or may become indebted to me [or us] by any ways or means whatsoever, all and every the debt and debts, sum and sums of money by them respectively due and owing, and to compound for any such debt or debts, and to take less than the whole for the same, or otherwise to adjust and settle the same in such manner and upon such terms as he the said Henry Brown shall in his discretion think fit; and for non-payment thereof, or of any part thereof, to take such course for recovering the same as to my [or our] said attorney shall seem meet; and upon receipt of the said debt or debts, sum or sums of money respectively, or any part thereof, acquittances or other sufficient discharges for me, and in my name [or for us, in our names], or in his own name, to make and give for what he shall so receive; and generally to do, negotiate, transact, and perform all such other acts, matters, and things, for me, and in my behalf [or for us, and on our behalf], in and about the premises, as fully in every respect as I [or we] might or could do if personally present; hereby ratifying and confirming, and agreeing further to ratify and confirm all and whatsoever my [or our] said attorney shall lawfully do, or cause to be done, in and about the said premises, by virtue of these presents. In witness whereof, I [or we] have hereunto set my hand and seal [or our hands and seals] this twentieth day of March, in the year of our Lord 1806, and in the forty-eighth year of the reign of our Sovereign Lord George the Third, &c. *

Edward Stokes, (E. S.)
 [or, *Edward Smith,* (E. S.)
John James,] (J. J.)

A letter of attorney may be executed by any person, if of full age; and a man may give a power of attorney to his wife,

* The words included in crotchets belong to the power when given by Edward Smith and John James; in other respects the power is the same as when given by Edward Stokes.

or a woman to her husband, in cases that do not belong to him. The nature of it is, to give the party who is to act the full power and authority of the maker, to accomplish the act intended.

In these instruments the authority must be strictly pursued; where the party acting does less than the authority mentions, the act is in most cases void; but where he doth more than his authority, it may be good for so much as he had power to do, if he does nothing unlawful.

If a letter of attorney be made to three jointly, two cannot execute it; or if it be made to two jointly, one cannot execute it. The death of the maker generally determines the power.

All the foregoing precedents must be stamped according to the several rates imposed by the late acts for increasing the stamp duty, or they are void by statute.

CHAP. VII.

OF MENSURATION.

SECT. I.

OF SUPERFICIAL MEASURE, WITH THE METHOD OF MEASURING MASONS, BRICKLAYERS, CARPENTERS, SAWYERS, PLASTERERS, PLUMBERS, PAINTERS, AND GLAZIERS WORK, AND THE PRICE OF EACH ; ALSO THE PRICE OF EACH COMMODITY, AND THE WAGES OF JOURNEYMEN.

BEFORE the learner proceeds to mensuration, it is necessary that he should understand duodecimals, or, as it is generally called, cross multiplication.

This rule is called duodecimals, because the numbers decrease from the left hand in a twelve-fold proportion ; the first number being feet, the next number inches, and the next number the twelfth parts of an inch, &c.

By this rule workmen and artificers cast up the contents of their work, and multiply feet, inches, and parts, by feet, inches, and parts, without reducing them to one denomination, as in common arithmetic. In this rule inches are sometimes called *primes*, the parts are called *seconds*, the next division *thirds*, &c.

Rule 1. Under the multiplicand, write the corresponding denominations of the multiplier : viz. feet under feet, inches under inches, &c.

S s 2

2. Multiply

2. Multiply every term in the multiplicand, beginning at the lowest, by the feet in the multiplier, and write the result of each under its respective term, observing to carry 1 for every 12 in the product to the next higher denomination.

3. In the same manner multiply every term in the multiplicand by the inches in the multiplier, and set the product of each term one place further to the right hand of that term in the multiplicand, observing to carry 1 to the next higher denomination for every 12 as before.

4. Proceed in the same manner with the seconds, and the other denominations, if there be any more, and the sum of the products will be the product required.

Example 1. Multiply 10 feet 4 inches and 5 parts by 7 feet 8 inches and 6 parts.

Feet.	Inches.	Parts.		
10	4	5		
7	8	6		
<hr/>				
0	2	11		
2	4	0		
70	0	0		
0	0	3	4	
0	2	8		
6	8	0		
0	0	0	2	6
0	0	2		
0	5	0		
<hr/>				
79	11	0	6	6
<hr/>				

Answer.

In this example the 5 parts are first multiplied by 7 feet, and the product is 2 inches 11 parts; then the 4 inches are multiplied by the 7 feet, and the product is 2 feet 4 inches; then the 10 feet are multiplied by the 7 feet, and the product is 70 feet. Then I multiply by the 8 inches, saying 5 times 8 is 40, which is 3 twelves and 4 over, the 4 I set one place further towards the right hand than the multiplicand 5, and the 3 I place under the seconds; then I multiply the 4 inches, saying 4 times 8 is 32, which is 2 twelves and

and 8, the 8 I set under the parts and the 2 under the inches; then I say 8 times 10 is 80, which is 6 twelves and 8, the 8 I set under the inches and the 6 under the feet. In the same manner I multiply by the third and last figure 6, saying 6 times 5 is 30, which is 2 twelves and 6, the 6 I place one place further towards the right hand than the first figure of the last product, and proceed in the same manner as before, and the sum of all the products will be the answer.

I have given this example at somewhat an unnecessary length, in order to instruct the learner more fully in the nature of the rule; but it is generally performed in three lines only when consisting of feet, inches, and parts, as in the following example:

Example 2. Multiply 6 feet 4 inches and 5 parts, by 4 feet 6 inches and 9 parts.

Feet.	Inches.	Parts.			
6	4	5			
4	6	9			
<hr/>					
25	5	8			
3	2	2	6		
0	4	9	3	9	
<hr/>					
29	0	5	9	9	<i>Answer.</i>
<hr/>					

Note. As the foot is divided into 12 inches, so each inch is subdivided into 12 parts, called seconds, and each second is again subdivided into 12 thirds, and each third into 12 fourths, &c.

Qn. 3. What is the superficial content of a square piece of ground, measuring 39 feet 10 inches and 7 parts in length, and in breadth 18 feet 8 inches and 4 parts?—*Ans.* 747 feet 6 inches 10 seconds 2 thirds and 4 fourths.

Qn. 4. What is the area of a floor in length 24 feet 10 inches 8 seconds 7 thirds and 5 fourths, and in breadth 4 feet 4 inches and 6 seconds?—*Ans.* 233 feet 4 inches 5 seconds 9 thirds 6 fourths 4 fifths and 6 sixths.

Second,



Seconds, thirds, &c. are usually marked by two or three short strokes placed over the figures: thus, the answer to the last question is expressed, 233 feet

1							
4	5	9	6	4	0		

Superficial Measure.

Mensuration, in general, is the art of measuring and estimating the magnitude and dimensions of bodies or figures, and is divided principally into three parts, called *lineal measure*, *superficial measure*, and *solid measure*.

Definitions.

1. *Lineal measure* is simple measure in length, without breadth or depth.
2. *Superficial measure* consists of length and breadth taken together.
3. *Solid measure* consists of length, breadth, and depth; of which hereafter.
4. A point has no parts nor dimensions, neither in length nor breadth.
5. A line has only length, without any other dimensions.
6. A right line lies all in the same direction, and is the shortest way between its two extremities.
7. A curve line continually changes its direction. See *fig. 1.*
8. Parallel lines are always at the same distance from each other. See *fig. 2.*
9. Oblique right lines change their distance and meet in an angle. See *fig. 3.*
10. An angle is the meeting of two lines. See *fig. 5.*
11. If the two lines which form an angle be perpendicular to each other, they then form a right angle. See *fig. 6.*

12. But if the two lines be not perpendicular to each other they form what is called an oblique angle, which is either greater or less than a right angle. See *fig. 9* and *10*.

13. If an angle be less than a right angle, it is called an acute angle. See *fig. 7*.

14. An angle that is greater than a right angle is called an obtuse angle. See *fig. 8*.

15. A triangle is a figure contained under three lines, and has various names, according to the size of its angles.

16. An equilateral triangle has its three sides, and consequently its three angles, equal to each other. See *fig. 9*.

17. An isosceles triangle has only two sides equal. See *fig. 10*.

18. A scalene triangle has its three sides and angles unequal to each other. See *fig. 11*.

19. A right-angled triangle has one right angle. See *fig. 12*.

20. An obtuse-angled triangle has one obtuse angle. See *fig. 13*.

21. An acute-angled triangle has all its angles acute. See *fig. 14*.

22. A figure of four sides is called a quadrangle, or a quadrilateral figure, and is either a parallelogram, a square, a rhomboid, a rhombus, a trapezium, or a trapezoid.

23. A square is an equilateral rectangle, having all its sides equal, and all its angles right angles. See *fig. 16*.

24. A rhomboid is an oblique-angled parallelogram, See *fig. 17*.

25. A rhombus is an equilateral figure, having all its sides equal, but its angles are oblique. See *fig. 18*.

26. A trapezium is a quadrilateral figure, but its opposite sides are not parallel. See *fig. 19*.

27. A trapezoid has only two opposite sides parallel. See *fig. 20*.

28. Plane figures having more than four sides, are generally called polygons, but they receive particular names, according

to the number of their sides : thus a polygon of five sides is called a *pentagon*, a figure of six sides an *hexagon*, a figure of seven sides an *heptagon*, eight sides an *octagon*, nine sides a *nonagon*, ten sides a *decagon*, eleven sides an *endecagon*, and twelve sides a *dodecagon*.

29. A circle is a plain figure bounded by one circular line, called the circumference, which is every where equally distant from the centre. See *fig. 21*.

30. The radius of a circle is a right line drawn from the centre to the circumference.

31. The diameter of a circle is a right line drawn through the centre, and bounded at each end by the circumference.

32. An arc of a circle is any part of the circumference.

33. A chord is a right line joining the two extremities of an arc.

34. A segment of a circle is any part thereof.

35. A semicircle is half a circle.

36. A sector is a part of a circle contained under part of the arc, and two radii drawn to the centre. See *fig. 22*.

37. A quadrant is a sector of a circle, having one quarter of the circumference for its arc, and its two radii are perpendicular to each other. See *fig. 23*.

38. The circumference of every circle in geometry is supposed to be divided into 360 equal parts, called degrees, and each degree subdivided into 60 minutes, each minute into 60 seconds, and so on ; hence a semicircle contains 180 degrees, and a quadrant 90 degrees, which form a right angle, and half a quadrant, called an octant, contains 45 degrees ; for the measure of every right-line angle is an arc of a circle contained between the two lines which form the angle, the point of the angle being in the centre of the circle, and the number of degrees contained in the arc of the circle gives the measure of the angle. See *fig. 25*.

39. In every right-angled triangle the side opposite the right angle is called the hypotenuse, and the other two sides, the legs of the triangle.

40. The height or altitude of a figure, is a line drawn from the uppermost side or angle, perpendicular to the base.

41. An angle is generally described by three letters: that letter which stands in the middle shews the angle intended. Thus, in figure 24, EAC represents an angle formed by the line EA , and the line AC ; and the letters DAE in the same figure shew the angle formed by the two lines DA and EA .

Problems.

PROBLEM I.

TO DIVIDE A GIVEN LINE INTO TWO EQUAL PARTS.

Fig. 26. From B as a centre with the compasses opened to any greater length than BC describe the arcs at m and n ; then (the compasses being still at the same width) from A , as the other centre, describe two arcs cutting the former arcs m n ; from the intersections of these arcs draw the straight line mn , and it will divide the given line AB into two equal parts, at the point G .

PROBLEM II.

TO DIVIDE A GIVEN ANGLE INTO TWO EQUAL PARTS.

Fig. 27. From B as the centre with the compasses describe the arc AC ; and from A and C with the radius describe two arcs intersecting each other at m , then draw the line mb , and it will divide the angle ABC into two equal parts.

PROBLEM III.

TO DIVIDE A RIGHT ANGLE INTO THREE EQUAL PARTS.

Fig. 28. From B as the centre describe the arc AC . Then with the same extent of the compasses, from A as the centre, cross the arc AC in n , and with the same radius from C , as the centre, cross the arc in m : from the points of in-



tersection m and n , draw the right lines mB and nB , and they will divide the right angle ABC as required.

PROBLEM IV.

TO DRAW A LINE PARALLEL TO ANOTHER GIVEN LINE, AND AT A GIVEN DISTANCE.

Fig. 29. Let the given line be AB : then from the points m and n with a radius equal to C , the given distance, describe the arcs r and o , upon which draw the line CD , and it will be parallel to the line AB .

PROBLEM V.

TO ERECT A PERPENDICULAR, FROM A GIVEN POINT IN A GIVEN LINE.

Fig. 30. Let the given line be BC , and the given point A . Take two equal distances Am and An , and with a radius greater than Am , from m and n as centres, describe two arcs, cutting each other at r : from A draw Ar , and it will be perpendicular to the line BC ; thus this perpendicular forms a right angle with each part of the line BC . There are many other ways of drawing perpendiculars to a given line, but the best way is to draw them by a square, or other mathematical instrument for that purpose: I shall, however, give another Problem, to draw a perpendicular from a given point out of the line.

PROBLEM VI.

TO LET FALL A PERPENDICULAR FROM A GIVEN POINT, NEARLY OPPOSITE TO THE END OF A GIVEN LINE.

Fig. 31. Let the given point be A , and the given line BD ; then from A draw any line Am , to meet BC : bisect Am at n , and from n as a centre, with the radius An , describe the arc ADm , cutting BC in D ; then draw a line AD , and it will be the perpendicular required.

PROBLEM VII.

TO DIVIDE A GIVEN LINE INTO ANY NUMBER OF
EQUAL PARTS.

Fig. 32. Let the given line be AB , then from A draw a line AC , and from B draw BD parallel to AC : on each of these parallel lines point off as many parts as the given line is to be divided into, beginning at A in the first line, and at B in the second line; then join the opposite points of division in each line, by the lines AS , 14 , 23 , &c. and they will divide the line AB as required.

PROBLEM VIII.

AT A GIVEN POINT IN A GIVEN LINE, TO MAKE AN
ANGLE OF ANY PROPOSED NUMBER OF DEGREES.

Fig. 33. Let the given point be A , and the given line AB , then from A , as a centre, with a radius of 60 degrees taken from a line of chords, describe an arc mn , cutting AB in m ; then take the proposed number of degrees with the compasses from the same line of chords, and apply the compasses from m to n ; through the point n , draw the right line AC , and it will form an angle with the line AB of the number of degrees required.*

Angles of more than 90 degrees cannot be made at the line of chords at once.

Or the angle may be made with a compass, by moving the centre to the point A , and as radius, in the line AB , and

* A line of chords is a very celebrated upon the plane table, which is an instrument of math, used, in which, there are several other lines, as is had at the mathematician's instrument maker's, and are all drawn upon the scale of the instrument, as is seen at the end of the page. It contains, semicircles, rhombs, and a line of equal parts: and is the figure of use of these lines, and the method of applying them to several lines upon it, with the also for which they are applied, which I come to treat of Trigonometry.



mark at n , the number of degrees proposed from the arch, and draw the line $A n$, as before.

PROBLEM IX.

TO MEASURE A GIVEN ANGLE.

Fig. 33. Describe the arc $m n$, with the compasses opened to the radius of 60 degrees in the line of chords, take the width of the arc $m n$, and apply that distance to the line of chords, and it will shew the number of degrees in the angle.

If the arc of the angle exceed 90 degrees, it must be taken off at twice, as before.

Or, the angle may be measured by means of a divided arch, as the angle in the last problem was constructed.

PROBLEM X.

TO FIND THE CENTRE OF A CIRCLE.

Fig. 34. In the circle $A B C$, describe any line, as $A B$, which is called a chord, from the middle of which line draw the perpendicular $C D$, which will be the diameter of the circle, and bisect $C D$ in the point O , which will be the centre.

PROBLEM XI.

TO DESCRIBE THE CIRCUMFERENCE OF A CIRCLE,
THROUGH THREE GIVEN POINTS.

Fig. 35. Let the given points be $A B C$: then from the middle point B , draw two chords, or two right lines to the other two points $A C$; bisect these two lines by perpendicular lines meeting each other in the point O ; O will then be the centre of the circle; and with the radius $O A$, or $O B$, describe the circle $A B C$, and it will pass through the three given points.

PROBLEM

PROBLEM XII.

TO MAKE A REGULAR PENTAGON ON A GIVEN LINE.

Fig. 36. Let the given line be AB . Draw Bm perpendicular to AB , and equal to half of it. Draw Am , and produce it till mn be equal to Bm . With a radius equal to Bn describe two arcs, intersecting each other in a point O from A and B , as centres. With the same radius, from the point of intersection O , describe the circumscribing circle ABn , and about the circumference of this circle apply the distance AB five times, and it will describe the figure required.

PROBLEM XIII.

TO DESCRIBE AN HEXAGON UPON A GIVEN LINE.

Fig. 37. Let the given line be AB . Then with a radius equal to AB , from A and B as centres describe two intersecting arcs. From the point of intersection O , with the same radius describe the circle ABC ; then apply the line AB six times round the circumference, and it will form the figure required.

If each side of the above figure be divided into two parts, a figure of twelve sides may be formed; and if each side be divided into three parts, a figure of eighteen sides may be formed; thus it appears that the radius of a circle will divide the circumference into six equal parts; half the radius will divide it into twelve equal parts; and twice the radius will divide it into three equal parts.

PROBLEM XIV.

TO FIND A RIGHT LINE EQUAL TO ANY GIVEN ARC OF A CIRCLE.

Fig. 38. Let the given arc be AB : through the point A , and the centre of the circle, draw Am , making ma equal to $\frac{1}{2}$ of the radius nO . Draw a line AP perpendicular to Am , then through mB draw mP ; then AP will be equal to the arc AB .

PROBLEM

PROBLEM XV.

TO MAKE ANY REGULAR POLYGON ON A GIVEN LINE.

Fig. 36. Let the given line be $C D$: draw $C O$ and $D O$, making the angles at C and D , each equal to half the angle of the polygon: from the centre O with the radius $O D$ describe the circle $A B C D$, then apply the line $C D$ continually round the circumference, and it is done.

Note. The angles of a polygon are found in this manner: divide the whole 360 degrees, by the number of sides in the polygon, and the quotient will be the angle at the centre, which angle subtracted from 180 degrees, the remainder will be the angle of the polygon.

A TABLE

Containing the Number of Degrees in the Angle, and at the Centre of every regular Polygon, from three Sides to twelve.

No. of Sides.	Names of the Polygon.	Angle at the Centre.	Angle of the Polygon.	Angle O D C or O C D.
3	Trigon	120°	60°	30°
4	Tetragon	90	90	45
5	Pentagon	72	108	54
6	Hexagon	60	120	60
7	Heptagon	51 $\frac{3}{4}$	128 $\frac{4}{5}$	64 $\frac{2}{5}$
8	Octagon	45	135	67 $\frac{1}{2}$
9	Nonagon	40	140	70
10	Decagon	36	144	72
11	Endecagon	32 $\frac{8}{11}$	147 $\frac{3}{11}$	73 $\frac{3}{11}$
12	Dodecagon	30	150	75

PROBLEM

PROBLEM XVI.

TO MAKE A FIGURE SIMILAR TO ANY OTHER GIVEN FIGURE.

Fig. 39. From any angle, as suppose A , draw diagonals to each of the other angles; then to bc , one side of the figure, draw a parallel line BC ; and CD parallel to cd , and ED parallel to ed ; and it will be the figure required.

PROBLEM XVII.

TO REDUCE A COMPLEX FIGURE FROM ONE SIZE TO ANOTHER, BY MEANS OF A SCALE.

Fig. 40 and 41. Divide the given figure into squares, by cross-lines, then divide another paper on which you intend to draw your figure, into the same number of squares, and observe what squares the several parts of the figure fall in, and draw similar parts in the corresponding square of the other figure.

PROBLEM XVIII.

TO DRAW A TRIANGLE EQUAL TO A GIVEN TRAPEZIUM.

Fig. 42. Let the trapezium be $ABCD$; draw the diagonal BD and CE parallel to BD , meeting AB , produced in the point E ; join DE : then the triangle ADE will be equal to the trapezium $ABCD$.

PROBLEM XIX.

TO MAKE A TRIANGLE EQUAL TO A FIGURE OF FIVE SIDES.

Fig. 43. Let the figure be $ABCDEA$; draw the two diagonals DA , DB , and the lines EF , CG , parallel to them, and produced till they meet the base AB , produced to F and G ; join DF and DG , and DFG will be the triangle required.

In the same manner a triangle may be made, equal to any right-lined figure whatever.

PROBLEM



PROBLEM XX.

TO MAKE A TRIANGLE EQUAL TO A GIVEN CIRCLE.

Fig. 44. Let the given circle be A , draw the radius $A O$, and the tangent $A B$, perpendicular to it; on the line $A B$, take $A B$ equal to the circumference of the circle; join $B O$; then will the triangle $A B O$ be equal to the circle.

PROBLEM XXI.

TO MAKE A RECTANGLE, OR A PARALLELOGRAM,
EQUAL TO A GIVEN TRIANGLE.

Fig. 45. Let the given triangle be $A B C$, bisect the base $A B$ in m , through C ; draw $C n$ parallel to the base of the triangle $A B$; draw the line $m n$ and $B O$ parallel to each other; and the rectangle $m n O B$ will be equal to the triangle $A B C$.

PROBLEM XXII.

TO MAKE A SQUARE EQUAL TO A GIVEN RECTANGLE.

Fig. 46. Let the given rectangle be $A B C D$: produce the side $A B$, till $B E$ be equal to $B C$: bisect $A E$ in the point O , on which as a centre, with the radius $O A$, describe a semi-circle $A G F F$, and produce $B C$ to F , on which describe the square $B F G H$, and it will be equal to the rectangle.

In this manner any right-lined figure may be reduced to a square.

PROBLEM XXIII.

TO MAKE A FIGURE EQUAL TO TWO OTHER SIMILAR
FIGURES.

Fig. 47. Let the two similar figures be P and Q , which are squares: let the two sides $A B$ and $B C$ be perpendicular to each other; join their extremities $A C$ by a right line, on which describe the square R , which will be equal to the two squares P and Q taken together.

ALL

All similar figures may be added together in the same manner: for any two similar figures, constructed upon the legs of any right angle, are equal to a similar figure constructed upon the hypotenuse.

PROBLEM XXIV.

TO MAKE A SQUARE EQUAL TO ANY NUMBER OF SQUARES TAKEN TOGETHER.

FIG. 48. Draw two lines $A m$, $A n$, perpendicular to each other: on one line mark $A B$ equal to the side of one of the given squares, and on the other line mark $A C$ equal to the side of another given square: then draw the line $B C$, which will be equal to the side of a square, equal to the two former given squares taken together: then mark $A D$ equal to $B C$, and $A E$ equal to the side of a third given square; then $D E$ will be the side of a square, equal to the sum of the three given squares taken together.

Thus *any number* of similar figures may be added together.

PROBLEM XXV.

TO MAKE PLAIN DIAGONAL SCALES.

FIG. 49. Draw a line of a convenient length, as $A B$, and divided into eleven equal parts $*$; form each of these parts into a rectangle of a sufficient height, by drawing parallel and perpendicular lines; divide the altitude into ten equal parts, if it be for a decimal scale—but if it be for feet and inches, divide into twelve parts; through these points of division draw parallel lines the whole length of the scale; then divide the length of the first large division $A C$ into ten equal parts, both at the top and the bottom of the scale, and connect these points of division by diagonal lines as shown in the figure, and the scale is finished.

* Only four parts are laid down in this scale for want of room.

The use of these diagonal scales is, to take off the dimensions of three figures; and if the first large divisions in the scale from B to C be units, the second set of divisions from C to A will be the tenth parts of an unit, and the divisions in the altitude from D to A will be hundredth parts: if B C be tens, A C will be units, and A D tenth parts: if C B be thousands, A C will be hundreds, and A D tens: for example—if it were required to take off 243 from the scale, fix one foot of the compasses, at the figure 2 of the largest divisions at the bottom of the scale, and extend the other foot to 4 of the second smaller divisions, and on the bottom of the scale; then for the three units slide up both points of the compasses in a perpendicular line, till they fall upon the third longitudinal line, and on that line, with one foot of the compasses remaining fixed, extend the other foot to the third diagonal line, and you will have the extent of the three figures as required.

To measure the Length of any Line.

Take the length of the line, between the compasses, and apply it to the scale: suppose it contains above three of the large divisions, then set one foot of the compasses on the point 3 of the large divisions, and suppose the other foot of the compasses to fall between 4 and 5 of the second divisions; slide up the compasses by a perpendicular motion, keeping one foot on the line of the large division 3, till the other foot fall on the intersection of one of the diagonal lines, which suppose to be four; this shows that the length of the line measured is 344.

P R O B L E M XXVI.

TO MAKE PLAIN SCALES FOR TWO FIGURES.

FIG. 50. If the scale be a decimal one, after dividing it into any proposed number of large divisions, one of those divisions must be subdivided into ten parts: but, if it be a duodecimal scale, one of the large divisions must be divided into twelve parts, and serve to take dimensions for feet and inches.

The

The most proper form for a scale of equal parts is that where the divisions are marked on the very edge; and are generally made of ivory, the edge being made thin for the purpose of pricking off divisions on the paper, without the help of the compasses.

PROBLEM XXVII.

TO MAKE A RIGHT LINE, THAT SHALL BE A MEAN PROPORTIONAL BETWEEN TWO GIVEN LINES.

FIG. 51. Let the two given lines be AC , CB : join them to each other so as to form a right angle at the point C : join the points $A B$, which will be the diameter of a circle; then describe the circle $B C A E$, draw $C D$ perpendicular to $A B$, and it will be the mean proportion required.

NOTE. The angle $A C B$, described in a semicircle, is always a right angle.

The chord $A C$ is a mean proportional between $A D$ and $A B$, and the chord $B C$ is a mean proportional between $B D$ and $A B$.

The square of the hypotenuse of a right-angled triangle is equal to the squares of both the sides, as seen in Problem XXIII.

The three inward angles of a triangle are equal to two right angles.

Triangles that have each angle equal to each other are called similar triangles.

To find the Area of superficial Figures.

The area of a figure is the measure of the surface, without any regard to its thickness or depth.

The area of a superficial figure is the number of square inches, feet, yards, &c. contained on its surface.

Square yards, feet, inches, &c. differ from the same measures in length, as seen in the following table:—

U U U

Linear



<i>Linear Measures.</i>		<i>Square Measures.</i>	
12 Inches 1 Foot	144 Inches 1 Foot
3 Feet 1 Yard	9 Feet 1 Yard
6 Feet 1 Fathom	36 Feet 1 Fathom
16½ Feet or } 5½ Yards } { 1 Pole { or Rod	272½ Feet or } 30½ Yards } { 1 Pole { or Rod
40 Poles 1 Furlong	1600 Poles 1 Furlong
8 Furlongs 1 Mile	.64 Furlongs 1 Mile

PROBLEM XXVIII.

TO FIND THE AREA OF A PARALLELOGRAM, A SQUARE, A RECTANGLE, A RHOMBUS, OR A RHOMBOID.

The area of any of these figures is found by the following rule:

RULE. Multiply the length by the breadth (or, if it be a rhombus or a rhomboid, by the perpendicular height *), and the product will be the area.

FIG. 40. What is the area of a square, one side whereof is 6 feet, and the other side 7 feet? Answer 42 feet.

$$\begin{array}{r} 6 \\ \times 7 \\ \hline 42 \end{array} \text{ feet.}$$

PROBLEM XXIX.

FIG. 18. What is the area of a rhombus, the length whereof is 12 feet, 5 inches, and the perpendicular height 6 feet, 10 inches?

Feet	12	5	inches
	6	10	
	74	6	
	10	4	2
Answer	84	10	2

* The perpendicular height of a parallelogram, a rhombus, &c. is a line drawn from the uppermost side, perpendicular to the base, and the perpendicular height of a triangle is also a line drawn from the uppermost angle, perpendicular to the base.

PROBLEM XXX.

FIG. 17. What is the area of a rhomboid, whose length is 5 chains and 20 links; and perpendicular height 20 chains 5 links?

In this Problem the chains and links are multiplied by each other as decimals, as the links are always decimal parts of the chain, as will be shown hereafter, and the product is the answer in square chains and links, which is divided by 10 to bring them into acres, as there are 10 square chains in an acre; and the decimals are multiplied by 4 for roods, and those decimals again multiplied by 40 for perches, because 4 roods make one acre, and 40 perches one rood.

$$\begin{array}{r}
 5,20 \\
 20,05 \\
 \hline
 2600 \\
 1040 \\
 10) 104,2600 \\
 \hline
 10,4260 \\
 4 \\
 \hline
 1,7040 \\
 40 \\
 \hline
 28,1600
 \end{array}$$

Acres.	Rood.	Perches.	
10	1	28,1600	Ans.

PROBLEM XXXI.

TO FIND THE AREA OF A TRIANGLE.

FIG. 9. The area of all triangles is found by the following rules:—

RULE 1. Multiply the base by the perpendicular height, and half the product will be the area. But when only the three sides of the triangle are given, the area is found by the following rule:—

RULE 2. Add the three sides together, and from half the sum subtract each side separately; multiply the half sum by the remainders of the three sides continually together, and the square root of the product will be the area of the triangle.

EXAMPLE

EXAMPLE 1. What is the area of a trapezoid, whose two parallel sides are 8 feet and 12 feet, and perpendicular distance 10 feet?

$$\begin{array}{r}
 8 \\
 12 \\
 \hline
 20 \\
 \times 5 \\
 \hline
 100
 \end{array}$$

Answer 100 feet.

PROBLEM XXXIV.

TO FIND THE AREA OF A TRAPEZIUM.

RULE. Divide it into two triangles, by a diagonal line drawn from any one angle to the opposite angle, then find the area of these two triangles, by Problem XXXII. and add them together.

Note. Two perpendiculars drawn from the diagonal line to the other two opposite angles will be the perpendiculars of the triangle.

PROBLEM XXXV.

TO FIND THE AREA OF ANY IRREGULAR FIGURE OF ANY NUMBER OF SIDES.

All irregular figures consisting of more than three sides are to be divided into triangles or trapeziums, then find the area of each of these triangles or trapeziums separately, and add them together, and the sum will be the area of the whole figure.

Note. Every figure may be divided into as many triangles, except two, as the figure has sides.

Thus, a figure of five sides may be divided into three triangles; a figure of six sides, into four triangles; a figure of eight sides, into six triangles; &c.

PROBLEM

PROBLEM XXXVI.

TO FIND THE AREA OF A REGULAR POLYGON.

CASE 1. When the centre can be found.

RULE. Multiply the perimeter of the figure, or sum of its sides, by half the perpendicular, drawn from the middle of one of its sides to the centre; and the product will be the answer.

CASE 2. When the centre cannot be found readily, as in pieces of water, where buildings are erected, &c.

RULE 2. Multiply the square of one side of the polygon, by the multiplier set against it, in the following Table; and the product will be the answer.

TABLE.

No. of Sides of the Pol.	Names of the Polygon.	Multipliers.
3	Trigon, or equi. tran.	0.4330127
4	Tetragon, or square	1.0000000
5	Pentagon	1.7204774
6	Hexagon	2.5980762
7	Heptagon	3.6339124
8	Octagon	4.8284271
9	Nonagon	6.1818242
10	Decagon	7.6942088
11	Undecagon	9.3656399
12	Dodecagon	11.1961524

EXAMPLE. What is the area of a regular pentagon, whose side is 25 feet, and perpendicular 17.204774 feet?

By the first rule,

8.602387 $\frac{1}{2}$ per.
125 perim.

43 011935
172 04774
860 2387

1075.298375 Ans.

By the second rule,

1.7204774
625

8 6023870
34 409548
1032 28644

1075.2983750 Ans.

PROBLEM XXXVII.

TO FIND THE DIAMETER OR THE CIRCUMFERENCE OF A CIRCLE, THE ONE FROM THE OTHER.

RULE 1. As 7 is to 22, so is the diameter to the circumference.—As 22 is to 7, so is the circumference to the diameter.

RULE 2. As 113 is to 355, so is the diameter to the circumference.—As 355 is to 113, so is the circumference to the diameter.

RULE 3. As 1 is to 3.1416, so is the diameter to the circumference.—As 3.1416 is to 1, so is the circumference to the diameter.

Q^u. 1. If the diameter of the earth be 7958 miles, as it nearly is, what is the circumference, supposing it were exactly round? **Answer** 25000.8528 miles.

Q^u. 2. What is the diameter of the earth, supposing the circumference 25000 miles? **Answer** 7957 $\frac{1}{2}$ miles.

PROBLEM XXXVIII.

TO FIND THE AREA OF A CIRCLE.

RULE 1. Multiply half the circumference by half the diameter, and the product is the area.

RULE 2. Multiply the square of the diameter by .7854.

RULE 3. Multiply the square of the circumference by .07958.

RULE 4. As 14 is to 11, so is the square of the diameter to the area.

RULE 5. As 88 is to 7, so is the square of the circumference to the area.

Q^u. 1. What is the area of a circle, whose diameter is 7 feet? **Answer** 38.4846 feet.

Q^u.

Q_n. 2. How many square feet does a circle contain, the circumference of which is 10.9956 yards? Answer 86.19266.

The operation of these problems is purposely omitted for the exercise of the learner.

PROBLEM XXXIX.

TO FIND THE LENGTH OF ANY ARC OF A CIRCLE.

RULE 1. As 180 is to the number of degrees in the arc, so is the product of the radius multiplied by 3.1416 to the length of the arc.

RULE 2. As 3 is to the number of degrees in the arc, so is the product of the radius multiplied by .05236 to the length of the arc.

Q_n. 1. What is the length of an arc of 12 degrees 10 minutes, the radius being 10 feet? Answer 2.1234 $\frac{2}{3}$ feet.

Q_n. 2. What is the length of an arc of 30 degrees, the radius being 9 feet? Answer 4.7124 feet.

PROBLEM XL.

TO FIND THE AREA OF A SECTOR OF A CIRCLE.

Fig. 22. RULE 1. Multiply the radius by half the arc of the sector, and the product is the area.

RULE 2. As 360 is to the degrees in the arc of the sector, so is the whole area of the circle to the area of the sector.

Note. For the semicircle take one half of the whole circle; for a quadrant, one quarter; for the sextant, one sixth, &c.

EXAMPLE. What is the area of a sector of a circle, whose radius is 15 feet, and the arc 30 feet?

$$\begin{array}{r} 15 \\ 15 \\ \hline 75 \end{array}$$

Answer 112.5

Q_n.

Q^{n. 2.} What is the area of a quadrant, and a semicircle, whose radius is 13 feet each? **Answer,** the quadrant 138.7326 feet, and semicircle 265.4652 feet.

PROBLEM XLI.

TO FIND THE AREA OF A SEGMENT OF A CIRCLE.

RULE. Find the area of the sector of the circle by the last Problem.

Find the area of the triangle formed by the chord of the segment, and the radius of the sector.

Then the sum of these two will be the answer, if the segment be greater than the semicircle; but if the segment be less than a semicircle, the difference will be the answer.

The foregoing Problems will be found fully sufficient to instruct the learner in the method of measuring every kind of superficies whatever, as there is no figure but may be reduced to one or more of the foregoing figures.

OF MEASURING ARTIFICERS' WORK.

Masons' Work.

To masonry belong all sorts of stone-work, and the measure of it is by the foot, either superficial or solid. Stone walls, columns, blocks of stone, &c. are measured by the solid foot; but slabs, chimney-pieces, pavement, &c. are measured by the superficial foot.

The Prices of Masons' Work.

	£.	s.	d.
Newcastle stone, per foot cubic	0	1	10
Workmanship to ditto, per superficial foot, from			
6d. to	0	1	6
Portland stone, per cube foot	0	4	6
Workmanship to ditto, per foot superficial, from			
1s. to	0	2	8
			Paving

Paving in Portland stones straight, $1\frac{1}{2}$ inch thick, <i>per foot superficial</i>	£. s. d.
Ditto 2 inch thick	0 2 0
Purbeck stone, paving, <i>per foot superficial</i> , from 11d. to	0 1 4
Bremen stone, paving in terraces, <i>per foot superficial</i>	0 1 6
Portland stone, chimney-pieces and slabs, from $1\frac{1}{2}$ inch thick to 3 inches, <i>per foot superficial</i> , from 1s. 11d. to	0 2 9
Marble chimney-pieces veined, <i>per foot cube</i>	1 15 0
Workmanship to ditto, <i>per foot superficial</i> , from 3s. 6d. to	0 8 6
Blue and white marble, an inch thick, <i>per foot superficial</i>	0 5 6

Bricklayers' Work.

Brick-work is measured by the square rod of 16 feet and a half, and the brick-work estimated at a brick and a half thick; if a wall be more or less than this standard thickness, it must be reduced to it, by the following rule:

RULE. Multiply the superficial content of the wall by the number of half bricks in the thickness, and divide the product by 3, and the quotient will be the superficial contents of the wall, at the true standard thickness of a brick and a half.

To take the dimensions of a building, it is usual to measure half round the outside, and also half round the inside; the sum of these two measures multiplied by the height, gives the superficial content, which must be reduced to the standard measure, in order to ascertain the price.

Chimnies are sometimes measured as if they were solid, deducting only the vacuity from the hearth to the mantle; and sometimes they are measured round for their breadth, and the height of the story is their height, and the depth of the jambs is their thickness; in this case no deduction is made for the fire-place.

The

The chimney-shafts, which appear above the building, are measured round for their breadth, and multiplied by their height for the superficial content; and the thickest is accounted half a brick more than it really is, on account of the scaffolding, &c.

All windows, door-ways, &c. are to be deducted in measuring for the materials, but not in measuring for workmanship. There are also some other allowances, as double measure for feathered gable ends, &c.

All the following brick-work is measured by the rod square of $16\frac{1}{2}$ feet, which contains $272\frac{1}{4}$ square feet; hence, when the contents of brick-work is found in square feet, it is divided by 272 (the fraction being neglected), and the quotient is the number of square rods; which is to be reduced to standard measure by the former rule.

Tiling is measured by the square of 10 feet, making one hundred square feet to each square.

Prices.

Brick-work at <i>per rod</i> .		£.	s.	d.
New place bricks, laid dry, without mortar, in wells, cefs-pools, &c.	10 10 0			
Rough greystock brick-work, in party-walls, &c. from 10 <i>l.</i> to	12 0 0			
Labour and mortar only, to brick-work, from 3 <i>l.</i> to	3 15 0			
Labour only to ditto, from 1 <i>l.</i> 8 <i>s.</i> to	1 10 0			
Plain tiling, at <i>per square</i> .				
New plain tiling, from 2 <i>l.</i> 12 <i>s.</i> to	2 15 0			
Labour only, from 6 <i>s.</i> to	0 7 0			

Price of Materials.

Lime, <i>per hundred</i>	0 14 0
Sand, <i>per load</i>	0 6 0
Mortar, <i>per load</i> , 16 <i>s.</i> — <i>per hod</i>	0 0 7
Pointing	

	£.	s.	d.
Pointing mortar, <i>per load</i>	1	12	0
Bricks, <i>per hundred</i> , according to the quality,			
from 3s. 6d. to	0	10	6
Bricklayer, <i>per day</i> , in summer	0	4	8
Ditto, in winter	0	4	3
Labourer, <i>per day</i> , in summer	0	3	4
Ditto, in winter	0	3	0

Carpenters' and Joiners' Work.

Most large and plain articles are measured by the square of 10 feet, each square being 100 feet; but mouldings and other ornaments are generally measured by the foot, running measure.

In measuring flooring, no deductions are made for hearths, on account of the trouble.

Partitions are measured from wall to wall, and from floor to floor, and no deductions made for door-ways.

In measuring roofs, the length of the house in the inside with two thirds of the thickness of one gable, is considered as the length; and the breadth is equal to twice the length from the ridge to the end of the rafters.

For staircases, take the breadth of all the steps by a string from the top of the staircase to the bottom, which multiply by the length of one step.

For wainscoting, take the compass of the whole room for the length, and the height from the floor to the ceiling for the breadth, deducting door-ways, windows, &c. from the materials, but not from the workmanship.

For doors, shutters, &c. These are measured by the *best* square, allowing for their thickness, the number of panels, &c.

Prices.

Flooring, at <i>per square superficial</i> .	£.	s.	d.
Labour and nails only, from 6s. 6d. to . . .	6	12	6

Best,

Roofs, at <i>per square</i> .		£.	s.	d.
Labour and nails only, from 5s. 6d. to . . .		0	18	0
Partitions, at <i>per square</i> .				
Labour and nails, from 5s. 6d. to . . .		0	12	6
Doors, at <i>per foot square superficial</i> .				
Deal ditto		0	1	0
Two-pannelled ditto, according to the thick-				
ness, from 10½d. to		0	1	1½
Four-pannelled ditto, from 1s. 0½d. to . . .		0	1	6½
Sash-frames and sashes, at <i>per foot superficial</i> .				
Deal ditto, single hung, with weights, pullies,				
&c. from 1s. 4d. to		0	2	0
Wainscot, 1s. 6d. to		0	3	3
Mahogany, 2s. 3d. to		0	4	4

Plasterers' Work.

Plasterers' work is of two kinds: viz. plastering upon laths, and rendering, which is plastering upon walls.

They are valued differently;—the contents are either found by the square foot, or the square of 10 feet, or the square yard; but mouldings, &c. are measured by running measure.

Deductions are always made for door-ways, fire-places, &c. but not for windows.

Prices.

	£.	s.	d.
Lime whitening once, <i>per yard</i>	0	0	1½
Colouring, common, <i>per yard</i>	0	0	3½
Rendering rough, one coat, <i>per yard</i>	0	0	3½
Lathing only, <i>per yard</i>	0	0	8
Lathing and plastering, <i>per yard</i> , from 10d. to . . .	0	1	6
Ditto, to ceilings, from 1s. 6d. to	0	2	0
Outside lime and hair, <i>per load</i>	0	1	0
Plaster, <i>per cwt.</i>	0	8	6
Fir laths, <i>per bundle</i>	0	2	6
Size, <i>per gallon</i>	0	0	5
Whitening, <i>per dozen</i>	0	0	2½

Sawyers'

Sawyers' Prices.

	£.	s.	d.
Twelve feet deals and battens sawed, <i>per dozen cuts</i> , from 2s. 9d. to	0	3	6
Ten-feet ditto, from 2s. 7d. to	0	3	0

All extra cuts are charged after the rate of three shillings per square of 10 feet.

Plumbers' Work.

Plumbers' work is rated by the cwt. and sometimes by the single pound, for there is no method yet discovered whereby lead can be cast of an uniform thickness.

Sheet-lead is of two sorts, called cast sheet-lead, and milled sheet-lead.

Solder is charged by the pound, and leaden pipes by the yard or foot running measure.

Sheet-lead used in roofing, guttering, &c. is from 7 to 12lb. weight in each square foot.

Prices.

	£.	s.	d.
New sheet-lead, cast at 7lb. weight to the <i>sq.</i> , <i>per cwt.</i> and labour to ditto	1	1	3
Sash weights, <i>per cwt.</i>	2	1	3
Solder, <i>per lb.</i> 1s.; or, <i>per cwt.</i>	5	12	3
Leaden pipes, <i>per cwt.</i>	2	1	3
Small cast pipe, from half an inch to two inches diameter, <i>per foot</i> , from 1s. to	3	4	3
The new patent copper sheets, covering, <i>per lb.</i>	1	1	3

Painters' Work.

Painters' work is measured by the square yard: and a measuring-line is forced into all the mouldings, and where the brush goes.

Windows are done by the piece, and sash-squares by the dozen.

Chimney-pieces, variegated or fancied colours, &c. by the foot square.

Ornamental figures, mouldings, &c. by the foot running.

Prices.

	£.	s.	d.
Common painting in oil			
Once, <i>per yard</i>	0	0	3½
Ditto, twice	0	0	5½
Ditto, three times	0	0	7½
Sash frames, once in oil, <i>each</i>	0	0	9
Ditto, twice in oil	0	1	2
Sash squares once in oil, <i>per dozen</i>	0	0	8
Ditto, twice	0	1	0
Casements	0	0	6

Glaziers' Work.

Glaziers take their dimensions in feet, inches, and parts, and compute their work by the square foot.

In taking the length and breadth of a window, the cross-bars between the squares are included; and windows of an oval or circular form are measured as if they were square, taking the greatest length and breadth, on account of the waste in cutting the glass.

Prices.

	£.	s.	d.
Plate-glass, cut from 1 foot to 2 feet, <i>per foot superficial</i>	0	5	6
Ditto from 2 feet to 3 feet	0	6	6
Best Ratcliffe crown glass <i>per foot superficial</i>	0	2	4

SECT.

S E C T. II.

OF MEASURING LAND; OR SURVEYING.

Before I proceed to the practice of surveying, it is necessary to mention the instruments used in measuring land.

I. *Of the Chain.*

The chain, called Gunter's chain, consists of 100 equal links, each link being $\frac{22}{100}$ of a yard, or $\frac{66}{100}$ of a foot, that is 7.92 inches long; and the whole chain is 4 poles or 22 yards in length.

An acre of land is equal to 10 square chains.

An acre is divided into 4 equal parts, called roods, and each rood into 40 parts, called perches or poles.

The length of lines measured with the chain is set down in links, as integers, every chain being 100 links in length; therefore, when the content of a piece of land is found in square links, cut off 5 of the figures on the right hand for decimals, and the rest will be acres: those decimals are then multiplied by 4, to bring them into roods, and the decimals of the product multiplied by 40, for perches, as in the following example:

EXAMPLE. What is the area of a rectangular piece of ground, the length of which is 1300 links, and the breadth 240?

$$\begin{array}{r}
 1300 \\
 \times 240 \\
 \hline
 52000 \\
 2600 \\
 \hline
 312000 \\
 4 \\
 \hline
 .48000 \\
 40 \\
 \hline
 \text{Answer } \underline{\underline{3120000}}
 \end{array}$$

3 Acres, 0 Roods, 19 Perches.

Y Y 2

2. *Of*

2. *Of the Plain Table.*

This instrument consists of a plain rectangular board of any convenient size, fixed to a stand of three legs, by means of a ball and socket, by which the table is inclined in any direction: it has also a frame of wood to fit round its edges, which can be taken off, for the purpose of fixing a sheet of paper on the table. One side of the frame is divided into several equal parts, for the purpose of drawing lines across the paper, and the other side of the frame is divided into 360 degrees from a centre in the middle of the table, for the purpose of taking angles, &c. There is also a needle and a compass on one side of the table, to point out the directions, and fix the table in the same position with regard to the points of the compass at every remove.

There is an index also, which is a brass two-foot scale, with two open sights, one at each end: these sights, and one edge of the scale, are in the same plane, and that edge is called the fiducial edge of the index.

In using this table, a sheet of paper is to be wetted and spread smooth, with the frame of the table pressed down close, to keep the paper steady, which is to remain till it is dry, that it may stretch itself smooth: upon this paper the plan is to be drawn.

In taking a survey of any place, a point is to be made on the paper to denote that spot where the table is fixed, which is called the *station*; then in that point fix a pin, or one foot of the compasses, and to it apply the fiducial edge of the index, moving the other part of the index about, till through the sights you perceive one angle of the field you survey, or some other remarkable object; and from the station point draw a line along the fiducial edge of the index. Then turn the index about upon the station point as a centre, till you perceive another angle, or some other object, and draw a line from the station point along the edge of the index, as before: continue to do the same, till you have drawn lines from the
station,

station, to represent the bearings of as many angles or objects as may be necessary, and no more; then measure from the station where the table is fixed to every object which you have viewed, and lay the measures down upon their respective lines on the paper.

1 If it be required, as it sometimes is, to take a survey from more than one station; remove the table, and by the help of the compass fix the table in the same position as before, and mark another point on the paper, for this second station point; and from thence draw lines to as many objects as may be necessary, marking the distance from the station as before.

In using the plain table, choose such a point for your station as shall have an object both before and behind the station point, if it be possible: and in moving the table from one station to another, it will be necessary to prove that it be straight in the line towards the object, and also that the distance be rightly laid down on the paper. To know whether the table be set straight in the line, move the table about till through the sights of the index you can perceive either the fore or back object; then go round the table, and look through the sights at the other end of the index, to see if the other object can be perceived; if it can, the table is in the line, but if not, the table must be shifted according to your judgment.

To know if the table be in the right part of the line, that is, if the distance has been rightly measured, fix the table in the same position as at first, and lay the index along the station line; then turn the table about, till the fore and back objects appear through the sights, and the needle will point to the same degree as at first; then lay the index over the station point, and any other point on the paper representing an object which is seen from the station, and if the same object appears straight through the sights, the station may be depended upon as right, but if not, the distance must be examined and corrected till the said object can be seen.

Of



Of shifting the Paper on the plain Table.

When the paper is full written, and it is necessary to continue the plan upon another sheet, draw a line through the farthest point of the last station line, then take the sheet off the table, and fix another sheet on, drawing a line upon it in the most convenient part for the rest of the work: then fold the old sheet back, close by the line drawn upon it; apply the edge to the line on the new sheet of paper, and as they lie in that position continue the last station line upon the new sheet, and also the rest of the measures, beginning at where the old sheet left off; and so on from one sheet to another. When the work is done, the sheets are to be fastened together into one piece, and the lines in each sheet to be accurately joined together.

It must here be noted, that the said joining lines upon the old and new sheets must have the same inclination with regard to the points of the compass.

3. Of the Theodolite.

This is a brass circular ring divided into 360 degrees, with an index, with open sights, or a telescope moveable upon the centre; also a compass to point out the bearings, &c. The whole is fixed by the centre upon a stand.

When this instrument is used, a field-book is necessary, to note down all measures, angles, &c. to be remembered when the plan is drawn.

In using this instrument, any station may be taken as is judged most convenient, but it is best to take a station from which most objects can be seen; and it is necessary at every new station to fix the Theodolite in the same position by means of viewing the fore and back objects, and the compass, as in using the plain table; registering in the field-book the number of degrees cut off by the index in viewing each object.

The best method of using this instrument is to draw a large circle, quarter it, and mark upon it the number of degrees cut by the index, in viewing each object; then by a parallel ruler draw from station to station, lines parallel to the lines drawn from the centre to the respective points of the circumference.

4. Of the Cross.

This instrument is only two pair of sights, set at right angles to each other upon a staff with a sharp point to stick in the ground.

The cross is very useful to measure small and crooked pieces of ground. The method of measuring them is this: measure a base or chief line in the longest direction of the piece, from corner to corner, and, while measuring it, find the places where perpendicular lines should fall upon this line, from the several corners and bends in the piece, with the cross, by fixing it upon such parts of the line that through one pair of the sights both ends of the line may be seen, and through the other pair of sights the corresponding angle or angles may be seen, and then measure the length, &c.

There are also several other instruments used in surveying; as the Circumferentor, which resembles the *Thornike* both in shape and use; the Semicircle, used for taking angles, &c.; and the Perambulator, for measuring roads and other great distances, on level ground; it has a wheel of $8\frac{1}{2}$ feet in circumference, upon which the machine turns, and the distance measured is pointed out by an index moved round by clock-work.

Levels are instruments with telescopes or other sights, used to find levels, or how much one place is higher than another.

The offset-staff is ten links in length, and used for measuring the offsets and other short distances.



There are also various scales used in protracting, and measuring on the paper; as plain scales, protractor, line of chords, line of equal parts, reducing scale, parallel and perpendicular rules, &c.

But of all the instruments used in surveying, the plain table is by much the best, both in point of accuracy and expedition; for by planning every part immediately upon the spot, while the objects are in view, the work is more correct, and a deal of writing is saved in the field-book; it also affords an opportunity of proving the work at every station: but there are many cases in which some of the other instruments will be found more proper; and some cases in which no instrument at all is necessary, but the chain itself, particularly in large open fields lying together.

In surveying with any instrument, except the plain table, some sort of a field-book is necessary to note down the angles, distances, &c.; but in using the plain table, a book is not requisite, as the whole plan is drawn upon the spot.

This book every person contrives for himself; I shall, however, give the form of one as it is generally used.

Form of the Field-Book.

<i>Offsets and Remarks on the left.</i>	<i>Stations, Bearings, and Distances.</i>	<i>Offsets and Remarks on the right.</i>
21 —	° 1	21
	10° 20'	
	° —	Day's hedge.
Red House 12	24	4°
Offset 20	50	°
	240	
	° 2	
	87° 14'	
	°	
A tree.	30	20 Brown's pond.
A stile 20	90	
	450	10 Sheep pens.

Here

Here, in this system, the book is ruled in three columns; the middle one is for the stations, angles, bearings, distances measured, &c. and those columns on the right and left are for the offsets on the right and left sides, which are set against their corresponding figures in the middle column, and also for such remarks as may be necessary to note.

Thus ° 1 in the middle column stands for the first station where the angle or bearing is $10^{\circ} 20'$. On the left side against 50 links is an offset of 12 links to the red house. On the left at 0, or the beginning of the principal line, is an offset of 21 links, and on the right of the same Day's hedge begins; at 24 on the right an offset to the same hedge of 40 links, and at 50 the same hedge terminates, and at 240 an offset of 20 links on the left.

A line is to be drawn under the work, at the end of every station line, to prevent confusion.

Notwithstanding the field-book is in very general use, yet a more ready method is to draw by hand a rough figure resembling that which is to be measured, and write the dimensions as they are found against the corresponding parts of the figure. This method may be practised in most cases, where the survey is not exceedingly large and intricate.

PROBLEM I.

TO MEASURE A LINE, OR DISTANCE, WITH THE CHAIN.

After having set up a picket or station-staff at the end of the line to be measured, two persons are to begin the measure at the other end of the line. One person is called the leader, and the other the follower.

The leader, having one end of the chain, proceeds forwards in a straight line towards the station-staff, or object, till he has advanced the length of the whole chain: the follower all the while standing still at the beginning of the line, holding the ring, at the other end of the chain, in his hands. When



the leader has arrived to the end of the chain, he is directed by the follower waving his hand to the right, or left, till the follower sees him in the same line with the mark to be measured to; then both of them stooping, and stretching the chain, the leader sticks an arrow in the ground at the end of the chain (he being provided with ten small arrows for that purpose). The leader leaving the arrow in the ground, as a mark for the follower to come to, advances another chain forward, being directed by the follower, waving his hand as before, and also by moving himself from side to side, till he brings the follower and the first mark both in one line; having then stretched the chain, and stuck down another arrow, as before, the follower takes up his arrow, and they both advance another chain length: thus they continue to do, till all the ten arrows are in the hands of the follower, and the leader is advanced to the end of the eleventh chain without an arrow: the follower then sends or brings the ten arrows to the leader, and they proceed on as before. Thus, the arrows are changed from the one to the other at the end of every ten chains.

PROBLEM II.

TO TAKE ANGLES AND BEARINGS.

(See Fig. 1. in Surveying.)

Let C D be two objects, and let it be required to take their bearings, or the angle at the station G.

1. *With the Plain Table.*

The table being covered with a paper, and fixed on its stand, plant it at the station G, and fix a pin, or the point of the compasses, in a point of the paper, to represent the point G: close by the side of this point lay the fiducial edge of the index, and turn it about, still touching the point G, till one object, D, can be seen through the sights, then by the fiducial edge of the index draw the line G D. In the very same

mann

manner draw the line $C G$ from the object seen at C , and the angle $C G D$ on the paper, will be equal to the angle measured.

2. *With the Theodolite.*

Turn the instrument about till you see the object D through the sights, and there screw the instrument fast: then turn the index about, till, through the sights, you see the object C ; then the degrees cut by the index upon the ring of the instrument will show the measure of the angle.

3. *With the Chain only.*

Measure one chain length, or any other length from the angle, to both objects C and D , then measure the distance $D C$, and it is done. This must be transferred to paper, by making a triangle $G C D$, with lengths proportional to the corresponding lengths in the figure.

PROBLEM III.

TO SURVEY A TRIANGULAR FIELD.

1. *By the Chain only.*

Let the field to be measured be represented by the triangle $A B C$, (fig. 1.) then having set up marks at the corners (which is always to be done, when there are no objects to serve as marks) as you measure along the line from A to C , when you have arrived about n , where you judge a perpendicular will fall from the angle B , plant the cross, or any other fit instrument; moving it from one place to another, till, through one pair of sights, you can perceive the marks $A C$, and through the other sights the mark B . Then measure the remainder of the line from n to C , and also the perpendicular $n B$; thus, having the base $A C$, and the perpendicular $n B$, the area is easily found.

Or the area may be found by measuring the three sides of the triangle, as taught in the foregoing section.

2. *By taking one or more Angles.*

This is done by measuring any two sides, and taking the angle between them, as AB , BC , and the angle B ; or, measuring one side, and the two adjacent angles, as AB , and the angles A and B .

PROBLEM IV.

TO SURVEY A FIELD OF FOUR SIDES BY THE CHAIN.

Let the field be represented by the four-sided figure $ABCG$ (fig. 1); measure from A to C , which will be a diagonal, dividing the field into two triangles, ABC , ACG . In measuring along this diagonal line, draw the perpendiculars nB , mG , as directed in the last Problem; then find the areas of the two triangles ABC and AGC , and add them together for the area of the trapezium $ABCG$.

PROBLEM V.

TO SURVEY A FIELD OF ANY NUMBER OF SIDES BY THE CHAIN.

Let the field be represented by the figure $ABCDEFG$ (fig. 1.) Take a view of the field, and consider how it may best be divided into triangles and trapeziums; and divide it into as many trapeziums, and as few triangles, as possible. Thus, this figure is divided into two trapeziums, $ABCG$ $GDEF$, and the triangle CGD . Then the areas of the two trapeziums are found by means of the diagonals AC , FD , and perpendiculars Bn , mG , Go , pE , as in the last Problem: and the area of the triangle, by the base CG , and perpendicular qD , as in Problem III. The sum of these two trapeziums, and triangle, is the area of the whole field.

Note.

Note. The measures of each line may be wrote against the corresponding sides of a similar rough figure, drawn upon paper, or in any other manner.

Figures of this sort are, in fact, divided into triangles; as each trapezium is divided into two triangles.

PROBLEM VI.

TO SURVEY ANY FIELD WITH THE PLAIN TABLE.

FIG. 2. Plant the table at any one angle, as at C, from whence all the other angles can be seen; turn the table about till the needle points to the flower de luce, and there screw it fast: make a point on the paper for C, and then lay the edge of the index to C, turning it about the point C, till through the sights you see the mark D; and by the edge of the index draw a line D C; then measure the distance from C, where the table is fixed, to the mark D, and lay that distance down on the paper, on the line C D. Then turn the index about the point C, till the mark E can be seen through the sights, and draw the line C E, and measure the distance from the station C to the mark E, laying the distance down on the paper, on the line C E: in the same manner find the positions of the points A and B; and lay the lengths of these lines down on the paper, on the line C A, C B: then join the points by the lines C D, D E, E A, A B, B C.

PROBLEM VII.

TO SURVEY A FIELD BY MEASURING BOUNDARY.

Let the field be represented by figure A B C D E F (fig. 3.) Having set up marks at the angles A B, &c. plant the instrument at any point A, and turn it till the fixed index be in the direction A B, and there screw it fast; then turn the moveable index to the direction A F, and the degrees cut off will be the measure of the angle A: measure the line A B, planting the instrument at B; and there in the same manner

obtain

Of shifting the Paper on the plain Table.

When the paper is full written, and it is necessary to continue the plan upon another sheet, draw a line through the farthest point of the last station line, then take the sheet off the table, and fix another sheet on, drawing a line upon it in the most convenient part for the rest of the work: then fold the old sheet back, close by the line drawn upon it; apply the edge to the line on the new sheet of paper, and as they lie in that position continue the last station line upon the new sheet, and also the rest of the measures, beginning at where the old sheet left off; and so on from one sheet to another. When the work is done, the sheets are to be fastened together into one piece, and the lines in each sheet to be accurately joined together.

It must here be noted, that the said joining lines upon the old and new sheets must have the same inclination with regard to the points of the compass.

3. Of the Theodolite.

This is a brass circular ring divided into 360 degrees, with an index, with open sights, or a telescope moveable upon the centre; also a compass to point out the bearings, &c. The whole is fixed by the centre upon a stand.

When this instrument is used, a field-book is necessary, to note down all measures, angles, &c. to be remembered when the plan is drawn.

In using this instrument, any station may be taken as is judged most convenient, but it is best to take a station from which most objects can be seen; and it is necessary at every new station to fix the Theodolite in the same position by means of viewing the fore and back objects, and the compass, as in using the plain table; registering in the field-book the number of degrees cut off by the index in viewing each object.

The

The best method of using this instrument is to draw a large circle, quarter it, and mark upon it the number of degrees cut by the index, in viewing each object; then by a parallel ruler draw from station to station, lines parallel to the lines drawn from the centre to the respective points of the circumference.

4. *Of the Cross.*

This instrument is only two pair of sights, set at right angles to each other upon a staff with a sharp point to stick in the ground.

The cross is very useful to measure small and crooked pieces of ground. The method of measuring them is this: measure a base or chief line in the longest direction of the piece, from corner to corner, and, while measuring it, find the places where perpendicular lines should fall upon this line, from the several corners and bends in the piece, with the cross, by fixing it upon such parts of the line that through one pair of the sights both ends of the line may be seen, and through the other pair of sights the corresponding angle or angles may be seen, and then measure the length, &c.

There are also several other instruments used in surveying; as the Circumferentor, which resembles the Theodolite both in shape and use; the Semicircle, used for taking angles, &c.; and the Perambulator, for measuring roads and other great distances, on level ground; it has a wheel of $8\frac{1}{2}$ feet in circumference, upon which the machine turns, and the distance measured is pointed out by an index moved round by clock-work.

Levels are instruments with telescopes or other sights, used to find levels, or how much one place is higher than another.

The offset-staff is ten links in length, and useful for measuring the offsets and other short distances.

There

2. *Of the Plain Table.*

This instrument consists of a plain rectangular board of any convenient size, fixed to a stand of three legs, by means of a ball and socket, by which the table is inclined in any direction: it has also a frame of wood to fit round its edges, which can be taken off, for the purpose of fixing a sheet of paper on the table. One side of the frame is divided into several equal parts, for the purpose of drawing lines across the paper, and the other side of the frame is divided into 360 degrees from a centre in the middle of the table, for the purpose of taking angles, &c. There is also a needle and a compass on one side of the table, to point out the directions, and fix the table in the same position with regard to the points of the compass at every remove.

There is an index also, which is a brass two-foot scale, with two open sights, one at each end: these sights, and one edge of the scale, are in the same plane, and that edge is called the *fiducial edge* of the index.

In using this table, a sheet of paper is to be wetted and spread smooth, with the frame of the table pressed down close, to keep the paper steady, which is to remain till it is dry, that it may stretch itself smooth: upon this paper the plan is to be drawn.

In taking a survey of any place, a point is to be made on the paper to denote that spot where the table is fixed, which is called the *station*; then in that point fix a pin, or one foot of the compasses, and to it apply the fiducial edge of the index, moving the other part of the index about, till through the sights you perceive one angle of the field you survey, or some other remarkable object; and from the station point draw a line along the fiducial edge of the index. Then turn the index about upon the station point as a centre, till you perceive another angle, or some other object, and draw a line from the station point along the edge of the index, as before: continue to do the same, till you have drawn lines from the
station,

station, to represent the bearings of as many angles or objects as may be necessary, and no more; then measure from the station where the table is fixed to every object which you have viewed, and lay the measures down upon their respective lines on the paper.

If it be required, as it sometimes is, to take a survey from more than one station; remove the table, and by the help of the compass fix the table in the same position as before, and mark another point on the paper, for this second station point; and from thence draw lines to as many objects as may be necessary, marking the distance from the station as before.

In using the plain table, choose such a point for your station as shall have an object both before and behind the station point, if it be possible: and in moving the table from one station to another, it will be necessary to prove that it be straight in the line towards the object, and also that the distance be rightly laid down on the paper. To know whether the table be set straight in the line, move the table about till through the sights of the index you can perceive either the fore or back object; then go round the table, and look through the sights at the other end of the index, to see if the other object can be perceived; if it can, the table is in the line, but if not, the table must be shifted according to your judgment.

To know if the table be in the right part of the line, that is, if the distance has been rightly measured, fix the table in the same position as at first, and lay the index along the station line; then turn the table about, till the fore and back objects appear through the sights, and the needle will point to the same degree as at first; then lay the index over the station point, and any other point on the paper representing an object which is seen from the station, and if the same object appears straight through the sights, the station may be depended upon as right, but if not, the distance must be examined and corrected till the said object can be seen.

Of

Of shifting the Paper on the plain Table.

When the paper is full written, and it is necessary to continue the plan upon another sheet, draw a line through the farthest point of the last station line, then take the sheet off the table, and fix another sheet on, drawing a line upon it in the most convenient part for the rest of the work: then fold the old sheet back, close by the line drawn upon it; apply the edge to the line on the new sheet of paper, and as they lie in that position continue the last station line upon the new sheet, and also the rest of the measures, beginning at where the old sheet left off; and so on from one sheet to another. When the work is done, the sheets are to be fastened together into one piece, and the lines in each sheet to be accurately joined together.

It must here be noted, that the said joining lines upon the old and new sheets must have the same inclination with regard to the points of the compass.

3. Of the Theodolite.

This is a brass circular ring divided into 360 degrees, with an index, with open sights, or a telescope moveable upon the centre; also a compass to point out the bearings, &c. The whole is fixed by the centre upon a stand.

When this instrument is used, a field-book is necessary, to note down all measures, angles, &c. to be remembered when the plan is drawn.

In using this instrument, any station may be taken as is judged most convenient, but it is best to take a station from which most objects can be seen; and it is necessary at every new station to fix the Theodolite in the same position by means of viewing the fore and back objects, and the compass, as in using the plain table; registering in the field-book the number of degrees cut off by the index in viewing each object.

The

The best method of using this instrument is to draw a large circle, quarter it, and mark upon it the number of degrees cut by the index, in viewing each object; then by a parallel ruler draw from station to station, lines parallel to the lines drawn from the centre to the respective points of the circumference.

4. *Of the Cross.*

This instrument is only two pair of sights, set at right angles to each other upon a staff with a sharp point to stick in the ground.

The cross is very useful to measure small and crooked pieces of ground. The method of measuring them is this: measure a base or chief line in the longest direction of the piece, from corner to corner, and, while measuring it, find the places where perpendicular lines should fall upon this line, from the several corners and bends in the piece, with the cross, by fixing it upon such parts of the line that through one pair of the sights both ends of the line may be seen, and through the other pair of sights the corresponding angle or angles may be seen, and then measure the length, &c.

There are also several other instruments used in surveying; as the Circumferentor, which resembles the Theodolite both in shape and use; the Semicircle, used for taking angles, &c.; and the Perambulator, for measuring roads and other great distances, on level ground; it has a wheel of $8\frac{1}{2}$ feet in circumference, upon which the machine turns, and the distance measured is pointed out by an index moved round by clock-work.

Levels are instruments with telescopes or other sights, used to find levels, or how much one place is higher than another.

The offset-staff is ten links in length, and useful for measuring the offsets and other short distances.

There

There are also various scales used in protracting, and measuring on the paper; as plain scales, protractor, line of chords, line of equal parts, reducing scale, parallel and perpendicular rules, &c.

But of all the instruments used in surveying, the plain table is by much the best, both in point of accuracy and expedition; for by planning every part immediately upon the spot, while the objects are in view, the work is more correct, and a deal of writing is saved in the field-book; it also affords an opportunity of proving the work at every station: but there are many cases in which some of the other instruments will be found more proper; and some cases in which no instrument at all is necessary, but the chain itself, particularly in large open fields lying together.

In surveying with any instrument, except the plain table, some sort of a field-book is necessary to note down the angles, distances, &c.; but in using the plain table, a book is not requisite, as the whole plan is drawn upon the spot.

This book every person contrives for himself; I shall, however, give the form of one as it is generally used.

Form of the Field-Book.

<i>Offsets and Remarks on the left.</i>	<i>Stations, Bearings, and Distances.</i>	<i>Offsets and Remarks on the right.</i>
21 —	° 1	21
	10° 20'	
	0 —	Day's hedge.
Red House 12	24	40
Offset 20	50	0
	240	
	° 2	
	870° 14'	
	0	
A tree.	30	20 Brown's pond.
A stile 20	90	
	450	10 Sheep pens.

Here

Here, in this system, the book is ruled in three columns; the middle one is for the stations, angles, bearings, distances measured, &c. and those columns on the right and left are for the offsets on the right and left sides, which are set against their corresponding figures in the middle column, and also for such remarks as may be necessary to note.

Thus ° 1 in the middle column stands for the first station where the angle or bearing is $10^{\circ} 20'$. On the left side against 50 links is an offset of 12 links to the red house. On the left at 0, or the beginning of the principal line, is an offset of 21 links, and on the right of the same Day's hedge begins; at 24 on the right an offset to the same hedge of 40 links, and at 50 the same hedge terminates, and at 240 an offset of 20 links on the left.

A line is to be drawn under the work, at the end of every station line, to prevent confusion.

Notwithstanding the field-book is in very general use, yet a more ready method is to draw by hand a rough figure resembling that which is to be measured, and write the dimensions as they are found against the corresponding parts of the figure. This method may be practised in most cases, where the survey is not exceedingly large and intricate.

PROBLEM I.

TO MEASURE A LINE, OR DISTANCE, WITH THE CHAIN.

After having set up a picket or station-staff at the end of the line to be measured, two persons are to begin the measure at the other end of the line. One person is called the leader, and the other the follower.

The leader, having one end of the chain, proceeds forwards in a straight line towards the station-staff, or object, till he has advanced the length of the whole chain: the follower all the while standing still at the beginning of the line, holding the ring, at the other end of the chain, in his hand. When

the leader has arrived to the end of the chain, he is directed by the follower waving his hand to the right, or left, till the follower sees him in the same line with the mark to be measured to; then both of them stooping, and stretching the chain, the leader sticks an arrow in the ground at the end of the chain (he being provided with ten small arrows for that purpose). The leader leaving the arrow in the ground, as a mark for the follower to come to, advances another chain forward, being directed by the follower, waving his hand as before, and also by moving himself from side to side, till he brings the follower and the first mark both in one line; having then stretched the chain, and stuck down another arrow, as before, the follower takes up his arrow, and they both advance another chain length: thus they continue to do, till all the ten arrows are in the hands of the follower, and the leader is advanced to the end of the eleventh chain without an arrow: the follower then sends or brings the ten arrows to the leader, and they proceed on as before. Thus, the arrows are changed from the one to the other at the end of every ten chains.

PROBLEM II.

TO TAKE ANGLES AND BEARINGS.

(See Fig. 1. in Surveying.)

Let C D be two objects, and let it be required to take their bearings, or the angle at the station G.

1. *With the Plain Table.*

The table being covered with a paper, and fixed on its stand, plant it at the station G, and fix a pin, or the point of the compasses, in a point of the paper, to represent the point G: close by the side of this point lay the fiducial edge of the index, and turn it about, still touching the point G, till one object, D, can be seen through the sights, then by the fiducial edge of the index draw the line G D. In the very same manner

manner draw the line CG from the object seen at C , and the angle CGD on the paper, will be equal to the angle measured.

2. *With the Theodolite.*

Turn the instrument about till you see the object D through the sights, and there screw the instrument fast: then turn the index about, till, through the sights, you see the object C ; then the degrees cut by the index upon the ring of the instrument will show the measure of the angle.

3. *With the Chain only.*

Measure one chain length, or any other length from the angle, to both objects C and D , then measure the distance DC , and it is done. This must be transferred to paper, by making a triangle GCD , with lengths proportional to the corresponding lengths in the figure.

PROBLEM III.

TO SURVEY A TRIANGULAR FIELD.

1. *By the Chain only.*

Let the field to be measured be represented by the triangle ABC , (fig. 1.) then having set up marks at the corners (which is always to be done, when there are no objects to serve as marks) as you measure along the line from A to C , when you have arrived about n , where you judge a perpendicular will fall from the angle B , plant the cross, or any other fit instrument; moving it from one place to another, till, through one pair of sights, you can perceive the marks A C , and through the other sights the mark B . Then measure the remainder of the line from n to C , and also the perpendicular nB ; thus, having the base AC , and the perpendicular nB , the area is easily found.

Or the area may be found by measuring the three sides of the triangle, as taught in the foregoing Section.

2. *By taking one or more Angles.*

This is done by measuring any two sides, and taking the angle between them, as AB , BC , and the angle B ; or, measuring one side, and the two adjacent angles, as AB , and the angles A and B .

PROBLEM IV.

TO SURVEY A FIELD OF FOUR SIDES BY THE CHAIN.

Let the field be represented by the four-sided figure $ABCG$ (fig. 1); measure from A to C , which will be a diagonal, dividing the field into two triangles, ABC , ACG . In measuring along this diagonal line, draw the perpendiculars nb , mg , as directed in the last Problem; then find the areas of the two triangles ABC and AGC , and add them together for the area of the trapezium $ABCG$.

PROBLEM V.

TO SURVEY A FIELD OF ANY NUMBER OF SIDES BY THE CHAIN.

Let the field be represented by the figure $ABCDEFG$ (fig. 1.) Take a view of the field, and consider how it may best be divided into triangles and trapeziums; and divide it into as many trapeziums, and as few triangles, as possible. Thus, this figure is divided into two trapeziums, $ABCG$ $GDEFG$, and the triangle CGD . Then the areas of the two trapeziums are found by means of the diagonals AC , FD , and perpendiculars Bn , mg , Go , pE , as in the last Problem: and the area of the triangle, by the base CG , and perpendicular qD , as in Problem III. The sum of these two trapeziums, and triangle, is the area of the whole field.

Note.

Note. The measures of each line may be wrote against the corresponding sides of a similar rough figure, drawn upon paper, or in any other manner.

Figures of this sort are, in fact, divided into triangles; as each trapezium is divided into two triangles.

PROBLEM VI.

TO SURVEY ANY FIELD WITH THE PLAIN TABLE.

FIG. 2. Plant the table at any one angle, as at C, from whence all the other angles can be seen; turn the table about till the needle points to the flower de luce, and there screw it fast: make a point on the paper for C, and then lay the edge of the index to C, turning it about the point C, till through the sights you see the mark D; and by the edge of the index draw a line D C; then measure the distance from C, where the table is fixed, to the mark D, and lay that distance down on the paper, on the line C D. Then turn the index about the point C, till the mark E can be seen through the sights, and draw the line C E, and measure the distance from the station C to the mark E, laying the distance down on the paper, on the line C E: in the same manner find the positions of the points A and B; and lay the lengths of these lines down on the paper, on the line C A, C B: then join the points by the lines C D, D E, E A, A B, B C.

PROBLEM VII.

TO SURVEY A FIELD BY MEASURING ROUND IT.

Let the field be represented by figure A B C D E F (fig. 3.) Having set up marks at the angles A B, &c. plant the instrument at any point A, and turn it till the fixed index be in the direction A B, and there screw it fast; then turn the moveable index to the direction A F, and the degrees cut off will be the measure of the angle A: measure the line A B, planting the instrument at B; and there in the same manner obtain

obtain the angle B; then measure B C, and observe the angle C; then measure the distance C D, and take the angle D; then measure D E, and take the angle E; then measure E F, and take the angle F; and lastly, measure the distance F A.

To prove the work;—add all the inward angles A B C, &c. together; and if the work be right, the sum will be equal to twice as many right angles as the figure has sides, except four right angles; but when there is an angle that bends inwards, as at F, and you measure the outward angles, subtract it from four right angles, or 360 degrees, and the remainder will be the inward angle.

Otherwise;

Instead of taking the inward angles, you may take the outward angles, by producing one side of each angle further out, as seen in the figure: and if the sum of all the outward angles be equal to 360 degrees, or four right angles, the work is right: but when one of the angles runs inwards, as F, subtract it from the sum of the rest, and the remainder will be 4 right angles.

PROBLEM VIII.

TO SURVEY A FIELD WITH CROOKED HEDGES.

Let A B C D E be the shape of the field, (fig. 4.) Set up marks within the field, at a b c d, dividing it into as few sides as possible. Then begin at any station, as at a, and measure along the imaginary lines a b, b c, c d, d a, taking the offsets as you measure along the lines, at a, m, n, o, p, &c. either with the offset staff, or any other instrument; which are to be added to the area of the figure. The area is found either by measuring a diagonal from b to d, with perpendiculars to a, &c. or by taking the angles at the four corners a, b, c, d, as before directed.

Note.

Note. The area of the offsets is found either by the rule for finding the area of a triangle; or taking the mean measure of the perpendiculars, for the mean distance of the hedge; thus, the offset from o to B forms a triangle whose perpendicular is p. But that from B to D must be measured by adding the two perpendiculars together, and taking half the sum for the mean distance of the hedge from the line b c, which is to be multiplied thereby.

If the measurement be taken outside the offsets, they must then be subtracted from the area.

PROBLEM IX.

TO SURVEY ANY PLACE BY TWO STATIONS.

Let the place to be measured be represented by the heptagonal figures A B C D E F G (fig. 5.); plant the plain table at m, for the first station; then turn the table about, till through the sights you perceive n, the other station. Draw the line m n on the paper, along which lay the fiducial edge of the index, and then screw the table fast; turn the sights round m, to all the objects A B C, &c. successively, drawing a dry line by the edge of the index to each, as m A, m B, &c. Next, measure the distance on the ground from m, the first station, to n, the other station, and there plant the table, and lay that distance down on the paper, on the line m n; then lay the index by the station line m n, and turn the table about, till through the sights you perceive the other station m, and there screw it fast. Lastly, direct the sights successively to all the objects A B C, &c. as before, drawing lines from each, as n A, n B, &c. and the intersections of these lines with the former at the points A B C D, &c. give the place of the objects, or bounds of the figure.

The same method must be pursued in surveying with the theodolite, or any other instrument for taking angles; namely, measuring the distance m n, placing the instrument first at one station, then at another; placing the fixed sights

in the direction $m n$, and directing the moveable sights to every object, noting the degrees, cut off at each time.

These observations being planned, the intersections of the lines give the bounds of the figure, as before.

If all the objects cannot be seen from two stations, then more stations may be used; always measuring the distance from one station to another, placing the instrument in the same position at every station, and from each station observing every object that can be seen from it, by taking its angular position; till the place of every object be determined by the intersections of two or more lines—the more lines the better.

The stations may be taken either within the bounds of the figure, or in one side, or at a distance, and without the bounds of the figure.

In this manner very extensive surveys may be taken, of large estates, without entering them; or a survey may be taken of a country, or any parts of a country, by taking two stations, on two towers or eminences.

Remarks.

1. Take all the angles between the stations that are necessary, always measuring the distance from station to station in a right line. In measuring any of the station distances, mark where these lines meet with any ditches, hedges, roads, rivulets, or other remarkable objects, by measuring the distance from the station line, and where the perpendicular from it cuts that line; and as you measure along any main station line, take the offsets to the ends of all hedges, and to any pond, house, rivulet, &c. and be careful to set up marks at the intersections of all hedges of the station line, that you may know where to measure from when you come to survey these particular places. By such means all your station lines are to be measured, and the situation of all places adjoining to them determined.

2. The

2. The inner parts of the survey must be determined in like manner, by new station lines; taking inner stations at those places where you have the greatest command or view. Measure these station lines as before, and note all the intersections with hedges or other objects. Thus you may survey the adjoining fields, by taking the angles that the sides make with the station line at the intersections; and measure the distances to each corner from the intersections; for every station line will be a basis to all the future operations, the situation of all parts being entirely dependant upon them; therefore the station lines should be as long as possible, and should be so contrived as to run along the hedge or bound of some field, or to pass through some angle. These things being adjusted, you must take more inner stations, and divide and subdivide till you come to single fields, repeating the same work at the inner stations as at the outer ones; and take notice how one field lies by another, to place them properly in the draft.

3. When an estate is so situated that one part cannot be seen from another, it must be divided into two or more parts, and each part surveyed separately, as if they belonged to different owners; and at last join them together in the plan.

4. In laying the lines of distances down upon the paper, it is necessary to have a scale of a proper length. For this purpose you must take the length of the whole estate in chains, and consider how many inches in length the map is to be, and from these you may know how many chains there must be in an inch on the scale: thus, if the length of the estate be 200 chains, and the map is to be 20 inches in length, a decimal scale should be used; and each inch on the map will contain ten chains in length, a tenth of an inch, 1 chain, &c.

5. When the stations are long, a good instrument should be used for taking angles; and the plain table may be used for the inner stations, as it is a quick and ready instrument.

6. When the draft is finished, place all the trees, and every other remarkable object, in their proper situation.

P R O B L E M X.

TO SURVEY A LARGE TRACT OF LAND ; OR A COUNTY.

1. In choosing your stations, let them be on the top of some eminences ; as, high hills, mountains, towers, church steeples, &c. and such as command the greatest view, and can be seen from each other ; and as far distant from each other as possible. Raise a beacon, or flag, of a different colour, at each, to be visible from the other stations.

2. Plant flags also at each of those places you should distinguish in your plan, fixing them upon the tops of church steeples, houses, &c. and changing them to other places when necessary.

3. Then from one station take all the angles between the other station and each of those marks, observing the different colours of the flags, and note them down in the plan ; from the other station take all the angles between the first station and each of the former marks, and note them down in the plan also against the same coloured mark ; if it be necessary, the angles may also be taken from a third station, which will serve to prove the work, if the three lines intersect each other in the same point. The instrument for taking angles must be an exceedingly good one. A circumferentor is the best of four or five feet radius with telescope sights.

4. It is not necessary to measure every distance, because any stationary line being exactly laid down from any scale, all the other lines will be proportional to it : but sometimes it is necessary to measure the distance of a place in miles : and in measuring long distances, it will not be exact enough to measure along the high roads, by reason of their windings ; but the best way is, to measure in a straight line, over hills and dales, and all obstacles, if possible ; but where it is not possible,

possible, such parts must be measured by the method of measuring inaccessible distances. A good compass is also necessary, to show the bearing of the two stations, and to direct you to go straight when you do not see the stations.

5. From all your stations, and in the whole progress, be very careful in observing all remarkable objects; as, towns, castles, churches, windmills, rivulets, bridges, &c. &c.

6. Having taken your observations from the main stations which command the whole country, then proceed to take inner stations, as in surveying large estates; and from these new stations determine the places of as many of the remaining towns as you can; and if any remain, the situation of which you cannot take, take more inner stations; and thus proceed through all the parts, taking one station after another as long as is necessary.

7. Lastly, take the position of the station line with regard to the points of the compass, by an astronomical process, as follows: hang a plummet in the sun, over some part of the station line, and when the shadow of the plummet line runs along the station line, at that moment take the sun's altitude; then, having his declination and the altitude of the place, the azimuth will be found by spherical trigonometry; for the azimuth is the angle which the station line makes with the meridian: and thus the meridian may be drawn through the map.

PROBLEM XI.

TO SURVEY A TOWN OR CITY.

The proper instrument for this purpose is the plain table; as the distances are not very great, and every part should be laid down while in sight.

Take the first station at the meeting of two or more of the principal streets, in order to get the longest station lines, then draw the lines of direction to represent these streets: having chosen objects, measure the streets, and lay the mea-

6. When the draft is finished, place all the trees, and every other remarkable object, in their proper situation.

PROBLEM X.

TO SURVEY A LARGE TRACT OF LAND; OR A COUNTY.

1. In choosing your stations, let them be on the top of some eminences; as, high hills, mountains, towers, church steeples, &c. and such as command the greatest view, and can be seen from each other; and as far distant from each other as possible. Raise a beacon, or flag, of a different colour, at each, to be visible from the other stations.

2. Plant flags also at each of those places you should distinguish in your plan, fixing them upon the tops of church steeples, houses, &c. and changing them to other places when necessary.

3. Then from one station take all the angles between the other station and each of those marks, observing the different colours of the flags, and note them down in the plan; from the other station take all the angles between the first station and each of the former marks, and note them down in the plan also against the same coloured mark; if it be necessary, the angles may also be taken from a third station, which will serve to prove the work, if the three lines intersect each other in the same point. The instrument for taking angles must be an exceedingly good one. A circumferentor is the best of four or five feet radius with telescope sights.

4. It is not necessary to measure every distance, because any stationary line being exactly laid down from any scale, all the other lines will be proportional to it: but sometimes it is necessary to measure the distance of a place in miles: and in measuring long distances, it will not be exact enough to measure along the high roads, by reason of their windings; but the best way is, to measure in a straight line, over hills and dales, and all obstacles, if possible; but where it is not possible,

possible, such parts must be measured by the method of measuring inaccessible distances. A good compass is also necessary, to show the bearing of the two stations, and to direct you to go straight when you do not see the station.

5. From all your stations, and in the whole progress, be very careful in observing all remarkable objects: as towns, castles, churches, windmills, rivulets, bridges &c. &c.

6. Having taken your observations from the main stations which command the whole country, next proceed to take inner stations, as in surveying large estates. And from these new stations determine the places of as many of the remaining towns as you can; and if any remain, the situation of which you cannot take, take more inner stations: and thus proceed through all the parts, taking one station after another as long as is necessary.

7. Lastly, take the position of the station line with regard to the points of the compass, by an astronomical method as follows: hang a plummet in the line over some part of the station line, and when the shadow of the plummet line runs along the station line, at that moment take the sun's altitude: then, having his declination and the altitude of the place, the azimuth will be found by spherical trigonometry. The azimuth is the angle which the station line makes with the meridian: and thus the meridian may be drawn through the map.

PROBLEM II.

TO SURVEY A TOWN BY THE

The proper instrument for this purpose is the plane table; as the distances are not very great, and every part being so laid down while in sight.

Take the first station at the meeting of two or more of the principal streets, as where it is the largest square is: then draw the lines of direction to the several other parts: having chosen any one, measure the distance and so on to the

asures down from a scale, on the lines on the paper, taking offsets with the staff, to all bendings and windings in the streets, and to all remarkable objects; then take another station in one of the foregoing lines, and repeat the process.

Thus, (fig. 6.) having chosen A for the first station, draw two lines in the direction of the two streets meeting there; and measure A B, marking the street on the left, and lay the same measure down on the plan: let B be the second station, from which draw the directions of the streets meeting there; measure from B to C, noting the ends of the streets at n and o as you pass them; and lay the same measure down on the plan. At C, the third station, take the direction of the streets meeting there, and measure from C to D. At D, the next station, do the same, and measure D E, noting the cross street at p; and in the same manner take the directions of all the principal streets, laying the measures down upon the plan. Then proceed to the next smaller streets; and lastly, to the lanes, alleys, courts, &c. using the same process throughout the whole.

Note. In taking the survey of a town there is no necessity for having objects fixed up, as there are generally marks enough to serve as objects; such as the doors or windows of houses, &c.

OF PLANNING, CASTING UP, AND DIVIDING.

PROBLEM I.

TO LAY DOWN THE PLAN OF A SURVEY.

When the survey is taken by the plain table, a rough draft of the plan is already drawn on the paper; but where the survey is taken with any other instrument, the plan is to be drawn from the measures taken in the survey: first, a rough draft is to be drawn on paper, in which all the lines and angles must be laid down in the same order in which they were

were taken in the survey; laying down first the angles, then the lengths of the lines, with the places of the offsets; then the offsets themselves. All these should be done with dry or obscure lines; then a black line drawn through the extremities of all the lines, and the offsets will be the bounding line of the whole plan.

After the principal lines and bounds are laid down, proceed to mark the smaller objects, till every thing be laid down that is necessary.

The north side of the map or plan is commonly placed uppermost. And in some vacant part a scale of equal parts must be drawn. All hills must be shadowed, and all hedges coloured with different colours; all hilly grounds should be drawn like broken hills and valleys: foot-paths are represented by single dotted lines, and roads by double ones.

In taking large surveys, all oblique lines, such as are measured up and down hills, must be reduced to straight lines, by making a proper allowance; for which purpose there is generally a table engraved on some of the instruments.

PROBLEM II.

TO CAST UP THE CONTENTS OF A FIELD.

This is no more than what has been taught in the first section of this chapter, for finding the contents of figures, whether they be triangular, square, four-sided, polygonal, or circular figures, by the proper rules there delivered; then, having found the area in links, after you have cut off five figures on the right hand, the remainder is acres. The five separated figures are to be multiplied by 4 for roods, and five figures cut off from this product are to be multiplied by 40 for perches.

In pieces of land bounded by winding hedges, in measuring the offsets, all the parts between the offsets are accurately measured separately, like small triangles or trapezoids; but
sometimes

lures down from a scale, on the lines on the paper, taking offsets with the staff, to all bendings and windings in the streets, and to all remarkable objects; then take another station in one of the foregoing lines, and repeat the process.

Thus, (fig. 6.) having chosen A for the first station, draw two lines in the direction of the two streets meeting there; and measure A B, marking the street on the left, and lay the same measure down on the plan: let B be the second station, from which draw the directions of the streets meeting there; measure from B to C, noting the ends of the streets at n and o as you pass them; and lay the same measure down on the plan. At C, the third station, take the direction of the streets meeting there, and measure from C to D. At D, the next station, do the same, and measure D E, noting the cross street at p; and in the same manner take the directions of all the principal streets, laying the measures down upon the plan. Then proceed to the next smaller streets; and lastly, to the lanes, alleys, courts, &c. using the same process throughout the whole.

Note. In taking the survey of a town there is no necessity for having objects fixed up, as there are generally marks enough to serve as objects; such as the doors or windows of houses, &c.

OF PLANNING, CASTING UP, AND DIVIDING.

PROBLEM I.

TO LAY DOWN THE PLAN OF A SURVEY.

When the survey is taken by the plain table, a rough draft of the plan is already drawn on the paper; but where the survey is taken with any other instrument, the plan is to be drawn from the measures taken in the survey: first, a rough draft is to be drawn on paper, in which all the lines and angles must be laid down in the same order in which they were

6. When the draft is finished, place all the trees, and every other remarkable object, in their proper situation.

PROBLEM X.

TO SURVEY A LARGE TRACT OF LAND; OR A COUNTY.

1. In choosing your stations, let them be on the top of some eminences; as, high hills, mountains, towers, church steeples, &c. and such as command the greatest view, and can be seen from each other; and as far distant from each other as possible. Raise a beacon, or flag, of a different colour, at each, to be visible from the other stations.

2. Plant flags also at each of those places you should distinguish in your plan, fixing them upon the tops of church steeples, houses, &c. and changing them to other places when necessary.

3. Then from one station take all the angles between the other station and each of those marks, observing the different colours of the flags, and note them down in the plan; from the other station take all the angles between the first station and each of the former marks, and note them down in the plan also against the same coloured mark; if it be necessary, the angles may also be taken from a third station, which will serve to prove the work, if the three lines intersect each other in the same point. The instrument for taking angles must be an exceedingly good one. A circumferentor is the best of four or five feet radius with telescope sights.

4. It is not necessary to measure every distance, because any stationary line being exactly laid down from any scale, all the other lines will be proportional to it: but sometimes it is necessary to measure the distance of a place in miles: and in measuring long distances, it will not be exact enough to measure along the high roads, by reason of their windings; but the best way is, to measure in a straight line, over hills and dales, and all obstacles, if possible; but where it is not possible,

possible, such parts must be measured by the method of measuring inaccessible distances. A good compass is also necessary, to show the bearing of the two stations, and to direct you to go straight when you do not see the stations.

5. From all your stations, and in the whole progress, be very careful in observing all remarkable objects; as, towns, castles, churches, windmills, rivulets, bridges, &c. &c.

6. Having taken your observations from the main stations which command the whole country, then proceed to take inner stations, as in surveying large estates; and from these new stations determine the places of as many of the remaining towns as you can; and if any remain, the situation of which you cannot take, take more inner stations; and thus proceed through all the parts, taking one station after another as long as is necessary.

7. Lastly, take the position of the station line with regard to the points of the compass, by an astronomical process, as follows: hang a plummet in the sun, over some part of the station line, and when the shadow of the plummet line runs along the station line, at that moment take the sun's altitude; then, having his declination and the altitude of the place, the azimuth will be found by spherical trigonometry; for the azimuth is the angle which the station line makes with the meridian: and thus the meridian may be drawn through the map.

PROBLEM XI.

TO SURVEY A TOWN OR CITY.

The proper instrument for this purpose is the plain table; as the distances are not very great, and every part should be laid down while in sight.

Take the first station at the meeting of two or more of the principal streets, in order to get the longest station lines, then draw the lines of direction to represent these streets: having chosen objects, measure the streets, and lay the mea-

6. When the draft is finished, place all the trees, and every other remarkable object, in their proper situation.

PROBLEM X.

TO SURVEY A LARGE TRACT OF LAND; OR A COUNTY.

1. In choosing your stations, let them be on the top of some eminences; as, high hills, mountains, towers, church steeples, &c. and such as command the greatest view, and can be seen from each other; and as far distant from each other as possible. Raise a beacon, or flag, of a different colour, at each, to be visible from the other stations.

2. Plant flags also at each of those places you should distinguish in your plan, fixing them upon the tops of church steeples, houses, &c. and changing them to other places when necessary.

3. Then from one station take all the angles between the other station and each of those marks, observing the different colours of the flags, and note them down in the plan; from the other station take all the angles between the first station and each of the former marks, and note them down in the plan also against the same coloured mark; if it be necessary, the angles may also be taken from a third station, which will serve to prove the work, if the three lines intersect each other in the same point. The instrument for taking angles must be an exceedingly good one. A circumferentor is the best of four or five feet radius with telescope sights.

4. It is not necessary to measure every distance, because any stationary line being exactly laid down from any scale, all the other lines will be proportional to it: but sometimes it is necessary to measure the distance of a place in miles: and in measuring long distances, it will not be exact enough to measure along the high roads, by reason of their windings; but the best way is, to measure in a straight line, over hills and dales, and all obstacles, if possible; but where it is not possible,

possible, such parts must be measured by the method of measuring inaccessible distances. A good compass is also necessary, to show the bearing of the two stations, and to direct you to go straight when you do not see the stations.

5. From all your stations, and in the whole progress, be very careful in observing all remarkable objects; as, towns, castles, churches, windmills, rivulets, bridges, &c. &c.

6. Having taken your observations from the main stations which command the whole country, then proceed to take inner stations, as in surveying large estates; and from these new stations determine the places of as many of the remaining towns as you can; and if any remain, the situation of which you cannot take, take more inner stations; and thus proceed through all the parts, taking one station after another as long as is necessary.

7. Lastly, take the position of the station line with regard to the points of the compass, by an astronomical process, as follows: hang a plummet in the sun, over some part of the station line, and when the shadow of the plummet line runs along the station line, at that moment take the sun's altitude; then, having his declination and the altitude of the place, the azimuth will be found by spherical trigonometry; for the azimuth is the angle which the station line makes with the meridian: and thus the meridian may be drawn through the map.

PROBLEM XI.

TO SURVEY A TOWN OR CITY.

The proper instrument for this purpose is the plain table; as the distances are not very great, and every part should be laid down while in sight.

Take the first station at the meeting of two or more of the principal streets, in order to get the longest station lines, then draw the lines of direction to represent these streets: having chosen objects, measure the streets, and lay the mea-

sures



6. When the draft is finished, place all the trees, and every other remarkable object, in their proper situation.

P R O B L E M X.

TO SURVEY A LARGE TRACT OF LAND ; OR A COUNTY.

1. In choosing your stations, let them be on the top of some eminences ; as, high hills, mountains, towers, church steeples, &c. and such as command the greatest view, and can be seen from each other ; and as far distant from each other as possible. Raise a beacon, or flag, of a different colour, at each, to be visible from the other stations.

2. Plant flags also at each of those places you should distinguish in your plan, fixing them upon the tops of church steeples, houses, &c. and changing them to other places when necessary.

3. Then from one station take all the angles between the other station and each of those marks, observing the different colours of the flags, and note them down in the plan ; from the other station take all the angles between the first station and each of the former marks, and note them down in the plan also against the same coloured mark ; if it be necessary, the angles may also be taken from a third station, which will serve to prove the work, if the three lines intersect each other in the same point. The instrument for taking angles must be an exceedingly good one. A circumferentor is the best of four or five feet radius with telescope sights.

4. It is not necessary to measure every distance, because any stationary line being exactly laid down from any scale, all the other lines will be proportional to it : but sometimes it is necessary to measure the distance of a place in miles : and in measuring long distances, it will not be exact enough to measure along the high roads, by reason of their windings ; but the best way is, to measure in a straight line, over hills and dales, and all obstacles, if possible ; but where it is not possible,

possible, such parts must be measured by the method of measuring inaccessible distances. A good compass is also necessary, to show the bearing of the two stations, and to direct you to go straight when you do not see the stations.

5. From all your stations, and in the whole progress, be very careful in observing all remarkable objects; as, towns, castles, churches, windmills, rivulets, bridges, &c. &c.

6. Having taken your observations from the main stations which command the whole country, then proceed to take inner stations, as in surveying large estates; and from these new stations determine the places of as many of the remaining towns as you can; and if any remain, the situation of which you cannot take, take more inner stations; and thus proceed through all the parts, taking one station after another as long as is necessary.

7. Lastly, take the position of the station line with regard to the points of the compass, by an astronomical process, as follows: hang a plummet in the sun, over some part of the station line, and when the shadow of the plummet line runs along the station line, at that moment take the sun's altitude; then, having his declination and the altitude of the place, the azimuth will be found by spherical trigonometry; for the azimuth is the angle which the station line makes with the meridian: and thus the meridian may be drawn through the map.

PROBLEM XI.

TO SURVEY A TOWN OR CITY.

The proper instrument for this purpose is the plain table; as the distances are not very great, and every part should be laid down while in sight.

Take the first station at the meeting of two or more of the principal streets, in order to get the longest station lines, then draw the lines of direction to represent these streets: having chosen objects, measure the streets, and lay the mea-

6. When the draft is finished, place all the trees, and every other remarkable object, in their proper situation.

P R O B L E M X.

TO SURVEY A LARGE TRACT OF LAND ; OR A COUNTY.

1. In choosing your stations, let them be on the top of some eminences ; as, high hills, mountains, towers, church steeples, &c. and such as command the greatest view, and can be seen from each other ; and as far distant from each other as possible. Raise a beacon, or flag, of a different colour, at each, to be visible from the other stations.

2. Plant flags also at each of those places you should distinguish in your plan, fixing them upon the tops of church steeples, houses, &c. and changing them to other places when necessary.

3. Then from one station take all the angles between the other station and each of those marks, observing the different colours of the flags, and note them down in the plan ; from the other station take all the angles between the first station and each of the former marks, and note them down in the plan also against the same coloured mark ; if it be necessary, the angles may also be taken from a third station, which will serve to prove the work, if the three lines intersect each other in the same point. The instrument for taking angles must be an exceedingly good one. A circumferentor is the best of four or five feet radius with telescope sights.

4. It is not necessary to measure every distance, because any stationary line being exactly laid down from any scale, all the other lines will be proportional to it : but sometimes it is necessary to measure the distance of a place in miles : and in measuring long distances, it will not be exact enough to measure along the high roads, by reason of their windings ; but the best way is, to measure in a straight line, over hills and dales, and all obstacles, if possible ; but where it is not possible,

possible, such parts must be measured by the method of measuring inaccessible distances. A good compass is also necessary, to show the bearing of the two stations, and to direct you to go straight when you do not see the stations.

5. From all your stations, and in the whole progress, be very careful in observing all remarkable objects; as, towns, castles, churches, windmills, rivulets, bridges, &c. &c.

6. Having taken your observations from the main stations which command the whole country, then proceed to take inner stations, as in surveying large estates; and from these new stations determine the places of as many of the remaining towns as you can; and if any remain, the situation of which you cannot take, take more inner stations; and thus proceed through all the parts, taking one station after another as long as is necessary.

7. Lastly, take the position of the station line with regard to the points of the compass, by an astronomical process, as follows: hang a plummet in the sun, over some part of the station line, and when the shadow of the plummet line runs along the station line, at that moment take the sun's altitude; then, having his declination and the altitude of the place, the azimuth will be found by spherical trigonometry; for the azimuth is the angle which the station line makes with the meridian: and thus the meridian may be drawn through the map.

PROBLEM XI.

TO SURVEY A TOWN OR CITY.

The proper instrument for this purpose is the plain table; as the distances are not very great, and every part should be laid down while in sight.

Take the first station at the meeting of two or more of the principal streets, in order to get the longest station lines, then draw the lines of direction to represent these streets: having chosen objects, measure the streets, and lay the mea-

asures

surcs down from a scale, on the lines on the paper, taking offsets with the staff, to all bendings and windings in the streets, and to all remarkable objects; then take another station in one of the foregoing lines, and repeat the process.

Thus, (fig. 6.) having chosen A for the first station, draw two lines in the direction of the two streets meeting there; and measure A B, marking the street on the left, and lay the same measure down on the plan: let B be the second station, from which draw the directions of the streets meeting there; measure from B to C, noting the ends of the streets at n and o as you pass them; and lay the same measure down on the plan. At C, the third station, take the direction of the streets meeting there, and measure from C to D. At D, the next station, do the same, and measure D E, noting the cross street at p; and in the same manner take the directions of all the principal streets, laying the measures down upon the plan. Then proceed to the next smaller streets; and lastly, to the lanes, alleys, courts, &c. using the same process throughout the whole.

Note. In taking the survey of a town there is no necessity for having objects fixed up, as there are generally marks enough to serve as objects; such as the doors or windows of houses, &c.

OF PLANNING, CASTING UP, AND DIVIDING.

PROBLEM I.

TO LAY DOWN THE PLAN OF A SURVEY.

When the survey is taken by the plain table, a rough draft of the plan is already drawn on the paper; but where the survey is taken with any other instrument, the plan is to be drawn from the measures taken in the survey: first, a rough draft is to be drawn on paper, in which all the lines and angles must be laid down in the same order in which they were

were taken in the survey; laying down first the angles, then the lengths of the lines, with the places of the offsets; then the offsets themselves. All these should be done with dry or obscure lines; then a black line drawn through the extremities of all the lines, and the offsets will be the bounding line of the whole plan.

After the principal lines and bounds are laid down, proceed to mark the smaller objects, till every thing be laid down that is necessary.

The north side of the map or plan is commonly placed uppermost. And in some vacant part a scale of equal parts must be drawn. All hills must be shadowed, and all hedges coloured with different colours; all hilly grounds should be drawn like broken hills and valleys: foot-paths are represented by single dotted lines, and roads by double ones.

In taking large surveys, all oblique lines, such as are measured up and down hills, must be reduced to straight lines, by making a proper allowance; for which purpose there is generally a table engraved on some of the instruments.

PROBLEM II.

TO CAST UP THE CONTENTS OF A FIELD.

This is no more than what has been taught in the first section of this chapter, for finding the contents of figures, whether they be triangular, square, four-sided, polygonal, or circular figures, by the proper rules there delivered; then, having found the area in links, after you have cut off five figures on the right hand, the remainder is acres. The five separated figures are to be multiplied by 4 for roods, and five figures cut off from this product are to be multiplied by 40 for perches.

In pieces of land bounded by winding hedges, in measuring the offsets, all the parts between the offsets are accurately measured separately, like small triangles or trapezoids; but
sometimes

sometimes such pieces are measured by finding a mean breadth, which is done by dividing the sum of the offsets by the number of them, accounting that for one where the boundary meets the station line, and the quotient is the mean breadth: but this last method is not very exact.

But in very large surveys, and where there are many fields, the best way of finding the contents of the whole is to make a rough plan of the whole, and divide it into several trapeziums and triangles, and find the area of each separately, and add them together; then, to prove the work, divide the whole estate into as few triangles or trapeziums as possible, by drawing new lines in the plan: and if the contents found by this last method be equal to the contents found by the former method, the work is right; but if they differ, the work must be examined and recomputed till they nearly agree.

But the chief difficulty in casting up the contents arises from the sides of the figure being curved, or of any other irregular shape; in which case such bounds of the figure must be reduced to straight lines, in the following manner: make a small bow with a piece of horsehair, and whalebone, or cane, or any other elastic substance, which will keep the horsehair extended at full stretch; then apply the horsehair to the crooked bounds of the figure, in such a manner, that those parts of the curve which are on one side the hair may be equal to the parts of the curve on the other side the hair; then, by making two points, draw a straight line in the direction of the horsehair, which will be a straight side to the figure, equal to the curve one. Do the same by every curve side of the figure; thus the figure may be reduced into a right-lined figure.

PROBLEM III.

TO TRANSFER A PLAN FROM ONE PAPER TO ANOTHER.

There are several methods of performing this; four of which I shall mention:

First.

First. Rub the back of the plan over with black lead powder, and then lay the black side upon the sheet of paper on which the plan is to be copied, keeping it steady: then with a blunt point of a tracer, trace over all the lines in the plan, pressing the tracer so that the black lead on the back of the paper may be transferred to the clean paper; then take off the plan, and you will see all the marks on the clean paper in black lead, which must be traced over with a pen and ink, &c.

Secondly. The plan may also be transferred to another paper, by dividing both ends and sides of the plan into any convenient number of equal parts, connecting the corresponding points of division with lines; which will divide the whole plan into a number of squares, or parallelograms: then divide the paper upon which the plan is to be drawn into the same number of squares or parallelograms; next copy the parts contained in the squares of the old plan, in the corresponding squares of the new ones. (See figures 40 and 41.)

Thirdly. Another method is by the instrument called a pentagraph, which will copy the plan in any size required.

Fourthly. The best method of any is the following: fix the old plan on the front of a copying frame of glass, with the face uppermost (which is a large square of the best window glass, set in a broad frame of wood, and constructed so as to be raised up to any angle whatever), the clean paper on the face of the old plan being fixed to the frame by several pins; then the frame being raised up facing the window, by means of the light shining through the paper, you will perceive every line of the plan through the clean paper, which is to be drawn thereon with a pencil: having copied that part which covers the glass, the other part is to be brought over the glass, and copied as before, and so on throughout the whole.

Then

Then take them asunder, and trace the lines with pen and ink; and finish the piece, writing such names as are necessary.

Thus the finest plan may be copied without the least injury.

The foregoing rules will be found fully sufficient to instruct any ordinary capacity in all the practical parts of Mensuration and Surveying.—I have avoided every thing that favours more of curiosity than real utility.

SECT. III.

OF SOLID MEASURE, MEASURING TIMBER, DIGGING, AND GAUGING.

Definitions.

1. Solids are such bodies as have length, breadth, and thickness.

2. A prism is a solid body whose ends are similar and equal plain figures, and its sides are parallelograms. It is called a triangular prism when its ends are triangles (as fig. 1. in the solids); when its ends are squares, a square prism; when pentagons, a pentagonal prism, &c.

3. A cube is a square prism, having six equal square sides perpendicular to each other. (Fig. 2.)

4. A parallelopipedon is a figure having six rectangular sides; each two opposite sides being equal, and parallel. (Fig. 3.)

5. A cylinder is a round prism, having two equal circles for its ends. (Fig. 4.)

6. A

6. A pyramid is a figure that has a right-lined figure for its base, and each of its sides is a triangle, whose vertices meet in a point at the top, which is called the vertex of the pyramid, as A (fig. 5.) The pyramid takes its name from the figure of its base, like the prism.

7. A cone is a round pyramid, having a circular base. (Fig. 6.)

8. A sphere is that solid body bounded by one continued convex surface, every part of which is equally distant from the centre. It may be supposed to be formed by the revolution of a semicircle about its diameter. (Fig. 7.)

9. The axis of any solid is a line drawn from the middle of one end to the middle of the opposite end. Thus the axis of a pyramid is a line drawn from the top, or vertex, to the middle of the base; and the axis of a sphere is a line drawn through the centre. When the axis is perpendicular to the base, it is a right prism, or pyramid; otherwise it is oblique.

10. The height or altitude of a solid is a line drawn from the vertex, or top, perpendicular to the base.

11. If the base of a prism or pyramid be a regular figure, it is called a regular prism or pyramid: but if the base be irregular, the prism or pyramid is called irregular.

12. The segment of a pyramid is a part cut off the top by a plane, parallel to the base, as C E, (fig. 6.)

13. A frustum, or trunk, is the part that remains after the segment is cut off, as E A, (fig. 6.)

14. A zone of a sphere is a part intercepted between two parallel lines.

15. The segment of a sphere is a segment less than a hemisphere, or half sphere; like a cone, whose base is the same as the base of the segment, and whose vertex passes through the centre of the sphere.

16. A circular spindle is formed by the revolution of the segment of a circle, about its chord, which remains fixed. (Fig. 8.)

1. A regular body is a solid contained under a certain number of equal and regular plane figures.

2. The face is a solid and the plane figures under which it is contained.

3. There are six five solid bodies which are regular, i. e. which have each face equal and similar: namely, first, the tetrahedron, which is a regular pyramid, having four triangular faces. Second, the hexahedron, or cube, which has six equal square faces. Third, the octahedron, which has eight triangular faces. Fourth, the dodecahedron, which has twelve pentagonal faces. Fifth, the icosahedron, which has twenty triangular faces.

Now, as square measure is greater than running measure, so cubic measure is greater than square measure.

The Table of Cubic Measures.

12	Inches make	1	Foot
27	Feet.....	1	Yard
216	Yards.....	1	Pole
1728	Poles.....	1	Furlong
512	Furlongs.....	1	Mile.

P R O B L E M L

TO FIND THE SOLIDITY OF A CUBE, PARALLELOPIPEDON, PRISM, OR CYLINDER.

The solidity of any of these figures is found by the following rule:—

RULE. Find the area of one end of the figure by the rules for finding the area of a superficies in the foregoing Section, and multiply such area by the length of the figure, and the product is the solid contents.

EXAMPLES

EXAMPLE 1. What is the content of a triangular prism, whose 3 sides are 3, 4, and 5 feet, and length 10 feet?

$$\begin{array}{r}
 3 \\
 4 \\
 \hline
 5 \\
 \hline
 12 \\
 \hline
 \hline
 \end{array}
 \begin{array}{r}
 6 \\
 \hline
 3 \\
 \hline
 \hline
 \end{array}
 \begin{array}{r}
 6 \\
 \hline
 4 \\
 \hline
 2 \\
 \hline
 \hline
 \end{array}
 \begin{array}{r}
 6 \\
 \hline
 5 \\
 \hline
 1 \\
 \hline
 \hline
 \end{array}
 \begin{array}{r}
 3 \\
 \hline
 2 \\
 \hline
 6 \\
 \hline
 6 \\
 \hline
 \hline
 \end{array}
 \begin{array}{r}
 10 \\
 \hline
 6 \\
 \hline
 \hline
 \end{array}
 \text{Ans. } 60 \text{ feet.}$$

Here the three sides of the triangular end are added together, and from the half sum each side is subtracted separately, and the remainders and half sum multiplied continually together: then the square root of the product is the superficial content, as taught in the foregoing Section; which, multiplied by the length, gives the solid content.

Cubes, parallelopipedons, and cylinders, have their solid contents found in the same manner.

Q_n. 2. What is the solid content of a cylinder, whose length is 20 feet, and circumference $5\frac{1}{2}$ feet? **Answer** 48.1459.

PROBLEM II.

TO FIND THE SOLIDITY OF A CONE OR PYRAMID.

RULE. Multiply the area of the base by the height, and one third of the product will be the content.

EXAMPLE 1. What is the solidity of a cone, whose height C D is 20 feet, and diameter A B of the base 3 feet? (Fig. 6.)

$$\begin{array}{r}
 .7854 \\
 \hline
 9 \\
 \hline
 7.0686 \text{ Area of the base.} \\
 \hline
 20 \\
 \hline
 3)141.3720 \\
 \hline
 47.1240 \text{ Answer.} \\
 \hline
 \hline
 \end{array}$$

2. *For the solid Content.*

Multiply the tabular solidity by the cube of the linear edge.

Surfaces and Solidities of regular Bodies.

No. of Sides.	Name.	Surface.	Solidity.
4	Tetraedron	1.73205	0.11785
6	Hexaedron	6.00000	1.00000
8	Octaedron	3.46410	0.47140
12	Dodecaedron	20.64573	7.66312
20	Icosaedron	2.66025	2.18169

EXAMPLE 1. If the linear edge of the tetraedron be 3 feet, what is its surface and solidity?

$$\begin{array}{r}
 0.11785 \\
 \times 27 \text{ Cube of the edge.} \\
 \hline
 82495 \\
 23570 \\
 \hline
 3.18195 \text{ Answer.}
 \end{array}
 \qquad
 \begin{array}{r}
 1.73205 \text{ Square of the edge.} \\
 \times 9 \\
 \hline
 15.58845
 \end{array}$$

Qⁿ. 2. What are the superficies and solidity of the octaedron, whose linear side is 2 feet? Answer, superficies 13.85640, and solidity 3.77120 feet.

Qⁿ. 3. What are the superficies and solidity of the icosaedron, whose linear side is 2 inches? Answer, superficies 10.64100, and solidity 17.45352 inches.

PROBLEM V.

TO FIND THE CONVEX SURFACE OF A CYLINDER, OR THE UPRIGHT SURFACE OF ANY PRISM.

RULE. Multiply the circumference by the height, and the product will be the answer.

Qⁿ.

Q_u. 1. What is the convex surface of the cylinder whose length is 20 feet, and diameter 2 feet? **Answer** 125.664 feet.

PROBLEM VI.

TO FIND THE CONVEX SURFACE OF A RIGHT CONE.

RULE. Multiply the circumference of the base by the slant height, or length of one side; and half the product will be the surface.

EXAMPLE 1. If the diameter of the base be 3 feet, and the side of the cone 10 feet, what is the convex surface?

3.1416	
<u>3</u>	
9.4248	Circumference.
<u>5</u>	Half of the height.
<u>47.1240</u>	Answer.

PROBLEM VII.

TO FIND THE CONVEX SURFACE OF A SPHERE OR GLOBE.

RULE. Multiply the diameter by the circumference, and the product will be the answer.

EXAMPLE 1. What is the surface of a globe, whose diameter is 12 inches?

3.1416	
<u>12</u>	Diameter.
37.6992	Circumference.
<u>12</u>	
<u>452.3804</u>	Answer.

In the same manner the convex surface of any zone, or segment of a sphere, is found: viz. by multiplying the height of the zone or segment by the whole circumference of the sphere.

Qu.

Qⁿ. 2. What is the superficies of the earth, its diameter being $7957\frac{3}{4}$ miles, and the circumference 25,000 miles?
Answer 198,943,750 square miles.

PROBLEM VIII.

TO FIND THE SOLIDITY OF THE SEGMENT OF A CONE
OR PYRAMID.

The segment of a cone, as C E (fig. 6.), or of a pyramid, may be considered as an entire cone or pyramid; and the solidity or superficies is found in the same manner, and therefore need not be repeated.

PROBLEM IX.

TO FIND THE SOLIDITY OF THE FRUSTUM OF A CONE
OR PYRAMID.

RULE 1. Multiply the mean area of the end by the height of the frustum, and the product will be the solid content.

2. The mean area of the ends is found by one of the following rules:—

Add the areas of the two ends, and the mean proportional between them, together, and one third of the sum will be the mean area.

Or, when the ends of the pyramid are regular plain figures, the mean area is found by multiplying one third of the tabular number belonging to the polygon (in the table of multipliers, in the superficial measure) by the sum arising by adding together the square of a side of each end, and the product of the 2 sides.

In the frustum of a cone, the mean area is found by multiplying the sum of the squares of the two diameters, added to the product of the diameters, by 2618, or one third of .7854.

But

~~_____~~ _____ ~~_____~~

~~CONFIDENTIAL~~

~~SECRET~~ ~~THIS IS AN UNCLASSIFIED DOCUMENT~~ ~~DATE 11-11-2011 BY 60322 UCBAW~~ ~~OF 2~~

~~ALL INFORMATION CONTAINED HEREIN IS UNCLASSIFIED~~

~~SECRET~~

[illegible]

106

~~SECRET~~

~~SECRET~~

[illegible]

ALL INFORMATION CONTAINED HEREIN IS UNCLASSIFIED

SECRET

五、研究结论

[illegible]

附錄一：研究工具

三、设计

SECRET

PROBLEM X.

TO FIND THE SUPERFICIAL CONTENT OF A BOARD OR PLANK.

RULE. Multiply the length by the mean breadth.

Note. When the board is tapering, add the breadths of the two ends together, and take half the sum for the mean breadth.

EXAMPLE 1. What is the superficial content of a board, whose length is 20 feet 3 inches, and mean breadth 10 inches?

By Decimals.

$$\begin{array}{r} 20.25 \\ 10 \\ \hline 12 \overline{)202.50} \\ \underline{16.875} \end{array} \text{ Answer.}$$

By Duodecimals.

	Feet.	Inches.
	20	.. 3
		10
Answer	16	.. 10 .. 6

When the board does not run regularly tapering, but is broader in some parts than others; take several breadths and add them together, and divide the sum by the number of them, and the quotient will be the mean breadth.

PROBLEM XI.

TO FIND THE SOLID CONTENT OF SQUARE OR FOUR-SIDED TIMBER.

RULE. Multiply the mean breadth by the mean thickness, and that product by the length; and the last product will be the solid content.

Note. If the timber taper regularly from one end to the other, the mean breadth and thickness is found either by

PROBLEM XII.

TO FIND THE SOLIDITY OF ROUND TIMBER.

The common rule is to multiply the square of the quarter girt, or one quarter of the mean circumference, by the length; and the product will be the content.

The quarter girt is a geometrical mean proportional, between the mean breadth, and thickness; that is, the square root of their product: unskilful measurers use the arithmetical mean instead of it, that is, half their sum; but this occasions a very great error in the contents, and particularly when the breadth and depth differ much from each other.

When the round timber or tree is tapering, take the mean dimensions from the dimensions of each end, as in the foregoing Problem, either by girting it in the middle for the mean girt, or half the sum of the two ends; but when the tree is irregular, find the content of each part separately.

Note. This rule gives the contents about $\frac{1}{4}$ less than the true quantity, so that it makes an allowance for hewing the tree square. But when the true quantity is required, it is found by the next rule.

EXAMPLE I. What is the content of a piece of round timber, being 9 feet 6 inches long, and its mean quarter girt 3 feet 6 inches?

<i>Decimals.</i>		<i>Quodecimals.</i>
3.5	quarter girt	3 6
<u>3.5</u>		<u>3 6</u>
17 5		10 6
<u>105</u>		<u>1 9</u>
12.25		12 3
<u>9.5</u>	length	<u>9 6</u>
612 5		110 3
<u>110 25</u>		<u>6 1 6</u>
<u>116.37 5</u>	content	<u>116 4 6</u>

3 c 2

PROBLEM

$$\begin{array}{r}
 2,0) 500,0 \\
 \underline{250,00} \quad 15,8 \text{ Answer.} \\
 1 \\
 25) 150 \\
 \underline{125} \\
 308) 25,00 \\
 \underline{36}
 \end{array}$$

Qⁿ. 2. A bowling-green, 300 feet long and 200 feet broad, is to be raised one foot higher by the earth to be dug out of a ditch that encompasses the green, and is 8 feet in breadth; to what depth must the ditch be dug? Answer $7\frac{2}{3}$ feet.

Of Gauging.

Gauging is most commonly performed by the sliding rule, or gauging rod. But I shall here give the practice of it, as wrought by calculation of the common measures only.

By gauging is always understood the method of finding the content of any hollow vessel, and is mostly used for finding the content of casks, coppers, &c.

The rule is to find the content of the vessel in inches, and divide them by 282, if for ale gallons; and by 231 for wine gallons; because 282 cubic inches contains an ale gallon, and 231 cubic inches is a wine gallon. But if any prefer multiplication to division, when the solid content is found, it may be multiplied by .003546 for ale gallons, instead of being divided by 282; and by .004329 for wine gallons, instead of being divided by 231; for dividing by 282 or 231 is the same as multiplying by these vulgar fractions $\frac{1}{282}$ or $\frac{1}{231}$; and the above multipliers are the decimal fractions, equal to these vulgar ones.

In vessels of a circular or elliptical form, to obtain the content in inches, it is necessary to multiply by .7854, and then to reduce them to ale or wine gallons. Therefore to multiply by .7854, and then divide by 282, or 231, is the same

PROBLEM XII.

TO FIND THE SOLIDITY OF ROUND TIMBER.

The common rule is to multiply the square of the quarter girt, or one quarter of the mean circumference, by the length; and the product will be the content.

The quarter girt is a geometrical mean proportional, between the mean breadth, and thickness; that is, the square root of their product: unskilful measurers use the arithmetical mean instead of it, that is, half their sum; but this occasions a very great error in the contents, and particularly when the breadth and depth differ much from each other.

When the round timber or tree is tapering, take the mean dimensions from the dimensions of each end, as in the foregoing Problem, either by girting it in the middle for the mean girt, or half the sum of the two ends; but when the tree is irregular, find the content of each part separately.

Note. This rule gives the contents about $\frac{1}{4}$ less than the true quantity, so that it makes an allowance for hewing the tree square. But when the true quantity is required, it is found by the next rule.

EXAMPLE 1. What is the content of a piece of round timber, being 9 feet 6 inches long, and its mean quarter girt 3 feet 6 inches?

<i>Decimals.</i>		<i>Quodecimals.</i>
3.5	quarter girt	3 6
<u>3.5</u>		<u>3 6</u>
17 5		10 6
<u>105</u>		<u>1 9</u>
12.25		12 3
<u>9.5</u>	length	<u>9 6</u>
612 5		110 3
<u>110 25</u>		<u>6 1 6</u>
<u>116.37 5</u>	content	<u>116 4 6</u>

3 c 2

PROBLEM

PROBLEM XIII.

TO FIND THE CONTENT OF A TREE, OR ROUND
PIECE OF TIMBER, MORE EXACTLY.

RULE. Multiply the square of $\frac{1}{2}$ of the mean girt by twice the length, and the product will be the contents.

EXAMPLE. What is the content of a tree, the length of which is 9 feet 6 inches, and mean girt 14 feet?

<i>Decimals.</i>		<i>Duodecimals.</i>
2.8	$\frac{1}{2}$ of mean girt	2 9 7
2.8		2 9 7
<u>22 4</u>		<u>5 7 2</u>
56		2 1 2
		<u>1 7</u>
<u>7.84</u>	square of $\frac{1}{2}$ of mean girt	7 9 12
19	- twice the length -	19
<u>79 56</u>		<u>148 8 5*</u>
78 4		
<u>148.96</u>	Answer.	

PROBLEM XIV.

TO FIND THE DEPTH, BREADTH, OR LENGTH OF ANY
VAULT, CELLAR, RESERVOIR, &c. TO BE MADE.

RULE. Divide the solid content by any one of the three dimensions (viz. length, breadth, or depth), and the quotient will be the product of the two other dimensions.

EXAMPLE. What will be the length and breadth of a square reservoir of water, to contain 5000 cubic feet of water, and the depth to be 20 feet?

* The answer by duodecimals is less in this question than the truth, as all the parts could not conveniently be included in the operation.

$$2,0)5000 \\ \text{LE}$$

$$\begin{array}{r}
 2,0) 500,0 \\
 \underline{250,00} \quad 15,8 \text{ Answer.} \\
 1 \\
 25) 150 \\
 \underline{125} \\
 308) 25,00 \\
 \underline{36}
 \end{array}$$

Qⁿ. 2. A bowling-green, 300 feet long and 200 feet broad, is to be raised one foot higher by the earth to be dug out of a ditch that encompasses the green, and is 8 feet in breadth; to what depth must the ditch be dug? **Answer** $7\frac{2}{3}$ feet.

Of Gauging.

Gauging is most commonly performed by the sliding rule, or gauging rod. But I shall here give the practice of it, as wrought by calculation of the common measures only.

By gauging is always understood the method of finding the content of any hollow vessel, and is mostly used for finding the content of casks, coppers, &c.

The rule is to find the content of the vessel in inches, and divide them by 282, if for ale gallons; and by 231 for wine gallons; because 282 cubic inches contains an ale gallon, and 231 cubic inches is a wine gallon. But if any prefer multiplication to division, when the solid content is found, it may be multiplied by .003546 for ale gallons, instead of being divided by 282; and by .004329 for wine gallons, instead of being divided by 231; for dividing by 282 or 231 is the same as multiplying by these vulgar fractions $\frac{1}{282}$ or $\frac{1}{231}$; and the above multipliers are the decimal fractions, equal to these vulgar ones.

In vessels of a circular or elliptical form, to obtain the content in inches, it is necessary to multiply by .7854, and then to reduce them to ale or wine gallons. Therefore to multiply by .7854, and then divide by 282, or 231, is the same

same as to multiply by the fraction $\frac{7854}{2111}$ or $\frac{7851}{2111}$, and the decimal fractions equal to these vulgar ones are .0027851 for ale, and .00339999 for wine.

PROBLEM XV.

TO GAUGE A CASK.

There are several methods of gauging casks according to their shapes, or the curvature of the sides, of which there are four general forms, but the content of any of them may be found by the following general rule:

RULE. To the square of twice the middle diameter add the squares of the head and bung diameters. Multiplying the sum by $\frac{1}{6}$ of the length, and that product multiplied or divided by the circular multipliers or divisors, will give the contents: or multiply the squares of the diameters by the length of the cask, and that product again by .0004 $\frac{2}{3}$ for ale gallons, or by .0005 $\frac{2}{3}$ for wine gallons.

EXAMPLE I. Required the contents of any cask, whose length is 40 inches, the bung diameter being 32, the head diameter 24, that between the bung and head 28 $\frac{1}{2}$ inches.

$$\begin{array}{r} 57.5 \\ 57.5 \\ \hline \end{array}$$

$$2875$$

$$4025$$

$$2875$$

$$\hline 3306.25$$

$$1024$$

$$576$$

$$\hline 4906.25$$

$$40$$

$$\hline 196250$$

$$.0004\frac{2}{3}$$

$$\hline 7850'00$$

$$130833$$

$$\hline 91.5833 \text{ Gallons.}$$

$$24$$

$$24$$

$$\hline 96$$

$$48$$

$$\hline 576$$

$$32$$

$$32$$

$$\hline 64$$

$$96$$

$$\hline 1024$$

$$196250$$

$$.0005\frac{2}{3}$$

$$\hline 981250$$

$$130833$$

$$\hline 111.2083 \text{ Wine.}$$

PROBLEM

PROBLEM XVI.

TO FIND THE CONTENT OF ANY CASK, FROM THREE DIMENSIONS ONLY.

RULE. Add into one sum 39 times the square of the bung diameter, 25 times the square of the head diameter, and 26 times the product of the two diameters: then multiply the sum by the length, and that product by .00034, and divide the last product by 9 for wine gallons, and divide by 11 for ale gallons, or multiply by .00003 $\frac{1}{11}$ for ale.

EXAMPLE. What is the content of a cask, whose length is 40, the bung diameter 32, and head diameter 24 inches?

32	24	32
32	24	24
<u>64</u>	<u>96</u>	<u>128</u>
96	48	64
<u>1024</u>	<u>576</u>	<u>768</u>
39	25	26
<u>9216</u>	<u>2880</u>	<u>4608</u>
3072	1152	1536
<u>39936</u>	<u>14400</u>	<u>19968</u>
	39936	
	<u>19968</u>	
	74304	
	40	
	<u>2972160</u>	2972160
	.00034	.00003 $\frac{1}{11}$
	<u>11888640</u>	<u>8916480</u>
	8916480	270196
	<u>9)1010.53440</u>	Ale <u>91.86676</u> gallons.
Wine gallons	<u>112.2816</u>	

PROBLEM XVII.

TO ULLAGE A STANDING CASK*.

RULE. Add together the square of the diameter at the

* The ullage of a cask is what it contains when only partly filled, and it is considered in two positions; namely, either standing on its end with its axis perpendicular to the horizon, or as lying on its side with the axis parallel to the horizon.

surface of the liquor, the square of the diameter of the nearest end, and the square of double the diameter taken in the mid-way between the other two; then multiply this sum by the length between the surface of the liquor and the nearest end, and multiply the product again by .00043 for ale gallons, or by .00052 for wine gallons, in the left part of the cask, whether empty or filled.

EXAMPLE. The three diameters being 24, 27, and 29, what is the ullage for ten wet inches?

$\begin{array}{r} 24 \\ \times 24 \\ \hline 96 \\ 48 \\ \hline 576 \end{array}$	$\begin{array}{r} 29 \\ \times 29 \\ \hline 261 \\ 58 \\ \hline 841 \end{array}$	$\begin{array}{r} 54 \\ \times 54 \\ \hline 216 \\ 270 \\ \hline 2916 \end{array}$	$\begin{array}{r} 2916 \\ \times 841 \\ \hline 576 \\ 4333 \\ 2916 \\ \hline 43330 \\ .00052 \\ \hline 216650 \\ 28886 \\ \hline 245536 \end{array}$
			Wine.

$\begin{array}{r} 43330 \\ .00043 \\ \hline 173320 \\ 28886 \\ \hline \text{Ale } 20.2206 \end{array}$
--

PROBLEM XVIII.

TO ULLAGE A LYING CASK.

RULE. Divide the wet inches by the bung diameter, and find the quotient in the column of versed sines in the table of circular segments at the end of this Section, taking out its correspondent segment; then multiply this segment by the whole content of the cask, and the product again by $1\frac{1}{4}$ for the ullage required.

EXAMPLE. If the bung diameter be 32 inches, and content 92 ale gallons, what is the ullage of 8 wet inches?

32)8(.25, whose tab. seg. is .153546

$$\begin{array}{r}
 92 \\
 \hline
 307092 \\
 1381914 \\
 \hline
 14.126232 \\
 \frac{1}{4} \text{ is } 3.531558 \\
 \hline
 \text{Answer } 17.657790
 \end{array}$$

A TABLE

Of the Areas of the Segments of a Circle,

Whose Diameter is Unity, and supposed to be divided into
One Thousand equal Parts.

Height.	Area Seg.	Height.	Area Seg.	Height.	Area Seg.
.001	.000042	.027	.005867	.053	.016007
.002	.000119	.028	.006194	.054	.016457
.003	.000219	.029	.006527	.055	.016911
.004	.000337	.030	.006865	.056	.017369
.005	.000470	.031	.007209	.057	.017831
.006	.000618	.032	.007558	.058	.018296
.007	.000779	.033	.007913	.059	.018766
.008	.000951	.034	.008273	.060	.019239
.009	.001135	.035	.008638	.061	.019716
.010	.001329	.036	.009008	.062	.020196
.011	.001533	.037	.009383	.063	.020680
.012	.001746	.038	.009763	.064	.021168
.012	.001963	.039	.010148	.065	.021659
.014	.002199	.040	.010537	.066	.022154
.015	.002438	.041	.010931	.067	.022652
.016	.002685	.042	.011330	.068	.023154
.017	.002940	.043	.011734	.069	.023659
.018	.003202	.044	.012142	.070	.024168
.019	.003471	.045	.012554	.071	.024680
.020	.003748	.046	.012971	.072	.025195
.021	.004031	.047	.013392	.073	.025714
.022	.004322	.048	.013818	.074	.026236
.023	.004618	.049	.014247	.075	.026761
.024	.004921	.050	.014681	.076	.027289
.025	.005230	.051	.015119	.077	.027821
.026	.005546	.052	.015561	.078	.028356

THE AREAS OF THE

<i>Height.</i>	<i>Area Seg.</i>	<i>Height.</i>	<i>Area Seg.</i>	<i>Height.</i>	<i>Area Seg.</i>
·079	·028894	·114	·044528	·149	·073161
·080	·029435	·115	·050167	·150	·073874
·081	·029979	·116	·050804	·151	·074589
·082	·030526	·117	·051446	·152	·075306
·083	·031076	·118	·052090	·153	·076026
·084	·031629	·119	·052736	·154	·076747
·085	·032186	·120	·053385	·155	·077469
·086	·032745	·121	·054036	·156	·078194
·087	·033307	·122	·054689	·157	·078921
·088	·033872	·123	·055345	·158	·079649
·089	·034441	·124	·056003	·159	·080380
·090	·035011	·125	·056663	·160	·081112
·091	·035585	·126	·057326	·161	·081846
·092	·036162	·127	·057991	·162	·082582
·093	·036741	·128	·058658	·163	·083320
·094	·037323	·129	·059327	·164	·084059
·095	·037909	·130	·059999	·165	·084801
·096	·038496	·131	·060672	·166	·085544
·097	·039087	·132	·061348	·167	·086289
·098	·039680	·133	·062026	·168	·087036
·099	·040276	·134	·062707	·169	·087785
·100	·040875	·135	·063389	·170	·088535
·101	·041476	·136	·064074	·171	·089287
·102	·042080	·137	·064760	·172	·090041
·103	·042687	·138	·065449	·173	·090797
·104	·043296	·139	·066140	·174	·091554
·105	·043908	·140	·066833	·175	·092313
·106	·044522	·141	·067528	·176	·093074
·107	·045139	·142	·068225	·177	·093836
·108	·045759	·143	·068924	·178	·094601
·109	·046381	·144	·069625	·179	·095366
·110	·047005	·145	·070328	·180	·096134
·111	·047632	·146	·071033	·181	·096903
·112	·048262	·147	·071741	·182	·097674
·113	·048894	·148	·072450	·183	·098447

SEGMENTS OF A CIRCLE.

<i>Height.</i>	<i>Area Seg.</i>	<i>Height.</i>	<i>Area Seg.</i>	<i>Height.</i>	<i>Area Seg.</i>
·184	·099221	·219	·127285	·254	·157019
·185	·099997	·220	·128113	·255	·157890
·186	·100774	·221	·128942	·256	·158762
·187	·101553	·222	·129773	·257	·159636
·188	·102334	·223	·130605	·258	·160510
·189	·103116	·224	·131438	·259	·161386
·190	·103900	·225	·132272	·260	·162263
·191	·104685	·226	·133108	·261	·163140
·192	·105472	·227	·133945	·262	·164019
·193	·106261	·228	·134784	·263	·164899
·194	·107051	·229	·135624	·264	·165780
·195	·107842	·230	·136465	·265	·166663
·196	·108636	·231	·137307	·266	·167546
·197	·109430	·232	·138150	·267	·168430
·198	·110226	·233	·138995	·268	·169315
·199	·111024	·234	·139841	·269	·170202
·200	·111823	·235	·140688	·270	·171089
·201	·112624	·236	·141537	·271	·171978
·202	·113426	·237	·142387	·272	·172867
·203	·114230	·238	·143238	·273	·173758
·204	·115035	·239	·144091	·274	·174649
·205	·115842	·240	·144944	·275	·175542
·206	·116650	·241	·145799	·276	·176435
·207	·117460	·242	·146655	·277	·177330
·208	·118271	·243	·147512	·278	·178225
·209	·119083	·244	·148371	·279	·179122
·210	·119897	·245	·149230	·280	·180019
·211	·120712	·246	·150091	·281	·180918
·212	·121529	·247	·150953	·282	·181817
·213	·122347	·248	·151816	·283	·182716
·214	·123167	·249	·152680	·284	·183619
·215	·123988	·250	·153546	·285	·184521
·216	·124810	·251	·154412	·286	·185425
·217	·125634	·252	·155280	·287	·186329
·218	·126459	·253	·156149	·288	·187234

THE AREAS OF THE

Height.	Area Seg.	Height.	Area Seg.	Height.	Area Seg.
289	188140	324	220404	359	253590
290	189047	325	221340	360	254550
291	189955	326	222277	361	255510
292	190864	327	223215	362	256471
293	191775	328	224154	363	257433
294	192684	329	225093	364	258395
295	193596	330	226033	365	259357
296	194509	331	226974	366	260320
297	195422	332	227915	367	261284
298	196337	333	228858	368	262248
299	197252	334	229801	369	263213
300	198168	335	230745	370	264178
301	199085	336	231689	371	265144
302	200003	337	232634	372	266111
303	200922	338	233580	373	267078
304	201841	339	234526	374	268045
305	202761	340	235473	375	269013
306	203683	341	236421	376	269982
307	204605	342	237369	377	270951
308	205527	343	238318	378	271920
309	206451	344	239268	379	272890
310	207376	345	240218	380	273861
311	208301	346	241169	381	274832
312	209227	347	242121	382	275803
313	210154	348	243074	383	276775
314	211082	349	244026	384	277748
315	212011	350	244980	385	278721
316	212940	351	245934	386	279694
317	213871	352	246889	387	280668
318	214802	353	247845	388	281642
319	215733	354	248801	389	282617
320	216666	355	249757	390	283592
321	217599	356	250715	391	284568
322	218533	357	251673	392	285544
323	219408	358	252631	393	286521

SEGMENTS OF A CIRCLE.

<i>Height.</i>	<i>Area Seg.</i>	<i>Height.</i>	<i>Area Seg.</i>	<i>Height.</i>	<i>Area Seg.</i>
·394	·287498	·430	·322928	·466	·358725
·395	·288476	·431	·323918	·467	·359723
·396	·289453	·432	·324909	·468	·360721
·397	·290432	·433	·325900	·469	·361719
·398	·291411	·434	·326892	·470	·362717
·399	·292390	·435	·327882	·471	·363715
·400	·293369	·436	·328874	·472	·364713
·401	·294349	·437	·329866	·473	·365712
·402	·295330	·438	·330858	·474	·366710
·403	·296311	·439	·331850	·475	·367709
·404	·297292	·440	·332843	·476	·368708
·405	·298273	·441	·333836	·477	·369707
·406	·299255	·442	·334829	·478	·370706
·407	·300238	·443	·335822	·479	·371705
·408	·301220	·444	·336816	·480	·372704
·409	·302203	·445	·337810	·481	·373703
·410	·303187	·446	·338804	·482	·374702
·411	·304171	·447	·339798	·483	·375702
·412	·305155	·448	·340793	·484	·376702
·413	·306140	·449	·341787	·485	·377701
·414	·307125	·450	·342782	·486	·378701
·415	·308110	·451	·343777	·487	·379700
·416	·309095	·452	·344772	·488	·380700
·417	·310081	·453	·345768	·489	·381699
·418	·311068	·454	·346764	·490	·382699
·419	·312054	·455	·347759	·491	·383699
·420	·313041	·456	·348755	·492	·384699
·421	·314029	·457	·349752	·493	·385699
·422	·315016	·458	·350748	·494	·386699
·423	·316004	·459	·351745	·495	·387699
·424	·316992	·460	·352742	·496	·388699
·425	·317981	·461	·353739	·497	·389699
·426	·318970	·462	·354736	·498	·390699
·427	·319959	·463	·355732	·499	·391699
·428	·320948	·464	·356730	·500	·392699
·429	·321938	·465	·357727		

In this table the diameter of the circle is 1, and the whole area .785398; the figures in the columns of heights are the height of the segment, or the versed sine of half its arc; and those in the columns of area segments are the areas of the circular segments, whose heights stand on the left hand in the column of heights.

The use of this table is to find the area of any segment of a circle, by the following rule:

RULE. Divide the height of the given segment by the diameter of the circle, and the quotient is the height of the segment: and opposite this height, in the table, is the tabular area, which is to be multiplied by the square of the diameter, and the product will be the area of the segment.

EXAMPLE. What is the area of a segment of a circle, whose height is 5 inches, and the diameter 30 inches?

$$\begin{array}{r}
 30 \overline{) 5.000} \begin{array}{l} (.166 \text{ Tab. height} \\ .085544 \text{ Tab. segment} \\ 900 \text{ Square of the diameter.} \end{array} \\
 \underline{00} \\
 \underline{000} \\
 \underline{000} \\
 \text{Answer. } \underline{\underline{76.989600}}
 \end{array}$$

$$\begin{array}{r}
 30 \\
 30 \\
 \hline
 900 \\
 \hline
 \hline
 \end{array}$$

Note. In dividing the given height by the diameter, if the quotient does not terminate in three places of decimals, then the area for that fractional part ought to be proportioned for as follows:

Having found the tabular area answering to the first three decimals of the quotient, take the difference between it and the next following tabular area; which difference multiplied by the fractional part remaining, the product will be the correspondent proportional part, to be added to the first tabular area.

Thus in the last example the quotient is .166, and there is a remainder of $\frac{2}{3}$, the tabular area of the segment is

.085544

.085544	
.086289	
<hr/>	
$\frac{2}{3}$ 745	The difference
248	
2	
<hr/>	
496	$\frac{2}{3}$ of the difference
.085544	
<hr/>	
.086040	The true tab. area.
<hr/>	

S E C T. IV.

OF INSTRUMENTAL ARITHMETIC.

Instrumental arithmetic is the method of calculation by instruments made for that purpose, for a quicker dispatch of business, and for the help of those who are deficient in common arithmetic. The most common instruments are, the *carpenter's rule*, *Coggeshall's sliding rule*, *Gunter's line*, the *gauging rule*, and the *diagonal rod*.

1. The carpenter's rule consists of two equal pieces, each a foot in length, connected together by a joint: one side of this rule is divided into inches, and eighths of an inch; on the same side are also several plane scales, divided into twelfth parts by diagonal lines, for planning dimensions in feet and inches.

2. On the other side of this rule, on one piece, are set all the principal lines which are on Coggeshall's sliding rule: namely, four lines marked A B C, and D respectively; the two middle ones being on a slider, which runs in a groove.

These

These four lines are logarithmic lines ; the three marked A, B, C, are all equal ; and are called double lines, because the numbers set upon them run from 1 to 10, twice : the lowest line D is a single line, the numbers running from 4 to 40. It is called the girt line, from its use in casting up the contents of trees, and timber. This line also may be used for gauging ; as upon it, at 17.15, is marked W. G. and at 18.95, A. G. for the wine and ale gauge points.

Upon the other part of this side is a table of pounds, shillings, and pence, showing the value of a load, or 50 cubic feet of timber, at all prices from sixpence to two shillings a foot.

In the use of those lines it must be strictly observed, that when 1 at the beginning of a line is accounted unity, then the 1 in the middle of the line will stand for 10, and the 10 at the end of the line will stand for 100 : and when the 1 at the beginning of the line stands for 10, the 10 in the middle of the line will stand for 100, and the 10 at the end of the line 1000, &c. and all the intermediate divisions are altered in proportion.

The edge of the rule is divided decimally, each foot being divided into an hundred equal parts : by this measure dimensions are taken in feet, and decimals of a foot, which is by much the best way.

3. Gunter's line is a line of figures, exactly the same as the three single lines on the carpenter's rule, and therefore needs no further description. Its use is to multiply, or divide numbers, to perform the rule of three direct, &c. It was formerly set by itself on the carpenter's rule.

The figures 1, 2, 3, 4, &c. sometimes stand simply for themselves, at other times they signify 10, 20, 30, &c. again at other times, 100, 200, &c. or 1000, 2000, &c.

4. The gauging rule serves to compute the contents of casks, or any other vessels, after the dimensions have been taken. It is a square rule, with logarithmic lines on the sides, having three sliders running in grooves in three of the sides.

Upon

Upon the first face of this rule are three lines : two, marked A and B, for multiplying and dividing ; and one, marked M D, for malt depth, as it serves to gauge malt. The middle line B is upon a slider, and is a double line, marked at each edge of the slider, like that on the carpenter's rule : these three lines are all of the same radius, or distance from 1 to 10, each line containing twice the length of the radius. A and B are numbered exactly alike, and the numbers may be increased or decreased at pleasure : thus the first number 1 may either stand for an unit, 10, or 100, &c. or 1, or 01, or 001, &c. : but whatever the first number is, the middle number will be ten times as much, and the last number 100 times as much, as on the carpenter's rule.

The 1 on the line M D is opposite 215 on the other lines, which are the cubic inches in a malt bushel ; and the divisions on this line are numbered retrograde to those of A and B.

Upon the lines A and B are several other marks and letters : thus, on the line A are M B, for malt bushel, opposite the aforesaid number 215 ; and A, for ale, opposite 282, the cubic inches in an ale gallon ; and upon the line B, opposite 231, is W, for wine, the cubic inches in a wine gallon ; also S I, for square inscribed at 707, which is the side of the square inscribed in a circle, whose diameter is 1 ; and S E, for square equal, at .886, being the side of a square equal to the same circle : also C, for circumference, at 3.1416, which is the circumference of the same circle.

Upon the second face, which is that opposite the first, are a slider, and four lines marked D, C, D, E, at one end, and at the other end marked *root, square, root, cubic* ; the lines C and E, containing respectively the squares and cubes of the opposite numbers of the lines D, D : the radius of the line D is double to that of C, and triple to that of E ; so that whatever the first one on D denotes, the first on C is the square of it, and the first on E the cube of it ; thus, if D begins with 1, C and E will begin with 1 ; but if D begins with 10, C will begin with 100, which is the square of 10, and E with 1000, the

which are placed the areas, in ale gallons, of circles to the corresponding diameters, in like manner as the lines on the under side of the three sliders in the sliding rule.

On the opposite face are the scales of ale and wine gallons, to show the contents of casks, from having the diagonal. All the other lines on this instrument are the same as those on the sliding rule, and to be used in the same manner.

The Use of the Carpenter's Rule.

PROBLEM I.

TO MULTIPLY NUMBERS TOGETHER.

Suppose the two numbers 24 and 12; draw the slider out, till 1 on the line B is opposite to 12 on the line A; then against 24 on the line B stands 288 on the line A, which is the product of the two numbers.

Note. In any operations, when a number runs beyond the end of the line, seek it on the other radius, that is, the other part of the line, which will be the tenth part of the required number.

PROBLEM II.

TO DIVIDE NUMBERS.

To divide 312 by 24; draw out the slider till the divisor 24 on B be opposite to the dividend 312 on A, then against 1 on B stands 13, the quotient on A.

PROBLEM III.

TO SQUARE ANY NUMBER.

Suppose to square 12: set 1 on the line B to 12 on A, then against 12 on B stands 144 on A.

PROBLEM IV.

TO EXTRACT THE SQUARE ROOT.

Set 1 or 100, &c. on C to 1 or 10, &c. on D; then against every number found on C stands its square root on D.

PROBLEM

PROBLEM IX.**TO DIVIDE BY GUNTER'S LINE.**

To divide 63 by 3; extend the compasses from 3 to 1 downwards, or towards the left hand; and that extent will reach the same way, from 63 to 21, the quotient.

Note. In multiplying, you must always extend upwards, or towards the right hand; and in dividing, extend the contrary way.

PROBLEM X.**TO PERFORM THE RULE OF THREE DIRECT; OR, TO FIND A FOURTH PROPORTIONAL.**

Suppose 5 bushels of barley cost 15 shillings, what will 25 bushels cost? Extend the compasses from 5 to 15 upwards, and that extent will reach the same way from 25 to 75 shillings, the answer.

*The Use of the Gauging Rule.***PROBLEM XI.****TO MULTIPLY, DIVIDE, PERFORM THE RULE OF THREE, EXTRACT THE SQUARE ROOT, OR TO FIND A MEAN PROPORTIONAL BETWEEN TWO NUMBERS.**

All these are wrought on the gauging rule exactly in the same manner as on the carpenter's sliding rule; the lines A, B, C, D, on each rule being used in the same way.

PROBLEM XII.**TO COMPUTE MALT BUSHELS BY THE LINE M D.**

To find the malt bushels in a floor, whose length is 230, breadth 58.2, and depth 5.4 inches.

Set 230 off B to 5.4 on M D, then against 58.2 on A ~~stands 33.6~~ bushels on B, for the answer.

PROBLEM

PROBLEM VII.

TO FIND THE AREA OF THE FLOOR OF A ROOM.

The room is a rectangle, and being divided into four equal parts, the area of the floor is the sum of the areas of the four parts. The area of a rectangle is found by multiplying the length by the breadth. The length of the room is 40 feet, and the breadth is 30 feet. The area of the floor is therefore 40 feet multiplied by 30 feet, which is 1200 square feet.

The area of the floor is 1200 square feet. The area of the walls is 1200 square feet. The area of the ceiling is 1200 square feet. The area of the floor is 1200 square feet.

12	in 1 foot	12	in 1	on C
12	in 1 foot	12	in 1	on C
The sum		12	in 1	
The sum		12	in 1	on C

Again, 12 in 1 in the wine gauge sq. 7 on D, and again

12	in 1 foot	12	in 1	on C
12	in 1 foot	12	in 1	on C
The sum		12	in 1	
The sum		12	in 1	on C

PROBLEM

PROBLEM XIV.

TO FIND THE CONTENT OF A CASK OF THE SECOND FORM.

Let the length, and head, and bung diameters be the same as in the last Problem: then, having set 40 on C to 32.82 on D, against 8 on D stands 2.4 on C, $\frac{4}{10}$ of which is 0.96. This, subtracted from 97.3, the contents of the last form, leaves 96.34 ale gallons.

Again, having set 40 on C to 29.7 on D, against 8 on D stands 2.9 on C, $\frac{4}{10}$ of which is 1.16, which, taken from 119.1, in the last form, leaves 117.94 wine gallons.

PROBLEM XV.

TO FIND THE CONTENT OF A CASK OF THE THIRD FORM.

The dimensions being the same as in the foregoing Problems, set 40 on C to 26.8 on D, then against

24 on D stands	32.0	on C
32 on D stands	57.3	on C
The sum	<u>89.3</u>	Ale gallons.

Again, set 40 on C to 24.25 on D: then against

24 on D stands	39.1	on C
32 on D stands	69.8	on C
The sum	<u>108.9</u>	Wine gallons.

PROBLEM XVI.

TO FIND THE CONTENT OF A CASK OF THE FOURTH FORM.

The dimensions being the same as before, set 40 on C to 65.64 on D, then against

8 on

Next, set 100 on B to the reserved number on A, and against the whole content on B will be found the ullage on A.

There are many solid figures besides the foregoing, the contents of which cannot be found by any actual dimensions taken on the figure, such as statues, &c. The contents of such bodies are found by immersing them in water, and measuring the rise of the water occasioned by such immersion.

S E C T. V.

A VIEW OF THE FIVE ORDERS OF ARCHITECTURE.

The origin of architecture is almost contemporary with that of civil society. When mankind felt the inclemencies of the weather, and, consequently, found the necessity of erecting habitations for shelter, ease and convenience soon improved itself into ornament and grandeur. A few trees perhaps growing round in a circle, and leaning together at the top, afforded the first habitations, being interwoven with twigs, and plastered over with mud. This gave rise, in after-ages, to the idea of columns.

It is probable that the inconvenience of these habitations rendered their owners desirous of inventing such as should answer every purpose of the former sort, and possess several advantages above those: this improvement naturally gave rise to the invention of the cross beams to support the roof; for this purpose, they had, no doubt, recourse to the trunks of trees: thus columns were at once introduced.

4. The **ABACUS** was no more than a tile or stone to throw off the water, and to prevent its sinking into the column.

5. The **ASTRAGALS** and **FILLETS** were bandages to bind the column.

6. The **ARCHITRAVE** was beams necessary to hold or unite the columns together.

7. The **FRIZE** was only an addition, when height was wanted in the inner parts.

8. The **CORNICE**, with its ornaments, were only the ends or edges of the timbers, necessary to support the roof.

9. The **DENTELES**, **TRIGLYPHS**, **MODILLIONS**, &c. were accidental improvements added in after-ages, when structures of wood gave place to those of stone.

The five orders of architecture are, the Tuscan, Doric, Ionic, Corinthian, and the Composite.

The **TUSCAN** order is distinguished by its plain and robust appearance: it is used only where strength and plainness are required. The Trajan column at Rome is an instance of its effect and elegance.

The **DORIC** has nearly the same character for strength as the Tuscan, but it is enlivened by several ornaments. It is used with good effect where united strength and grandeur are required.

The **IONIC** has more delicacy in the construction than either of the former orders; it partakes of a middle order, between the masculine Tuscan and Doric, and the slender Corinthian: the capital is distinguished by its boldness, and the shaft by its beauty, which makes it applicable to porticoes, frontispieces, entrances to houses, &c.

The **CORINTHIAN** has more ornament and delicacy than any other order: the capital is distinguished by its beauty and richness, and the pillar by its delicacy: which renders it proper, where elegance only is required. It is frequently used for internal decorations, in state rooms, &c.

2. **CATOPTICS** is that part which treats of reflected vision, or that which relates to viewing objects by reflected light, as in mirrors. This is called *reflex vision*.

3. **DIOPTRICS** treats of viewing objects through a different medium or body; as through *glass, water, &c.*

4. **CHROMATICS** treats of the different colours of light.

5. **LIGHT** is that property by which bodies are rendered visible. It consists of an infinite number of extremely small particles, which flow from every body exposed to it, and in straight lines.

6. A **RAY** or **BEAM OF LIGHT** is a very small part of it, proceeding from the luminous body.

7. The **RADIANT** is the object from which the rays of light proceed.

8. The **SPECIES** of an object is the image of it, made by the rays of light, in the *focus* of the reflecting or refracting medium.

9. **PARALLEL RAYS** are such as proceed from a body, parallel to each other.

10. **CONVERGING RAYS** are those rays, which, proceeding from a body, approach nearer each other in their progress, and tend to one certain point: thus the rays, proceeding from LN to the point O, are said to converge to that point. (Fig. 5. plate 5.)

11. **DIVERGING RAYS** are those which, proceeding from any point, do continually recede from each other in their progress: thus the rays proceeding from O towards LN are said to diverge. (Fig. 5.)

12. The **Focus** of rays is that point to which all converging rays tend, as O (fig. 5): this is called the *real focus*.

13. The **VIRTUAL** or **IMAGINARY FOCUS** is a point, as f, to which the rays AB tend, and where they would unite, if they were not intercepted by the obstacle CD (which suppose a mirror), by which means they are turned aside, made to converge in a real focus, E. (Fig. 6.)

14. The REFLECTION of rays is their regrefs, or returning from the surface of such bodies, on which they fall, and cannot enter: thus, the rays $A B'$ falling on the surface $C D$, are reflected or turned back again in the direction E . (Fig. 6.)

15. MIRRORS, or Speculums, are those bodies whose surfaces are so very smooth, as to be impervious to the rays of light which fall on them; and which, therefore, they reflect, so as to represent the images of the objects exposed to them: they are generally made of metal, highly polished, or of glass polished on one side, and silvered on the other; and are either plain, convex, or concave.

16. PLAIN MIRRORS are those whose surfaces are perfect planes, and whose section is a right line, as $C D$ (fig. 6): these are vulgarly called looking-glasses.

17. CONVEX MIRRORS are those whose surfaces do every way equally rise above the plane of their bases; the section of these sort of mirrors is a curve, and is either circular, elliptical, parabolical, or hyperbolical. $A C D$ (fig. 4) is a circular section; and the mirror is the segment of a globe, or spherical surface, which is that mostly used.

18. CONCAVE MIRRORS are those whose surfaces sink down with an uniform hollownes below the upper parts; whose section also is a curve, as various as the convex; but the circular form is the most common.

19. REFRACTION of rays is their being bent, or turned out of their course, in passing out of one medium into another: as the ray $B C$ (fig. 2), in passing into the dense medium $A I H D$, is refracted, or turned out of its natural course $C K$, in the direction $C E$, which is called the refracted ray.

20. The INCIDENT RAY is that which comes from any object, and falls on the refracting or reflecting surface, as $B C$ (fig. 2), or $A B$ (fig. 6).

21. The **ANGLE OF INCIDENCE** is that which is contained between BC (fig. 4), the incident ray, and FC , perpendicular to the reflecting surface, that is, the angle BCF .

22. The **ANGLE OF REFLECTION** is that contained between the said perpendicular FC and the reflected ray CE , that is, the angle FCE .

23. The **ANGLE OF INCIDENCE IN REFRACTION** is the angle BCF (fig. 2), as in reflection FCE (fig. 4) is the angle of reflection.

24. A **LENS** is a medium, generally of glass, made to disperse, or collect, the rays of light which pass through it: of these there are various forms, of which the most common are the eight following:

First, a *Plano-convex*, which has one side plain, the other convex; as A (fig. 3).—Second, *Plano-concave*, which is plain on one side, and concave on the other; as B .—Third, a *Double Convex*, which is convex on both sides; as C .—Fourth, a *Double Concave*, which is concave on both sides; as D .—Fifth, a *Meniscus*, which is convex on one side, and concave on the other; as E .—Sixth, a *Plain Lens*, which is flat on both sides, and of equal thickness in all its parts; as F .—Seventh, a *Flat Plano-convex*, whose convex side is ground into several flat surfaces; as G .—Eighth, a *Prism*, which has three flat sides; and, when viewed end-ways, appears like an equilateral triangle; as H .

Glasses ground into any of the foregoing shapes are called *lenses*: and a line going through the middle, as IK , is called the axis of the lens.

25. The **VISUAL or OPTIC ANGLE**, is that angle contained under two lines, drawn from the extreme points of an object to the eye; thus AIB (fig. 8) is the optic angle, or the angle under which the object AB appears to the eye at I .

26. A **PENCIL OF RAYS** is a double cone of rays, as $LONF$ joined together at the base in the lens LN ; of which

which one cone, LON , has its vertex O in one point of an object; and the other cone, LFN , has its vertex F in that point of the object. The middle line OF is called the axis of that pencil, (fig. 5.)

S E C T. I.

OF VISION.

Direct vision is the faculty of sight, and is occasioned by the rays of light proceeding from an object, and passing through the humours of the eye, where they form the image of the object on the back part, or bottom of the eye. In order to understand which, it will be necessary to explain the figure and construction of the human eye.

$ABCE$ is the eye. It is of a spherical figure, by which means it is easily moved any way in its socket, by muscles appointed for that purpose; the fore part at A (fig. 1) is more convex than any other part. The eye is enclosed in three membranes: the outermost is called the *Sclerotica*; the second, the *Tunica Choroides*; the fore part of which is called the *Iris*, which consists of many fibres, like so many radii: the third membrane, or innermost coat, is called the *Retina*, which is nothing but the optic nerve, spread over the bottom of the eye: upon this membrane the images of visible objects are formed.

In these three membranes are contained the three humours of the eye: the first, HAI , is called the *aqueous* humour,

humour, which is a thin watery liquor; the second, F G O, is the *crystalline*, in the form of a double convex lens, and more convex in the back part: behind this is the *vitreous* humour K L.

The crystalline is more dense than the vitreous, and the vitreous more dense than the aqueous humour: the three humours together form a compound lens, which refracts the rays of light, issuing from an object P R, to the bottom of the eye; and there paints its image p r, upon the retina, in an inverted position.

The aqueous humour is in the form of a meniscus; as is also the vitreous humour. The fore part of that membrane called the sclerotica is called the *cornea*, as at A, and that part adjoining is called the white of the eye. Within the cornea is that coat called the *veat*; in the middle of this is a hole O called the pupil, to let in the rays of light: this pupil is contracted or dilated by several muscular fibres, in order to let in more or less light, as found convenient.

D is the optic nerve, which, coming from the common sensorium in the brain, is expanded all over the concave back surface of the eye, and thus forms the retina. This nerve is not situated in the middle of the eye, but lies nearer the side E, in that part next the nose.

The crystalline F G has a ring of fibres round its edge, by which means it can be drawn more or less convex, and the distance A C is thereby made greater or less, in order to form the image p r, upon the retina, for distinct vision. This ring of fibres is called the *ligamentum ciliare*, the back part of which is black, in order to stifle the rays which fall upon it. The eye is moved in the head by several muscles in the sclerotica.

If the image of an object do not fall upon the retina at p r, the vision will be confused; if it fall short, or nearer F G, as is the case with short-sighted people, then a concave lens that makes the rays more diverging will bring it to the retina.

If the rays of light do not unite, so as to form the image of the object, till they get beyond the retina, as is the case with most old people, then a common convex lens of a proper form will make them converge sooner, and so form the image upon the retina; therefore long-sighted people must use convex glasses; and short-sighted people concave ones.

The ray of light Pp , flowing from the point of the object P , and the ray Rr , flowing from the point R , cross each other at O , and proceeding in the same straight lines, paint the image of the object PR on the retina, in an inverted position, as pr .

Note. Though the rays of light are in the figure represented by single lines, yet it must be observed that every visible point of the object sends forth a *pencil of rays*, which cross each other at O , and paint the image of the object on the retina.

There are many experiments made by philosophers to demonstrate the truth of this theory of vision; the most common of which is the following:—take a bullock's eye, while it is fresh, from a newly killed beast, and having cut off the three coats from the back part, quite to the vitreous humour, put a piece of paper behind that part, and hold the front of the eye towards any bright object, and there will be an inverted image of the object upon the white paper; which in this case serves as a retina to the eye.

Though the image of the object is inverted in the bottom of the eye, yet we judge it to be erect, being always used to that position of the object. By an attentive perusal of the figure, the position of the object, with regard to that of the image, may be easily accounted for. Thus, to view that point of the object P , the pupil of the eye at O must be turned upwards towards A , in order that the ray Pp may fall on the axis of the eye, opposite C , where alone distinct vision is performed. And to view that point of the object R , the pupil of the eye must be turned downwards, to take
in

in the rays of light $R r$, so that r may fall on the axis of the eye at C .

The diameters of objects are proportioned to the diameters of the images at the bottom of the eye; thus, the angle $P O R$ is equal to the angle $p O r$.

Some of the most common properties of the eye are the following:—

1. The eye can only see a very small part of an object distinctly at once; therefore the eye must be turned successively to the several parts of the object, that each part may fall in or near the axis of the eye.

2. When an object is seen distinctly with both eyes, the axes of both eyes are directed to that point; thus the object appears single, though it be seen by both eyes at once: but if the axes of both eyes are not directed to the same point of the object, it will appear double.

3. Few eyes can distinguish a particle of matter, that subtends, at the eye, an angle less than half a minute; and very few can distinguish it when it subtends a minute. If the distance of two stars in the heavens be not greater than this, they will appear as one.

4. The eyes of young people are more convex than those of old people; and this is the reason the former can see an object nigher than the latter.

5. The eyes of short-sighted people are too convex to admit of distinct vision, when the object is placed at the distance of six or eight inches, which is the common distance of an object for distinct vision. And the eyes of long-sighted people are not convex enough to admit of distinct vision at that distance. In the former case, the rays of light converging from an object through the humours of the eye, unite too soon, and before they reach the retina: to remedy which, concave glasses are used, which render the rays more diverging. And in the latter case the rays of light do not unite soon enough, in their passage through the eye, to paint the image on the retina: the remedy of which is the convex glasses.

6. Whatever light falls upon the optic nerve at D makes no impression; therefore if the image of an object fall thereon it is lost. This place is not at the bottom of the eye, but on one side, and towards the nose in each eye; so that no rays that come from any object can fall upon that part in both the eyes at the same time.

7. The dimness of sight which generally attends old people arises from two causes:—First, by the eye growing more flat, and thereby not uniting the rays of light at the retina. Secondly, by the opacity of the humours of the eye, which, in time, lose somewhat of their transparency.

8. As the rays of light flowing from an object are the immediate cause of vision; it must necessarily follow, that where there is no light there can be no vision.

Of the Properties of Light in general.

It may be necessary to give a general description of the nature and properties of light. Sir Isaac Newton has demonstrated, beyond a doubt, that the phenomena of vision are produced by light; which is a matter emitted from the shining body with immense velocity, moving uniformly in straight lines, and acted on by other bodies, so as to be reflected, refracted; or inflected, in various ways, by means of forces, which act on it in the same manner as on other inert matter.

Every visible body exposed to the light, emits, or reflects a great number of these inconceivably small particles of matter from each point of its surface; which issue from it continually, in straight lines, and in all directions, forming a sphere, of which the illuminated body is the centre. These rays of light, as before observed, entering the eye, and striking upon the retina, are the cause of vision. And as these rays differ in substance, density, velocity, or magnitude, they will produce in the mind the idea of different colours.

That the particles of light are exceedingly small, appears from hence: viz. That if there be a candle lighted, and nothing to obstruct the progress of its rays, it will fill all the

the space within two miles of it, every way, with luminous rays of light, before the tenth part of a grain of its matter is consumed.

The velocity with which the rays of light proceed from the surface of a body is surprising. The method by which the swiftness of the rays of light was determined first, was by an observation on the eclipses of Jupiter's satellites. These eclipses appear to us about seven minutes sooner than the precise time when the earth is situated in that part of its orbit between the sun and Jupiter; and the eclipse appears about seven minutes later, when the sun is between him and us: from which it is plain, that the rays of light require about seven minutes to pass through a space equal to the distance between the sun and us; that is, about ninety-five million of miles.

The rays of light proceeding from a visible body, as they proceed in all directions, and extend themselves upwards and downwards, as well as sideways, must necessarily become thinner and thinner: and that they do, in proportion to the squares of their diameters from the luminous body; that is, at the distance of twice a certain space they are four times thinner than at the distance of one such space; and at the distance of three times such a space, nine times thinner; and so on.

Of Refraction.

All light proceeding from any luminous body, without being reflected from any opaque substance, or inflected by passing very near one, is always found to proceed in straight lines. But if the rays pass obliquely from one medium to another, they always leave the direction they had before, and assume a new one; and this change of direction is called their *refraction*: and after having suffered this refraction, they again proceed in straight lines, till they meet with a different medium, when they are again turned out of their course.

The phenomena of refraction are occasioned by an attractive power in the medium through which the light passes.

When

When a ray of light passes out of a rarer into a denser medium (if the latter has a greater attractive force than the former, which is mostly the case), the ray, just before its entrance into the denser medium, will begin to be attracted towards it; and this attraction will continue to act upon the ray till some time after it has entered the medium. When a ray enters the denser medium obliquely, its direction will also become less oblique to the surface of the medium; and approach nearer to a perpendicular, drawn through the surface of the medium.

But when the ray of light passes from a denser into a rarer medium obliquely, its direction will be made more oblique to the surface of the rarer medium; and depart further from a perpendicular drawn through the surface of the rarer medium.

Thus (in fig. 2) let BC be a ray of light passing through the air into the denser medium $ADHI$; at entering the denser medium at the point C , it will begin to be attracted towards that medium, which attraction will change its course from the direction CK to that of CE . Let FG be a perpendicular drawn through the denser medium; then L will be the sine of the angle of refraction, and MN the sine of the angle of incidence. Now the sine of the angle of incidence is always proportionate to the sine of the angle of refraction in the same medium.

From this property of refracting rays of light arises the magnifying power of the glass lenses used in optical instruments. For the rays of light proceeding from any object are too diverging to admit of distinct vision at a less distance than from 6 to 8 inches; if, therefore, by interposing a glass lens between the eye and the object, the object can be viewed at a less distance, the rays of light proceeding from the object, will be made to converge to a point sooner, and by that means the object will be seen under a larger angle.

Thus,

Thus, let O (fig. 10) be an object to be viewed by the eye at F : which object, to be viewed by the naked eye, must be removed to the distance of six inches at least from the eye. But by interposing a lens, whose focal distance is an inch and an half, the eye will have distinct vision of the object at the distance FO ; consequently the object will appear magnified four times in diameter, to what it would do to the naked eye. And the object appears under the angle IFM .

If an object AB (fig. 9) be placed in one focus of the lens DE , and the eye in the other focus F , the eye will see just so much of the object as is equal to the diameter of the lens: for the rays AD and BE , which go from the extremities of the object to the extremities of the lens DE , and are united at the focus F , must necessarily proceed from the object to the lens, parallel to the axis FC , and to each other; therefore, the part of the object AB , seen by the rays DF , EF , will be equal to the diameter of the lens. If only one part de of the lens be open, then only so much of the object AB as is equal thereto, will be seen by the eye. Now as AB is equal to DE , or ab to de : the angle $D F E$, or $d F e$, is the optic angle; that is, the angle under which the part of the object AB , or ab , appears to the eye at F : and as GF is supposed but half the distance of CF ; the angle $D F E$, or $d F e$, is double to that under which the part AB , or ab , would appear to the naked eye at the distance FC ; thus the eye sees the object twice as large in diameter through the lens, as it would do without it.

If it be required to see a part of an object larger than the lens, the eye must be placed nearer the lens than its focus. Let the lens be D (fig. 8); its two foci F and C . In the focus C , let there be an object AB larger than the lens: suppose the rays AD and B proceed from the extremities of the object to those of the lens, then by the lens they will be converged into the point K , between the lens and its focus F : then,

then, if the eye be placed at K, it will see all the object A B, which is larger than the lens.

But if the eye be placed further from the lens than its real focus, it can never see any object, or part of an object, at once, near so large as the lens, but always smaller. Let the eye be placed at I, beyond the focus F; it will then only see the part of the object G H, which is less than the lens.

Thus it is evident that the nature of a convex lens is such, as will render an object distinctly visible to the eye at the distance of its focus. The reason why these lenses are used as microscopes is exceedingly plain; for if the distance F I (fig. 10) be six inches, which is the least distance an object can be seen by the naked eye, and the focal distance of the lens c F two inches, then the object O will appear three times magnified in length, as I M: and if it were the surface, it would be magnified nine times; and the solidity or bulk twenty-seven times.

If the focal distance F c of the lens be one fourth part of an inch, then will that be but one twenty-fourth part of six inches, the distance of naked sight: and so the length of an object seen through such a lens will be magnified twenty-four times; the surface five hundred and seventy-six times; and the solidity thirteen thousand seven hundred and twenty-four times; for those are the square and cube numbers of twenty-four.

To find the principal Focus of any Lens.

Hold the lens perpendicularly against the rays of the sun, or a candle; hold a white paper behind it, to receive the refracted rays, which will make a round white spot upon the paper; move the paper backwards or forwards, till this spot, which is the image of the sun or candle, be the least possible, there fix the paper; then the distance of the paper from the lens is the focal distance required, or the burning point.

Or,

Or, secondly. Cover the side of a lens with a paper, having several small holes made in it with a pin; and placing the lens directly against the beams of the sun, or a candle, the light passing through the holes will form so many white spots upon the paper held behind it. Move the paper backwards and forwards, till all the spots coincide in one. That point is the focus; and the distance of the paper from the glass is the focal distance.

I shall here give some of the most general propositions and properties in the science of optics, leaving the process and demonstration for the exercise of the reader.

1. Wherever the rays, which come from all the points of an object, meet again in so many other points, to which they converge by refraction, there they will make a picture of the object upon any white body on which they fall.

2. Rays flowing from all points of an object, and passing through a lens, paint the image of the object in the real focus of the lens, and may be seen there upon a white paper: or without the paper, the image may be seen hanging pendulous in the air, by an eye placed six or eight inches behind the focus.

3. The object, and its image made by a lens, subtend equal angles at the lens.

4. The length of an object is to the length of the image made by a lens, as the distance of the object from the centre of the lens is to the distance of the image from it.

5. A convex lens magnifies an object, when it is nearer than twice the principal focal distance; but if further off, it lessens it.

6. A concave lens always diminishes an object.

7. The apparent magnitude of an object is the angle under which the object appears to the eye.

8. If the object and its image be both on the same side of the lens, the image will be erect; but if they be on different sides, the image will be inverted.

9. When the object and its image are on different sides of the lens, as the object approaches the lens, the image recedes from it: or, if the object recedes from the lens, the image approaches it.

10. If the object and image be both on one side of the lens, and the object move towards the lens, the image also moves towards the lens: and if the object moves from it, the image also moves from it.

11. In a convex lens, if the object be further off than the principal focus, its image will be on the other side of the glass, inverted. But if the object be nearer than the principal focus, the image will be on the same side of the glass, erect.

12. In a concave lens, the image and object are always on one side of the lens.

13. If an object be placed in the principal focus of a lens, its apparent magnitude at any place whatever, beyond the lens, will be invariably the same; and equal to the apparent magnitude, when seen from the centre of the lens, with the naked eye.

14. The apparent magnitude of a body, placed in the principal focus, will always continue the same, however the eye is moved backward or forward, from the lens.

15. The nearer the eye is to the lens, the more of the object appears; and the farther off the eye is from the lens, the less of the object is seen.

16. If the object be nearer than the principal focus, its apparent magnitude becomes less, in going from the glass. But if the object be further than the focus, the apparent magnitude will increase, in going from it.

17. If the eye be fixed in the principal focus, the apparent magnitude of an object will be invariably the same, wherever the object is placed before the glass.

18. If the eye and object be fixed, and a convex lens be moved from the object to the eye, the apparent magnitude

nitude of the object increases to the middle, and then decreases to the eye. When it comes into such a position that the eye and object are conjugate foci, the object is infinitely confused.

19. If the eye and object be fixed, and a concave lens be moved from either of them to the other; the apparent magnitude of the object will decrease to the middle, and then increase again. The apparent distance is reciprocally as the apparent magnitude; and, in general, we judge the apparent distance of an object to be the very same as we suppose the real distance of an object to be, from which the rays come to our eyes, with the same degree of confusion.

20. Converging rays are made more converging by a convex lens: and diverging rays are made more diverging by a concave.

21. Parallel rays falling on one side of a convex lens will be refracted to the focus on the other side. But parallel rays falling on a concave lens will, by refraction, diverge from the focus on the same side.

22. In a convex lens, rays diverging from the focus will emerge parallel on the other side.

23. In a concave lens, rays converging to the focus will emerge parallel, going out of the lens on the same side.

24. In a double convex lens of equal radii, the principal focus is distant from the lens, the radius of the sphere, of which the lens is a segment.

25. When the lens is a perfect sphere, the principal focus is distant from the lens half the radius.

26. When the lens is an hemisphere, the principal focus is distant four thirds of the radius, when the convex side is exposed to the rays; and twice the radius, when the plane side is exposed.

27. When the lens is a plano-convex, the distance of the focus is equal to the diameter, or twice the radius of the sphere when the plane side is exposed to the rays. But when the

the convex side is exposed to the rays, the focus is distant twice the radius, except two fifths of the thickness.

28. In double concaves of equal radii, the principal focus is distant the radius of the sphere; and it is virtual.

29. In a plano-concave, the principal focus is distant twice the radius; and is virtual.

30. If the radiant point and the focus be equidistant from a lens, they will each of them be distant twice the principal focal distance.

31. If the radiant point be nearer the lens than the principal focus, the rays after refraction will diverge; but if the radiant point be in the principal focus, they will after refraction emerge parallel; and if the point be farther off than the focus, the rays will converge after refraction.

32. The distance of the radiant point and its focus, made by a convex lens, is the least possible, when they are equidistant from the lens.

33. If a convex lens be held directly to the rays of the sun, and a combustible body be held in the principal focus; the heat of the rays of the sun, converging to a point in the focus, will set the body on fire.

34. The heat of the focus of the lens is to the sun's direct heat, as the area of the glass to the area of the image in the focus.

35. The degrees of heat in the foci of different lenses are as the squares of the diameter directly: and the squares of the focal distances reciprocally.

The foregoing propositions contain all the phenomena of refracting lenses; and may be proved for the most part by actual experiments.

It is necessary just to observe, for the sake of those who may use lenses of a different medium, that different mediums have different refracting powers: I shall, therefore, exhibit a table of the refracting power, as given by Sir Isaac Newton, and proved by many later experiments.

In

In the first column are the names of the bodies; in the second column, the sines of incidence and refraction; in the third column, the refracting power; in the fourth column, their density, or specific gravity; in the fifth column, the ratio.

<i>Bodies.</i>	<i>Sines of Incidence, and Refraction.</i>		<i>Refracting Power.</i>	<i>Density.</i>	<i>Ratio.</i>
Pseudo topaz	23	14	1.699	4.27	0.39
Air	3201	3200	0.000625	0.0012	.52
Glass antimony	17	9	2.568	5.28	.48
A selenitis	61	41	1.213	2.252	.54
Glass	31	20	1.4025	2.58	.54
Rock crystal	25	16	1.445	2.65	.54
Island crystal	5	3	1.778	2.72	.65
Sal gemma	17	11	1.388	2.143	.64
Alum	35	24	1.1267	1.714	.65
Borax	22	15	1.1511	1.714	.67
Nitre	32	21	1.345	1.9	.70
Vitriol	303	200	1.295	1.715	.75
Oil vitriol	10	7	1.041	1.7	.61
Rain water	529	396	0.7845	1.	.78
Gum Arabic	31	21	1.179	1.375	.85
Spirit wine	100	73	0.8765	0.866	1.01
Camphire	3	2	1.25	0.996	1.25
Oil olive	22	15	1.1511	0.913	1.26
Linseed oil	40	27	1.1948	0.932	1.28
Spirit turpentine	25	17	1.1626	0.874	1.32
Amber	14	9	1.42	1.04	1.36
Diamond	100	41	4.949	3.4	1.45

From this Table it appears, that those bodies which contain oily sulphurous particles differ from the constant ratio, and have a greater refractive power.

eye as great as if it were only the eightieths of an inch (which is the one hundredth and sixtieth part of six inches) distant: therefore, the length of an object seen through such a lens will be magnified one hundred and sixty times; the surface, twenty-five thousand six hundred times; and the solidity, four million and ninety-six thousand times; which is so great a magnifying power as to surpass conception.

But spherule lenses, of so small a diameter as the latter, are of no use but in viewing transparent objects; for if an opake object were to be viewed by such a lens, the eye must be applied almost close to the surface of the lens, by which the lens and object would be so overshadowed, as to render an opake object too obscure to be viewed.

These spherule lenses were used by the famous Mr. Leeuwenhock, by which he made such wonderful discoveries in the minuter parts of nature. And it must be by a proper arrangement of these, that the particles of matter are to be discovered, if ever they can, which Sir Isaac Newton thought was possible.

But there are great difficulties attending the use of these very small spherules: first, the difficulty of making very good ones, of a very small diameter. Secondly, the prejudice done to the eyes in using them. Thirdly, the trouble of placing objects at the proper focus. And lastly, the very small part of the object, which can be seen at one time. These inconveniences render this sort of microscopes of very little service for common use; they have therefore been superseded by others, particularly by the following; in which both transparent and opake objects may be viewed with less trouble.

In this instrument (fig. 11.) F is a piece of brass turning round in a socket, at the end of which is a small spring tube moving upon a rivet, through which there runs a steel wire, terminated at one end by a sharp point, G, and the other end hath a pair of piers, H. The point and piers are to thrust into, or take up any object. Either of them may be turned

M (fig. 13) is a round object plate, black on one side, and white on the other. N, a steel spring, turning down on each side, to hold any object fast. A dark-coloured object is to be placed on the white side; and a light-coloured object on the black side, to render them more distinct. This plate also has a small pipe to screw on the needle G (fig. 11).

This microscope is of late invention: and by means of the polished mirror, any opaque object may be viewed with a very small magnifier. Transparent objects may also be viewed by it: but in this case, it will not be proper to throw on the object all the light reflected from the speculum; lest the light transmitted through the object, meeting the reflected light, produce too great a glare. For this purpose, a bit of paper may be interposed between the object and some part of the speculum. A little practice will enable a person to know how to regulate the light.

As opaque objects are more numerous than others, very great discoveries may be made in such objects.

A Microscope on a Stand, which will answer all the Ends of the large Double Reflecting Microscope.

B (fig. 17) is a round frame of wood, on which is fixed the brass scroll A for the purpose of holding the microscope steady. C is a brass screw that passes through a hole in the scroll, into the microscope D, and serves to screw it fast to the scroll.

E is a concave looking-glass, or speculum, of metal, set in a brass box, and hanging in the arch G by two small screws: at the bottom of this arch is a small brass pin h, which goes through the stand to the arch into the wooden frame B, by which means the arch is turned round in an horizontal direction; and the mirror swings in a vertical direction. By these two motions the speculum may be so adjusted as to reflect the light directly upwards, through the body of the microscope D, which is fixed perpendicularly over it.

also, but on the other side; thus, the rays of each pencil will converge at the eye k , where they cross each other, and after passing through the humours of the eye, they will form the large inverted image $A B$ on the retina.

This image, seen by such a microscope, will be inverted with regard to direct vision: for the object $a b$ lies in the same direction as the image $A B$ on the retina, as seen in the figure; which is not the case when the object is seen by the naked eye, or by a single microscope.

To calculate the magnifying power of these double microscopes, we reckon how many times the object is magnified by the object glass, and then how many times the image of this object is magnified by the eye glass: thus, if the object $a b$ be magnified six times in length by the object glass $c d$, as it will when the image $g h$ is six times the distance from the object glass, as the object $a b$ is: and if the eye glass $e f$ be only of half an inch focus, the image $g h$ can be viewed at half an inch distance, which being only the twelfth part of six inches, the image $g h$ will be magnified twelve times in length to what it will appear to the naked eye without the eye glass: and the image $g h$ being six times the length of the object $a b$, the object $a b$ is therefore magnified in length seventy-two times, and the superficies and solidity in proportion.

The field of view is very small in a microscope of this sort; therefore, there are generally two eye glasses used, which are placed sometimes close together, and sometimes an inch or more asunder: by which means the visible area is much enlarged, and a larger field of view is obtained, though the object appears somewhat less magnified.

A Compound Microscope.

A (fig. 18) is the body of the microscope, which is moveable up or down in the external case $C D$, which is fixed on the foot of the instrument I , by three pillars. F is a plate fixed horizontally to the three pillars, and is called the stage; in the centre of which is a hole, on which a piece of glass,

or any other implement is placed, with the object to be viewed. H is a reflecting speculum, which reflects the light upwards, through the hole in the stage. K is a pipe, to which is screwed the brass button Q, which holds the magnifier, or the object glass.

There are generally three, and sometimes four glasses used in this microscope: first, the magnifying lens, or object glass at Q. Second, a large lens at B, called the body glass, by which a large field of view is obtained. And third, an eye glass at A, by which the magnified image of the object is magnified still more.

From what has been said, it is evident, that the magnifying power of all optical instruments depends upon this one principle; that the nearer the object can be brought to the eye, to be distinctly visible, the more it will be magnified; and that in proportion as the distance of the magnified object from the eye, is less than the distance at which the object can be seen distinctly by the naked eye, which is in general from six to eight inches.

SECT. III.

OF REFRACTING, REFLECTING, AND SOLAR TELESCOPES.

BEFORE the reader enters into the description of telescopes, it will be necessary to explain the nature of concave lenses, and their property in rendering rays divergent. Also the nature of reflection by plain and concave mirrors.

When

When parallel rays pass through a concave glass, they will diverge after passing through it: thus, let a, b, c, d, e, f, g, h , (fig. 1, plate 6,) be parallel rays: after passing through the glass, AB , they will diverge, as if they had come from the point C , which is the centre of the glass's concavity: this point C is called the negative, or virtual focus of the glass. Thus, the ray a , after passing through the glass, will proceed in the direction kl , and the ray h , in the direction nn , and the ray c in the direction op , &c. The middle ray C , falling directly upon the middle of the glass, suffers no refraction.

This lens being equally concave on both sides, the rays diverge, as if they came from the centre of the glass's concavity. But if the glass had been concave on one side only, and the other side plain, the rays would have diverged, as if they had come from a point at double the distance of C , or the centre of the glass's concavity.

If the rays, before they pass through the glass, converge more than parallel rays diverge after they have passed through the glass, such rays will continue to converge after passing through it; but will not meet so soon as if no glass had been in the way; and will incline towards the same side to which they would have diverged, if they had come parallel to the glass: thus, the rays fh , proceeding towards the glass in a converging direction, and converging more than the parallel rays diverge, after they have passed the glass; the rays fh will go on converging, after passing through the glass, though in a less degree; and will at length meet at I .

If parallel rays fall upon a concave mirror, they will be reflected back, and meet in a point at half the distance of the centre of the mirror's concavity. Thus, let $dfa, Cnl, e/c$, (fig. 2,) be parallel rays falling upon the concave mirror AB : they will be reflected back, and meet in the point m , half the distance of the mirror from C , which is the centre of its concavity; and they will be reflected at as great an

Let AB be a mirror whose centre of concavity is C (fig. 3). DE is an upright object placed beyond the centre C ; from the upper end D , of which flows a pencil of diverging rays to every point of the concave surface of the mirror AB .

From the centre C , draw CA , Cc , CB , touching the mirror in the same points where the pencils of rays touch it, which flow from D . These three lines drawn from C to the mirror will be perpendicular to the surface of the mirror. Make the angle $CA\alpha$ equal to the angle DAC ; and draw the right line $A\alpha$ for the reflection of the ray DA . Make the angle $CB\alpha$ equal to the angle DBC , and draw the right line $c\alpha$ for the course of the incident ray Dc . Make the angle $Cb\alpha$ equal to the angle DBC , and draw the right line $B\alpha$, for the course of the incident ray DB ; all these reflected rays will meet in the point α , where they will form the extremity D of the image DE .

If the pencils of rays be drawn from the other point of the object E , in the direction Ef , Eg , Ek , and be continued to the surface of the mirror, they will be reflected back from the mirror to the other end of the object α ; and their angles of reflection will be equal to their incident angles: thus, they form an inverted image of the object, at the point α .

Though I have only drawn three lines in the figure, to represent rays flowing from the point D ; yet it must be observed, that this point, as well as every other point in the object, sends forth a *pencil of rays* to every part of the mirror; and the rays of each pencil are reflected back, and meet in all the intermediate points of the object α , as has been shown, with regard to the point D .

When the object is farther from the mirror than its centre C , the image will be less than the object, and will be between the object and the mirror; but when the object is nearer than the centre of concavity, the image will be further off, and bigger than the object.

If

side the table, nothing will appear but a long beam of light.

If, instead of a fire, a large candle be placed opposite the mirror, a person standing by the candle will see the appearance of a large bright star over the table: thus, by fixing a large candle in a frame, with a small wax taper to turn round it, in a circle, an appearance will be made in the focus of the glass, representing a bright planet, and its satellite; the satellite performing a revolution round the planet.

The Refracting Telescope.

The first form of refracting telescopes that I shall mention, is that which consists of two lenses only. The object glass must be convex; but the eye glass may be either convex or concave.

Let $c d$ (fig. 4) be a convex object glass, in a long tube, and have its focus at D . $A B$ is a remote object: from the extremity A flows a pencil of rays $g b i$, which will be so refracted by passing through the glass $c d$, as to meet in the point f ; and the pencil of rays $k l m$, flowing from the other point of the object B , by passing through the lens will be refracted so as to unite in the point e : thus the points A and B of the object will be formed by refraction in the points $f e$: and all the intermediate points of the object send forth pencils of rays, which form corresponding points in the small figure $f e$, the inverted image of the object. But as this image is small, a concave glass $n c$ is placed at the end next the eye, so that its virtual focus may be at F . Then as the rays of the pencils come to the concave glass converging, they will converge less after passing through it, as hath been demonstrated: and they will proceed to b and a before they unite, which is on the retina, and there form the large image $a b$.

The only inconvenience attending the use of this telescope is, that the field of view is very small, by reason of the rays

diverging so much, after passing through the concave glass.

For, from a view of the figure, it is evident that when the telescope lies directly towards the object, none of the rays which flow from the extremities A and B can enter the pupil of the eye; but fall upon the iris, above and below the pupil. So that, to view the different parts of an object, the telescope must be moved upwards or downwards, unless the object be a very remote one; and then it is but indistinctly seen.

Therefore to remedy this inconvenience, a convex eye glass is used instead of a concave, as $a b$ (fig. 5), which is placed so, that its focus may be coincident with the focus of the object glass $c d$, whose focus is at E. Then the rays of the pencils flowing from the object A B, and passing through the object glass $c d$, will meet in its focus, and form the small inverted image $m p$. And as the image is formed in the focus of the eye glass $a b$, the rays of each pencil, after passing through this glass, will be parallel, but the pencils themselves will converge to each other, and cross each other in the focus of the glass at e : the pupil of the eye being in this focus, the image will be viewed through the eye glass, under the angle D e C.

This telescope also has an inconvenience, which is, that it inverts the object with respect to it, when seen by the naked eye: that is, the image is painted in the eye in the same position in which the object lies; we therefore judge it to be inverted. This telescope is, therefore, only fit for viewing celestial bodies, in many of which their position is immaterial. But it must be observed, in the use of this telescope, that whatever way the object *seems* to move, the telescope must be moved the contrary way; for as the object is inverted, so will be the motion.

The magnifying power in this telescope, is as the focal distance of the object glass is to the focal distance of the eye glass. Therefore, if the former be divided by the latter, the quotient will express the magnifying power.

Therefore,

Therefore, this telescope can be constructed so, as to magnify in any proportion, provided it be of a sufficient length: for the greater the focal distance of the object glass is, the less may the focal distance of the eye glass be. Thus, an object glass of ten feet focal distance will admit of an eye glass, whose focal distance is little above two inches and a half, which will magnify the length of an object near forty-eight times, and its surface and solidity will be magnified in proportion; as in the microscope.

Thus, if the focal distance of the object glass be equal to the focal distance of the eye glass, the telescope would not magnify the object in the least.

Again, if the focal distance of the object glass be twenty inches, and the focal distance of the eye glass one inch, the length of an object seen by such a telescope will be magnified twenty times, its surface four hundred times, and its solidity eight thousand times.

To remedy the inconveniences of both the former kinds of telescopes, the following one is generally used for terrestrial objects, which gives a larger field of view than that with the concave eye glass, and shows the object in its natural posture, as seen by the naked eye. In this telescope there is one object glass cd (fig. 6), and three eye glasses, ef , gh , ik . They are so placed, that the distance between every two glasses, which are next each other, may be equal to the sum of their focal distances: thus, the focus of one glass is coincident with the focus of the next glass. The foci of the two glasses cd and ef , meet at F ; those of the two glasses ef , gh , meet at l ; and those of gh and ik , at m ; and that of ik at n , where the pupil of the eye is situated. Then it is evident, from the figure, that the pencils of rays which flow from the object AB will pass through the object glass cd , where they cross each other, and meet, and form an inverted image, in the focus of the glass at $C F D$; this image being also in the focus of the next glass ef , the rays of the pencils, which flow from the image $C F D$, will

become parallel after passing through this glass, and cross each other at L , which is the focus of the next glass $g k$; therefore, the rays proceeding from this focus and passing through the next glass $g k$, are converged to the focus of this glass, where they form the erect image $E F$ of the object $A B$; and as this image is the focus of the eye glass $i k$, and the eye on the opposite side of the glass in the other focus, the image is viewed through the eye glass in this telescope, in the same manner as in the former one, but in a contrary position; that is, in the same position as when seen by the naked eye. The object also appears under the angle $i n k$.

The three eye glasses have generally all their focal distances equal. The magnifying power of this telescope is found in the same manner as that of the last; viz. by dividing the focal distance of the object glass by the focal distance of any one of the eye glasses, as the three latter are all equal.

Objects seen through such a telescope appear coloured about the edges, the reason of which is, that the rays of light coming from the object are unequally refracted through the glass lens, and particularly those rays which pass through a convex glass near its edges. These rays, being unequally refracted, do not all meet again in the same points exactly. Therefore, in this telescope, there must be a plate with a small round hole in the middle, fixed at m , parallel to the glasses. By this plate, the wandering rays about the edges of the glasses will be intercepted, and prevented from coming to the eye; and none admitted, but those which come through the middle of the glasses, or, at least, at a good distance from the edges. But this somewhat lessens the field of view, which would be much larger if the plate could be dispensed with.

The Binocular, or Double Telescope.

The binocular, or double telescope, is only two telescopes of an equal magnifying power, fixed in a frame, parallel to each

each other, and adjusted by the means of screws, so as to be at the same distance from each other as the pupils of the two eyes. Then by looking through them both at once, through one with each eye, the object will be seen by both eyes, and appear brighter and more distinct than through a single telescope.

The Reflecting Telescope.

Refracting telescopes require to be of a considerable length, to magnify to any great degree, and the great length of some of them has rendered them very inconvenient. Sir Isaac Newton, therefore, invented the first telescope of the reflecting kind, and which has received considerable improvements since his time. Since the invention of these, refracting telescopes are very little used for celestial objects; for a reflecting telescope, of only six feet in length, can be brought to magnify the object as much as one of the other sort of a hundred feet in length.

They are generally constructed now in the following manner: T T T T (fig. 7) is a large tube, at the bottom of which is placed a large concave mirror, D U V F, whose principal focus is at m . In the middle of this speculum is a round hole P, opposite which is placed the small mirror L, concave towards the great mirror, and fixed on the wire M, by which it may be brought nearer to the great mirror, or moved further off, having a long screw on the outside of the tube for that purpose, always keeping its axis in the same line with that of the great mirror; viz. P m n . In using this telescope, we may suppose the rays of light to fall parallel to each other upon the great mirror; for no portion of a celestial object can be seen, which is less than the surface of the mirror.

Let A B be the distant object, from whose two extremities flow the pencils of rays C and E, which we here suppose parallel: the rays C, falling upon the great mirror at D, are from thence reflected in the direction D G, and

distance of the small mirror from the image which is next the eye; and multiply the focal distance of the small mirror, by the focal distance of the eye glass; then divide the product of the former multiplication, by the product of the latter, and the quotient will be the magnifying power.

When the first image $I K$ is formed a little further from the great mirror than its principal focus, as at n ; the focus of the great mirror is never coincident with that of the small one: therefore the rays of the pencils reflected from the small mirror will not be parallel, but rather converging, so as to meet in the points $q e r$, where they would form a larger erect image than $a b$, if the glass R was not in the way. This image might then be viewed by means of a single eye glass, properly placed; but in that case, the field of view would be less; therefore, the glass R is used to enlarge the view.

The adjusting screw, to which the small mirror L is fixed, renders this telescope applicable to every sight, as the small mirror may be brought either nearer to the eye, or removed farther from it; by which means the rays may be made to diverge a little, for short-sighted eyes; or to converge, for persons of a long sight.

In the use of this telescope, it must be observed, that the nearer an object is, the more the pencils of rays will diverge before they fall upon the great mirror; and consequently they will be the longer before they meet after reflection; therefore, the first image $I K$ will be formed at a greater distance from the large mirror. But this image must always be formed further from the small mirror than its principal focus n ; therefore, the small mirror must be set at a greater distance from the large one in viewing a near object than when viewing an object more remote; this is easily performed by turning the adjusting screw.

The reflecting telescope will admit of an eye glass, of a much shorter focal distance (and consequently of a greater magnifying

magnifying power) than a refracting telescope will; therefore, those telescopes magnify to a greater power. And the rays of light are not coloured by the reflections of the concave mirror, if it be ground to a true figure, as they are by passing through a convex glass, though it be ground ever so true.

In these reflecting telescopes, the observer never sees the object itself, but only the image of it, which is formed next the eye. This will be demonstrated by the following experiment:

If the observer fix a stick across the mouth of the telescope before the object glass, it will not hide any part of the object from his view, unless it covers the whole mouth of the telescope; but it will only make the object appear dimmer, by intercepting part of the rays. But if he place only a piece of wire across the inside of the tube, between the eye glass and his eye, it will hide part of the object, which he thinks he sees; which proves, that it is not the real object, but the image, which is perceived. This is evident, from the small mirror L, which is made of opaque metal, and stands directly between the eye and the object, and would hide the whole object from the eye, if the two glasses R and S were taken out of the tube.

The Solar Telescope.

The solar telescope is the most useful instrument for viewing the face of the sun of any hitherto invented. It is formed of a scioptic ball and socket, which is fastened against a hole in the window-shutter of a darkened room; in the cylindrical hole of which is placed the end of a common refracting telescope, which is to be drawn out to its proper length; then the ball and telescope are moved about till the
rays

rays of the sun fall perpendicularly on the object glass, through the hole of the ball: the tube which contains the eye glass is then to be adjusted, by drawing it in, or out, till the image of the sun be formed on a white paper, in the focus of the telescope, very distinctly and large.

By this telescope, the face of the sun may be represented in almost any size, which affords the mathematician an opportunity of viewing all the phenomena to be seen in that planet; such as the spots on the sun's disc, which are seldom seen when viewed through small telescopes, in the common way. But they may here be seen in all their different appearances; as their increase, division, union, beginning and end, &c.; also the solar eclipse may be seen to great advantage. And the transits of Mercury and Venus over the face of the sun are most delightfully exhibited by this instrument; as the planets here appear truly round, and very black; their comparative diameters to that of the sun may be taken, with the true direction of their motions, their ingresses, egresses, &c. The circle of the sun's disc may be divided by lines and circles, that the quantity and time of the eclipse may be exactly determined.

The *heliostata* was the invention of Dr. Gravesand: by this excellent invention, the motion of the telescope, in viewing the solar light, is greatly taken off. It consists of two parts: first, a plain metallic speculum, supported on a stand; and a clock, which directs the speculum according to the motion of the earth, and thereby preserves the image of the sun in the same point of view.

The Acromatic Telescope.

This telescope has a double, and sometimes treble object glass, to correct the aberration of the rays of light; for all rays of light passing through a single lens will be somewhat decomposed, and divided into separate colours, which renders the object coloured; to remedy which, the double

in an angle of forty-five degrees from the perpendicular $I g$; then the pencils of rays flowing from the outward objects, and passing through the convex glass to the plane mirror, will be reflected from it upwards, and meet in points, and form the image $I K$, of the object $A B$, which is at the same distance from the mirror $E F$, as the image $G H$, which image they would have formed if no mirror had been in the way. This image $I K$ will be formed on an oiled paper, stretched horizontally in the direction $I K$. On this paper the outlines of the images may be drawn with a black lead pencil, and then copied on a clean sheet; but it is usual to place a plain unpolished glass in the direction $I K$ to receive the images of the outer objects; and their outlines may be traced upon this, as on the paper.

The tube in which the glass $C D$ is fixed must be moveable, that it may be drawn out to adjust the distance of the glass lens, from the plane mirror, in proportion to the distance of the focus. For this purpose, the tube must be moved backward or forward, till the images appear distinctly on the horizontal glass $I K$.

If the mirror $E F$ be reclined in the opposite direction, that is, leaning forwards towards the lens, in an angle of forty-five degrees, the images of the objects will then appear in an horizontal position, but below the box; and inverted with respect to the image $I K$.

To form an horizontal image, as $I K$, of an upright object $A B$, it is necessary that the mirror be reclined from a perpendicular, in an angle of forty-five degrees. Then the rays of the pencils $a b c$ flowing from the point A of the object $A B$, will fall upon the mirror at the points $k f g$, and from thence will be reflected in the lines $k I$, $f I$, $g I$, and form the point A of the object $A B$, at the point I in the image $I K$. And the rays of the pencils $q r s$, flowing from the point B , and passing through the convex lens $C D$, will fall upon the mirror at the point $m l k$, from whence they will be reflected to the point K , where they form the image

fracted in the direction fH ; by which the object will appear in the line fm , as if it were at D . If the glass be turned round the axis H/g , the object C will remain in the same place, but the other objects D and E will appear to go round C ; because the planes on which the oblique rays fall, will be turned round by the motion of the glass.

From what has been said of the properties of different lenses, the intelligent reader may be able to form a combination of them, for any ordinary optical purpose; but it may be necessary to say a few words more, concerning the reflecting telescope.

The great mirror of the reflecting telescope, as invented by Sir ISAAC NEWTON, was truly spherical; which telescopes are of essential service at this day, particularly in the *minutiae* of astronomy, when small apertures and long foci are used. Dr. *Herschell* uses a reflector of this form, in his seven feet reflecting telescope. And the highest power that a telescope of a spherical metal will bear with perfect distinctness, may be found by multiplying the diameter of the great mirror by seventy-four. Thus, in Dr. Herschell's seven feet reflector, the aperture or diameter of the great mirror is 6.25 inches, which, multiplied by 74, gives 462 the magnifying power; and the focal distance of the single eye glass may be found, by dividing the focal distance of the great mirror by the magnifying power: thus, 7 feet, the focal distance, multiplied by 12, and divided by 462, quotes 0.182 of an inch, the focal distance of the eye glass.

But the concave surface of the great mirror, in common reflecting telescopes, is of a parabolic form; which form Sir Isaac Newton was unable to give to his metal, though he recommended it; and imagined it might be effected by mechanical devices.

A TABLE

A TABLE
OF THE
Focal Distances, Apertures, and magnifying Powers
OF THE
NEWTONIAN REFLECTORS.

Focal Distance of the concave Metal	Aperture of the concave Metal	Sir Isaac Newton's Numbers.	Focal Distance of the single Eye Glass.	The magnifying Power.
<i>Feet.</i>	<i>In. Dec.</i>		<i>In. Dec.</i>	
1	0.86	100	0.167	36
1	1.44	168	0.199	62
2	2.45	283	0.236	102
3	3.31	383	0.261	138
4	4.10	476	0.281	171
5	4.85	562	0.297	202
6	5.57	645	0.311	232
7	6.24	—	0.323	266
8	6.89	800	0.334	287
9	7.54	—	0.344	314
10	8.16	946	0.353	340
11	8.76	—	0.362	365
12	9.36	1084	0.367	390
13	9.94	—	0.377	414
14	10.49	—	0.384	437
15	11.04	—	0.391	460
16	11.59	1345	0.397	483
17	12.14	—	0.403	506
18	12.67	—	0.409	528
19	13.20	—	0.414	550
20	13.71	1591	0.420	571
21	14.23	—	0.425	593
22	14.73	—	0.430	614
23	15.21	—	0.435	635
24	15.73	1824	0.439	656

A TABLE

A TABLE

OF THE

*Dimensions and magnifying Powers of the Gregorian
Reflecting Telescopes,*

As constructed by

MR. SHORT.

Magnifying Power.	Distance between the two Glasses.	Focal Distance of the Glass next the Eye.	Focal Distance of the Glass next the Metals.	Distance between the little Speculum, and the first Eye Glass.	Breadth of the Hole in the great Speculum.	Focus of the little Speculum.	Breadth of the great Mirror.	Focal Distance of the great Mirror.
	In. Dec.	In. Dec.	In. Dec.	In. Dec.	In. Dec.	In. Dec.	In. Dec.	In. Dec.
39	1.68	.81	2.44	8.54	.31	1.10	1.54	5.65
60	2.08	1.04	3.13	14.61	.39	1.50	2.30	9.60
86	2.93	1.31	3.94	23.81	.50	2.14	3.30	15.50
165	3.41	1.71	5.12	41.16	.65	3.43	6.26	36.00
241	4.28	2.48	6.43	68.17	.85	5.00	9.21	60.00

Since the invention of these reflecting telescopes, the refracting ones are very little used for celestial observations. For a refracting telescope, even of a thousand feet focus (supposing it possible to be used), could not be made to magnify more than one thousand times; whereas a reflecting telescope, not exceeding ten feet, will magnify twelve hundred times.

Mr.

of the glass would be of little service, if this latter property of transmitting the light be wanting; for the same quantity of light, spread over a larger surface, becomes proportionably diminished in force; and though the objects may appear bigger, they would appear proportionably dimmer, without an increase of light, as well as magnitude. Thus, though a glass should magnify the surface of an object an hundred times, yet if the focal distance of the glass were only eight inches (if this be possible), and its diameter only the size of the pupil of the eye, the object will appear an hundred times more dim through the glass, than to the naked eye; even though the glass transmitted all the light which fell upon it, which no glass can do. But if the focal distance of the glass be four inches, and the diameter the same as before; the object will appear far more bright, because the glass could be placed twice as near the object, as before, and consequently would receive four times as many rays of light from the object, as in the former case. Thus, diminishing the focal distance of the glass, and keeping its diameter as large as possible, we should perceive the object more and more magnified, and at the same time very distinct and bright.

With regard to the comparative merit of telescopes, for terrestrial objects, those of the refracting kind have evidently the advantage of all others, where the aperture is equal, and the aberrations of the rays are corrected according to Mr. Dollond's method; because the image is more perfect; and a greater quantity of light is transmitted, than can be reflected from the best materials hitherto known. A telescope for terrestrial objects should never magnify above an hundred times; if it do, the vapours arising from the earth will be so magnified, as to render vision very obscure and imperfect.

However, the imperfections in every kind of glass very much retard the improvement of these telescopes, in which there are five eye glasses; so that they cannot be made above three feet and an half long.

telescope must be kept constantly directed to the same point of the object : for if the telescope be moved gently from its position, the object will seem to move in the same, or in the opposite direction, according as the telescope shows the image inverted, or erect.

If the instrument shake ever so little, it makes the object dance before our eyes ; and causes an indistinctness in the vision. In the Gregorian telescopes it is hardly possible to prevent this quivering motion of the small speculum, as it is fixed to the tube by one arm only. And though the tube of the instrument be fixed ever so motionless, this speculum will retain a tremulous motion, particularly if the telescope has a great magnifying power. This motion is an imperceptible tremor, like that of an harpsichord wire, while it is sounding, and produces a similar effect to the sight. A person walking in the room where the instrument is fixed, prevents us from seeing the object distinctly ; or even the pulsation in the body of the observer agitates the ground enough to produce this effect. The more rapid this motion is, the greater is the optical indistinctness ; or the object appears more confused. Therefore, the more firm, elastic, and well bound together, the frame-work and apertures of the telescope are, the more hurtful will be the consequences. Just as it is to a rider on a wheel-carriage—the more firmly the carriage is fixed on the wheels, the more violent will be the motion to the rider. A telescope, therefore, mounted with lead (if it were practicable), would be preferable to one mounted on either wood, iron, or brass. This is the principal cause of that indistinctness of vision in the very best reflecting telescopes. Refracting telescopes have not this inconvenience.

From hence we may easily judge of the merits of any particular apparatus for the support of a telescope : and in general, all forms where the tube is supported only in or near the middle, or where it remotely depends upon one joint, are very destructive of distinctness of vision.

by a kind of *teasing*. Its length is thirty-nine feet four inches; and it measures four feet ten inches in diameter. It was formed of sheet iron, in order to be of less weight, and of greater durability; for a wooden one must have exceeded this iron one, at least three thousand pounds, in weight.

Very great mechanical skill is displayed in the construction of the apparatus by which the telescope is supported. In order to direct the telescope to every altitude, the point of support is moveable, and its motion is effected by mechanism: so that the telescope may be moved from its most backward point of support, to the most forward; and by means of the pulleys G G suspended from the great beam H, be set to any altitude, even up to the very zenith. The tube is also made to rest, with the point of support, in a pivot, which permits it to be turned sideways.

The concave face of the great speculum is forty-eight inches of polished surface in diameter: the thickness now is about three inches and a half: it is of equal thickness in every part. Its weight, when it came from the cast, was two thousand one hundred and eighteen pounds, of which it must have lost a small quantity in polishing. When this speculum is to be placed in the tube, it is suspended vertically by a crane in the laboratory, and placed on a small narrow carriage, which is drawn out, rolling upon planks, till it comes near the back of the tube: here it is again suspended; and placed in the tube by a very curious contrivance.

The method of observing a celestial object by this telescope, is by the *front view*, as it is called by Dr. Herschell. The observer is placed in the seat C, suspended at the end of the tube, with his back towards the object he views: then looking down the tube, he sees the image of the object reflected from the mirror. There is no small speculum in this telescope; but the magnifiers are applied to the first focal image.

At

At the opening of the telescope near the place of the eye glass, which the observer looks through, at the focal image, is a speaking pipe, which runs down to the bottom of the tube, where it goes into a turning joint; and after several other inflections, it at length divides into two parts, one of which goes into the observatory D, and the other into the work-room E. By this speaking-pipe, the observer conveys his observations to the assistant in the observatory: and the workmen are also directed by it to perform the necessary motions.

In the observatory there is a valuable sidereal time-piece, made by Mr. Shelton. On one side of this is also a polar distance-piece, with a dial-plate of the same dimensions with the time-piece: this piece may be made to show the polar distance, zenith distance, declination or altitude, by setting it differently.

This noble instrument, with proper eye-glasses, magnifies above six thousand times; and is the largest instrument, and has the greatest magnifying power of any that has been made. By it Dr. Herschell has been able to observe the lightning in the atmosphere of the moon; and has discovered several celestial bodies, unknown to all preceding astronomers. Those who wish for a fuller account of this instrument, and the admirable machinery which supports it, viz. the stairs, ladders, and platform B, may see the particulars of every part in the Second Part of the Transactions of the Royal Society, for 1795, where, by means of eighteen plates, and sixty-three pages of letter-press, an ample detail is given of every circumstance relating to joiner's, carpenter's, and smith's work. The whole was finished on August the 28th, 1769, on which day the sixth satellite of Saturn was discovered.

S E C T. V.

THE IMPROVED SOLAR AND LUCERNAL MICROSCOPES.

The Improved Solar Microscope, which is constructed to view both transparent and opaque Objects.

FIG. 4, plate 8, is the solar microscope, mounted for exhibiting opaque objects. Fig. 5 is the single tooth and pinion microscope, used for showing transparent objects. The cylindrical tube Y being made to fit into the tube E F, fig. 4. Fig. 6 is the slider, which contains the magnifying lenses: it fits into a dovetail slit, at the upper end of the microscope, fig. 5. A D E F, fig. 4, is the body of the solar microscope, one part whereof A D is conical, the other part C E F is cylindrical. The cylindrical part receives the tube G of the opaque box H L, or the tube Y of the single microscope, fig. 5. At the large end A B, on the conical part, is fixed a lens, to receive the rays from the mirror O N, and to refract them towards the box H L. The mirror N O P is fixed to a moveable circular plate *a b c*; and may be moved into the most convenient position for reflecting the light, by means of the nuts Q and R. By the nut Q, it may be moved from east to west, and by the nut R, it may be raised or depressed to any angle. *d e* are two screws, to fasten the microscope to a window-shutter. In the box H L is a plane mirror M, for reflecting the light, that it receives from the large lens, to the object, and thereby illuminating it. S is a screw to adjust this mirror M to the proper distance and angle. V X are two tubes of brass, sliding one within the other, to carry the magnifying lenses:

To use the Solar Microscope, (Fig. 4.)

Make a round hole in the window-shutter, a little larger than the circular plate $a b c$: pass the mirror $O N P$ through this hole, and apply the square plate to the shutter, screwing it thereto by the two milled screws $d e$: the screws are to pass through the shutter into their nuts in the microscope, and thereby to hold the body of the microscope firm. Screw the conical tube $A B C D$ to the circular plate $a b c$; and then slide the tube G of the opaque box into the cylindrical plate $C D E F$, if opaque objects are to be viewed; but if transparent objects are to be viewed, place the tube Y (fig. 5) in the cylindrical tube (fig. 4). The room should be made as dark as possible, and no light suffered to enter, but what comes through the body of the microscope; for, on this circumstance, together with the brightness of the sun-shine, the distinctness of the object, in a great measure, depends. The mirror $N O P$ is to be then so adjusted by means of the two screws Q and R , as to reflect all the sun's light through the lens, at $A B$, in the body of the microscope; the mirror is to be so adjusted, till you have formed a clear round spot of light upon a screen, or sheet of white paper, placed at some distance.

In viewing an opaque object, place the object between the plates at H : open the door $i k$ of the opaque box, and adjust the small mirror M , till the object is strongly illuminated. If this cannot be effected by the screw S , you must move the two screws Q and R , in order to get light reflected strongly from the great mirror $N O P$, to the small one M . The object being strongly illuminated, shut the door $i k$, and a distinct image of the object will soon be obtained on the screen, by adjusting the tubes V and X . In the northern latitudes a clear large spot of light cannot always be obtained, nor when the sun is perpendicular to the front of the room; as the light then falls upon the back of the mirror.

And Note. As the sun is continually changing its place, it will be necessary to change the position of the mirror, by

kingdom, may be exhibited in great perfection, in all their native beauty; the lights and shades, the prominences and cavities, and all the varieties of different hues, tints, and colours, heightened by the reflection of the solar rays condensed upon them."

The improved lucernal Microscope.

The lucernal microscope is represented (fig. 1), where **A B C D E** is a large mahogany pyramidical box, which is the body of the microscope; and supported on the brass pillar **F G**, by the socket **H**, and the curved piece. **I K M N** are two tubes, one within the other: to the inner tube is fixed the vertical piece **M L**, which is a guide to the eye, to direct it to the axis of the instrument. This piece may be raised, or depressed, pulled further out, or pushed further in, to adjust it to the focus of the glasses, and make it coincide with the centre of the field of view. The tubes **M N** are fixed to the box in a dovetailed piece of brass. **O P** is a small tube which carries the magnifiers; **O** is one of the magnifiers screwed into the end of the tube. **Q R S T V X** is a long square bar, passing through the sockets **Y, Z**, and serves to hold the stage for the objects: this bar may be moved backward or forward, by means of the pinion *a*, and the handle *b c*, in order to adjust the stage to the focus. *d e* is a brass bar to support the curved piece **I K**, and keep the body of the microscope firm and steady. *f g r s* is the stage to hold the sliders with the opaque objects; it slides upon the bar **Q R S** by means of the socket *h i*, by which means it is brought nearer to, or removed farther from, the magnifying lens. The objects are placed in the front side of the stage, next the tube of the microscope, between four small brass plates (which cannot be shewn in the figure), the edges of two of the plates are seen at *k l*; the two upper plates are moveable, and are pressed together by a spiral spring, to confine the slider with the objects: these two plates may be

To examine transparent Objects.

For this purpose, the upper part of the opake stage *f g r s* (fig. 1) must be removed; and the stage for transparent objects (fig. 3) inserted in its place, with the end 9 and 10 next the lamp. Place the gray glass in its groove at the end A B (fig. 1), and the objects to be viewed in the slider-holder at the front of the stage; then transmit as strong a light as possible to the object, by raising or lowering the lamp; and the object will be beautifully depicted on the gray glass, being regulated to the focus of the magnifier by turning the pinion *a*.

If the gray glass be taken out of the microscope, the image of the object may be received on a paper screen.

There is no instrument invented by the art of man capable of affording so great entertainment as the microscope. By this instrument we can pry into the minutest recesses of nature; and discover truths which have lain hidden from the creation of the world. It is from the use of this instrument that we can confidently assert, that there is hardly any part of nature which is not the abode of animal existence. By it we discover thousands of living creatures in a single drop of water; and whole nations of animated beings in the bloom which surrounds a plum, a pear, and most other fruit; not to mention numberless insects in every part of nature, so small, that a thousand of them might be crushed by a single grain of sand.

From the discovery of these truths, the philosophic mind is led to make still further improvements: thus, in the formation of a mite, we can see the same proportion and symmetry in the structure of its limbs, as in those of any of the larger quadrupeds; as free a circulation of blood in the one as in the other; as many members adapted to different purposes; as numerous veins, muscles, arteries, sinews, &c. &c. with a proboscis to take in its food, like that of the elephant.

And

And if we descend to insects of a yet smaller size, we shall always find in each, all the properties and attributes essential to animal life: if we descend to those insects, whose bulk, compared to that of a mite, is as that of the mite to the elephant.

Others are of so small a size, as to be beyond the reach of any magnifying power; and whose whole existence is circumscribed within the space of a few hours, or a summer's day at the most. Myriads there are in the vegetable world, to whom the morning gives birth, and who expire with the setting sun, besides numberless others who have their habitations on the bodies of other animals, such as the ichneumon, whose peculiar habitation is on the body of the caterpillar.

In short, there is no part of nature which does not afford to the microscopical observer an abundant source of wonder and admiration, and display the attributes of that Supreme Being, whose wisdom is as conspicuous in the construction of the smallest atom, to be discovered by our assisted sight, as his power is evident in the formation of the planetary systems.

But there are some objects which have more immediately engaged the attention of the curious observer: such as, what is called the silver tree; and the crystallization of salts. The silver tree is formed by dissolving a little silver in a small quantity of aqua fortis; and then adding to it twice the quantity of common water. Drop a little of this upon a piece of plain glass, and put upon it a small piece of brass wire; and immediately the wire is put upon the mixture, trees will appear to grow, and display their branches, which will be extended, as far as the liquor extends on the glass; and has all the appearance of real trees, but of a silver cast. To observe the crystallization of salts, dissolve a little sal-ammoniac in common water, drop a little of this mixture on a piece of glass, as in the former case, and when viewing it through the microscope, hold a hot iron near it, to make it evaporate the quicker. As soon as the evaporation takes place, it will
rise

rise up in a round cylindrical form like the trunks of trees, and divide and subdivide, so as to represent branches and boughs; and has a truly wonderful and beautiful appearance. And every different kind of salt forms a different figure.

There are several other objects, which I must recommend to the attention of the young observer: as, 1. The *common Fly*, the vibration of whose wings is repeated several hundred times in a second of time. The great quantity of eyes with which this animal is favoured renders it worthy of notice, each having a distinct optic nerve. 2. The *Louse*. 3. *Mites* in cheese. 4. The *cuticular Pores* in the human skin, so close and numerous, that a single grain of sand will cover hundreds of them. 5. The construction of the *Scales* of *Fishes*. 6. The *Animalcules* in several sorts of infusions and liquids. 7. The construction of *common Feathers*. 8. *Hair*. 9. The *Sting* of a *Bee*, with the form of its barbs. 10. The *Configuration* of *Wood*. 11. The *common Mildew*, which displays a numerous group of vegetable substances. 12. *Small vegetable Seeds*. And several other articles.

I shall close this chapter with a few words concerning the method of making small spherule lenses for microscopes, as recommended by the late celebrated Mr. *George Adams*. And also the method of mixing the metals for the great speculum in the reflecting telescope.

To make the small spherule lenses, a piece of window-glass is to be cut into slips, about an eighth of an inch in breadth. Then, holding one of these slips of glass at each end in the flame of a lamp, as the glass begins to melt, it is to be drawn by each hand into a fine thread, and at length it will break. Then one end of this thread being held in the flame of the lamp, it will run up into a small globule, which is to be taken off; and is a small spherule lens. Several of these lenses are to be made, and examined; and those that are the best, are to be preserved for use. For some will always prove faulty, though every precaution be taken.

In

In this process, the lamp is to be supplied with spirits of wine instead of oil, and the flame is to be blown in an horizontal direction by a blow-pipe, or a pair of bellows for that purpose; and the glass held in the whitest part of the flame, lest the smoke sully the glass.

The metal for the speculum of the reflecting telescope is generally formed of copper and grain tin; and in the proportion of two pounds of Swedish copper, to fourteen ounces and an half of grain tin; and this mixture is to be melted twice over, before it be cast into the mould.

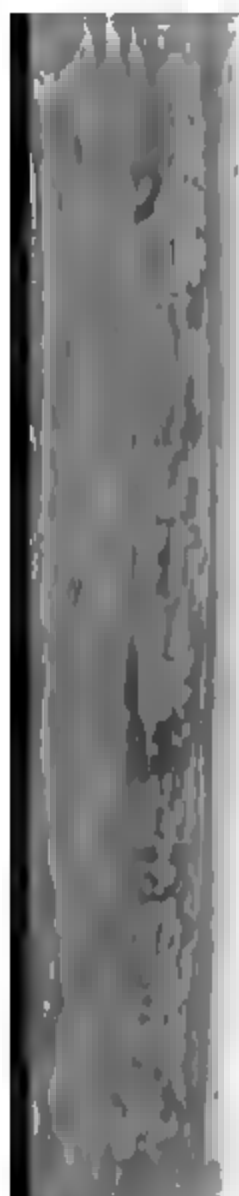
When the metal is cast into a concave mirror, it is ground upon, what is called, the rough grinder, or even a common grind-stone, of the same radius as the concavity of the metal, to take off the rough face. Then it is ground on a brass convex grinder, to give a true spherical figure; and lastly, upon a convex bed of hones, which is to perfect that figure, and give the metal a fine smooth face. Then the concave face of the metal is to be polished by a convex tool, covered with pitch. And, lastly, it is to be brought into the parabolic form by a merely mechanical method of grinding it on the polisher in a different direction.

But the best metal for speculums is that proposed by the Rev. Mr. *Edwards*, and which was proved by Dr. *Maskeelyne* to excel, in brightness and distinctness of the image, every other metal then known. It consists of thirty-two ounces of copper, with fifteen or sixteen ounces of grain tin (according to the purity of the copper), with one ounce of brass, one ounce of silver, and one ounce of arsenic. I once was present at the casting of a speculum of this metal; and, when finished, it reflected more light than any speculum I have ever seen.

END OF THE FIRST VOLUME.

S. GOSNELL, Printer,
Little Queen Street, Holborn.





THE NEW YORK PUBLIC LIBRARY
REFERENCE DEPARTMENT

**This book is under no circumstances to be
taken from the Building**

[illegible]

JAN 27 1944

**E NEW YORK PUBLIC LIBRARY
REFERENCE DEPARTMENT**

**book is under no circumstances to be
taken from the Building**

[illegible]



